Educational program in Data Analytics

MDA102 – Mathematics II

✍ 13th Set of Problems – MDA102

✎ material of 13TH WEEK

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Exercises

Problem 1. (**5 points**) Determine the convolution $f_1 * f_2$ of the functions

$$f_1(x) = \begin{cases} 1-x\,, & x \in [-2,1]\,, \\ 0\,, & \text{otherwise}\,, \end{cases} \qquad f_2(x) = \begin{cases} 1\,, & x \in [0,1]\,, \\ 0\,, & \text{otherwise}\,. \end{cases}$$

Hints: See Problems 7.B.1-7.B.3 in the newer version of the BG book.

Problem 2. (**5 points**) Verify each step in the computation of the Fourier transform $\mathcal{F}(f)$ of the function $f(t) = \mathrm{e}^{-at^2}$, with $t \in \mathbb{R}$, where $a > 0$ is a fixed parameter, presented in Problem 7.B.7 of the BG book. Next, use the `desolve` command in SageMath to solve the differential equation

$$\frac{dy}{d\omega} = -\frac{\omega}{2a} y\,,$$

appearing within the given solution, where $y(\omega) = \mathcal{F}(f)(\omega)$. Moreover, combine the `assume` function with the `integral` command in Sage to verify that

$$\int_{-\infty}^{\infty} \mathrm{e}^{-at^2}\, dt = \sqrt{\frac{\pi}{a}}\,.$$

Please present the Sage solutions in a PDF file exported via CoCalc.

Hints: For the Fourier transform see Section 7.2.5 and Problems 7.B.5-7.B.8 in the newer version of the BG book. In the same version, see also Problem 6.B.64 for an implementation of the `dsolve` command.