Educational program in Data Analytics

MDA103 – Script Languages

✍ 8th Set of Problems – MDA103

✎ material of 8TH WEEK

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This set of exercises is based on Sections 3.3-3.5 in the file " Chapter 3 (Linear Algebra).ipynb" shared via CoCalc.

Exercises

Problem 1. (**10 points**) Given the matrix

$$A = \begin{pmatrix} 4 & 1 & 2 \\ 0 & 3 & -1 \\ 0 & 0 & 2 \end{pmatrix}$$

use Sage to answer the following tasks:

1. Show that A is *not* an orthogonal matrix.
   Hint: Show for example that $AA^T \neq E$ where E is the identity 3×3 matrix. To introduce the identity 3×3 matrix one can use appropriately the built-in function `identity_matrix()`. As an alternative to check the orthogonality of A, one may use the function `is.unitary()` that we met for example in Chapter 2 of the BG book, see for example Problem 2.D.12 on page 139 of the BG book.

2. Find the characteristic polynomial of A and present its factorization.
   Hint: Combine the `characteristic_polynomial()` function with the `factor()` function.

3. Find the eigenvalues of A and their algebraic multiplicity.
   Hint: Use the `eigenvalues()` function.

4. Find eigenvectors corresponding to the eigenvalues of A.
   Hint: Use the `eigenvectors_right()` function.

5. Provide an explanation of what the following Sage cell is designed to accomplish and describe the goal of each line in the program:

```
B = matrix([[5, 4], [2, 1]])
eigenvectors_B = B.eigenvectors_right()
dim_B = B.nrows()
independent_vectors_B = sum([len(eig[1]) for eig in eigenvectors_B])
is_diagonalizable_B = independent_vectors_B == dim_B
print("Is B diagonalizable? ", is_diagonalizable_B)
```

   Next apply a similar method for the given matrix A and explain the mathematical meaning of Sage's output.

6. To illustrate your answer for the mathematical meaning of *diagonalization*, use the function **eigenmatrix_right**() in order to determine the invertible matrix P and the diagonal matrix D such that $P^{-1}AP = D$.