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Osvald Vašíček and Karel Musil

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Manager prof. Ing. Antonín Slaný, CSc., Lipová 41a, 602 00 Brno, e-mail: slany@econ.muni.cz, tel.: +420 549491111

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BEHAVIOR OF THE CZECH ECONOMY: NEW OPEN ECONOMY MACROECONOMICS DSGE MODEL

Abstract:

The paper introduces a New Keynesian Dynamic Stochastic General Equilibrium (NK DSGE) model. This model is derived from the New Open Economy Macroeconomics (NOEM). It is strictly based on microeconomic foundations and consists of sectors with representative agents. They are representative households and firms, a central monetary authority and a foreign economy. The foreign sector is exogenous in the model.

The representative household maximizes its utility function with respect to its budget constraints. Consumption contains a habit formation factor and is divided between domestic and imported goods. The behavior is influenced by terms of trade, uncovered interest parity and purchasing power parity that includes some rigidities.

The representative firm optimizes its behavior by setting prices of the production in a Calvo style. The result is relation called the New Keynesian Phillips Curve (NKPC).

Monetary policy of the central bank is represented by a generalized Taylor rule. It changes the interest rate with respect to the output gap and a difference between inflation and the inflation target.

The economic model is log linearized and transformed to a rational expectations model, which is solved. Parameters of the solved model are estimated by Bayesian method with Monte–Carlo simulation technique, which is using a priory set information.

The estimated model together with the impulse responses gives a suitable approximation of behavior of the Czech economy. The model is also used for a forecast.

Abstrakt:

Studie představuje novokeynesiánský dynamický stochastický model všeobecné rovnováhy (DSGE). Model vychází z konceptu nové otevřené makroekonomie (NOEM). Je důsledně odvozen na teoretických mikroekonomických základech a skládá se ze sektorů představující reprezentativní agenty. Jsou to sektory representativní domácnosti a firmy, sektory centrální banka a zahraniční ekonomika. Zahraniční sektor je exogenní.

Reprezentativní domácnost maximalizuje užitkovou funkci s ohledem na rozpočtové omezení. Spotřeba obsahuje prvek setrvačnosti a je rozdělena mezi domácí a importované zboží. Chování je ovlivňováno směnnými relacemi, nekrytou úrokovou paritou a paritou kupní síly, která obsahuje rigidity.

Representativní firma optimalizuje své chování stanovením ceny produkce Calvova typu. Výsledkem je novokeynesiánská Phillipsova křivka (NKPC).

Monetární politika centrální banky je představována zobecněným Taylorovým pravidlem. Vývoj úrokové sazby se mění v závislosti na vývoji mezery výstupu a na vývoji mezery inflace od stanoveného vývoje inflačního cíle.

Ekonomický model je log linearizován a restrukturován na soustavu rovnic lineárního modelu racionálních očekávání, který je řešen. K odhadu parametrů řešeného modelu je užita Bayesovská metoda s Monte–Carlo simulací, která vyžaduje zadanou apriorní informaci o parametrech.

Odhadnutý model a simulované impulzní odezvy představují přijatelnou aproximaci chování české ekonomiky. Model je použit také pro předpověď'.

Recenzoval:
prof. RNDr. Ing. Jan Kodera, CSc.

CONTENTS

1 INTRODUCTION

To describe all significant relationship in a economy it is not possible to concentrate only on the behavior within the economy. A connection with the foreign countries could be in some respects important. The smaller is the economy, the more open it is. If the economy is small, it tries to join the world business. The participation gives some advantages. According to Bhagwati, Panagariya and Srinivasan (1998), there can be higher welfare especially due to a possibility of international trade in open economies.[1]

The previous theory can be supported with the situation of the Czech economy as a small open economy. The import or export ratio to GDP is relatively high[2] (but comparable with similar countries). If we want to analyze behavior of the Czech economy it is necessary and inevitable to take the foreign sector into account.

The first step to make a model is usually to make a closed economy model. These models use a condition of a closed economy with no connection to the rest of the world. They are able to describe some basic characteristic of the economy (see for example the model of the Czech economy in Vašíček and Musil (2005) where is more detailed analysis of the behavior and the model indicates a suitable approximation). Then we can adapt the model to the conditions of a small open economy.[3] The easiest way how to introduce the foreign sector is to take it as exogenous. It is simplifying but it allows to cover a connection to outside of the economy. In this paper we use this approach.[4]

The basic goal of this paper is to analyze and describe the behavior of the small open Czech economy with employing a suitable New Key-

[1] We use this approach to introduce a small open economy that is connected to the world economy through the international trade of consumption goods. It means that a positive effect of the openness influences mainly (but not only) consumers in the end.

[2] The average ratio of import and export to GDP is for the Czech economy about 0.68 and 0.65 respectively. The calculation is based on year–to–year data from 1995 to 2005 from Czech Statistical Office (a).

[3] This procedure can be shown e.g. on the closed economy model of Ireland (2005) and its extension to the small open economy model of Dib (2003). Some theoretical notes to this topic are included in Musil (2006).

[4] The model structure has some parts identical to a closed economy model. For a comparison see e.g. Clarida, Galí and Gentler (1999) or Woodford (2000).

nesian Dynamic Stochastic General Equilibrium (NK DSGE) model which is based on the knowledge of New Open Economy Macroeconomics (NOEM). We respect especially the conditions connected to the inflation targeting monetary policy and a tight linkage of the Czech economy to the German economy. The model is used for a prediction of the behavior of the Czech economy.

The estimated model and impulse response functions can be compared with similar model. The values of parameters or an analysis of behavior can indicate some specific characteristics of the Czech economy or give some advices to the policymakers.

In this respect interesting can be a comparison with e.g. a DSGE model of T. Monacelli (2002) aimed to the optimal monetary policy design, a little bit extended small open model with a Taylor rule of Amber, Dib and Rebei (2004), a model of Adolfson, Laséen, Lindé and Villani (2005) created for behavior analysis of the Swedish monetary policy based on inflation targeting, similarly Ghironi (2000a) does for the Canadian economy. These models reflect basic characteristics of the presented approach to the small open economy models and all of them put stress on the monetary policy role in the economy. In this respect, their structure and results are comparable with our model.

For our purposes, we go out from a model of P. Liu (2005) and (2006) and the next part draws from his paper.[5]

The paper is divided in separate sections. The first section introduces the model and describes important features connected to the behavior of a representative household, firm, central bank and a foreign sector.

Next section is devoted to the model equilibrium and the linearized model. Then there is a part aimed to solving and estimation of the linearized model.

In the following section, we assess and analyze the estimated result and the behavior of the model. We try to interpret some shocks to the economy and a prediction ability of the model. The last part concludes the working paper.

Detailed calculations of some equations and relationships together with the original data are attached as supplements.

[5] Authors thank to Phillip Liu for his permission to use his model for our purpose and applications on the condition of the Czech economy.

2 NEW OPEN ECONOMY MACROECONOMICS

New Open Economy Macroeconomics (NOEM) is relatively new approach to the economic modeling. Our presented model can be classified as a NOEM model, so we try to outline some basic properties of these type of models.[6]

At the beginning of the 1990s, P. Krugman stressed 3 problems open-economy macroeconomics faced. He pointed out a necessity to use:

- open economy macroeconomic model with nominal rigidities that makes possible the integration with modern economics (especially sticky–price open economy macroeconomic model),
- expectations that *make sense* and are incorporated into macroeconomic models (especially to help explain the behavior of asset markets),
- better understanding of the microeconomic foundations of an open economy macroeconomic model.

As a reaction Obstfeld and Rogoff (1995) wrote the publication *Exchange Rate Dynamics Redux*, which was marked as a beginning of a new class of open–economy macroeconomic models. Later the NOEM approach started to be interested in the following key features:

- optimization-based dynamic general-equilibrium modeling,
- sticky prices and /or wages in at least some sectors of the economy,
- incorporation of stochastic shocks,
- evaluation of monetary policies based on household welfare.

The main objective of the New Open Economy Macroeconomics is a development of a new fundamental model for open–economy macroeconomic analysis.

In the last decade not only the NOEM has introduced a number of important developments with substantial contributions to the theory but as well as to the empirics. New Open Economy Macroeconomics has made some progress in each of the quoted areas but number of challenges and open questions remain unsolved and continue to

[6] Surveys about this topic can be found in Bowman and Doyle (2003), and Sarno (2000).

stimulate economists (e.g. how monetary policy should operate in an open economy[7]).

NOEM models offer some advantages, such as a higher standard of a behavior analysis coming from fully specified microeconomic foundations (NOEM models are created in the context of dynamic general equilibrium models), a welfare analysis based on households' behavior, some policy evaluations stemming from the microeconomic behavior of representative agents together with an influence of foreign economy (especially foreign shocks), relatively sophisticated and realistic representation of real world markets (in particular financial markets and markets with imperfections and nominal rigidities), and so on.

[7] Because the NOEM incorporates sticky prices or wages into optimization–based general equilibrium models (NK DSGE models), it combines some components of the long run properties of international real business cycle models with short-run Keynesian features that cause a discussion about monetary policy and its effects on the economy.

3 THE MODEL

In this section, we introduce a small open economy model. The model is a New Keynesian Dynamic Stochastic General Equilibrium (NK DSGE) model strictly based on microeconomic foundations. First we describe behavior of representative agents. Then we aggregate this behavior to outline basic relations that create the model and explain behavior of the whole economy. We suppose all representative agents optimize their behavior – a household maximizes its utility function subject to its budget constraint, a firm maximizes its profit subject to its constraint represented by a demand restrictions. The same behavior is valid not only for domestic agents but for the foreign agents as well.

The model is a small open economy model of the Czech economy and its structure is closely related to the New Open Economy Macroeconomic (NOEM) approach.

The model consists of representative agents. They are households, firms, a central monetary authority, and a foreign sector. Representative households and firms optimize its behavior from present to future (for $t = 0, 1, 2, ..., \infty$). The optimizing process of some firms is also influenced by the last period development – especially in case of setting new prices by indexation to the last period inflation. Generally, firms can be divided into two groups: representative firms producing some goods in the domestic economy for either domestic consumption or export and representative firms importing goods from abroad for the domestic consumption.

A conduction of monetary policy by a central monetary authority is subordinate to an inflation targeting regime. The central bank influences short run interest rate as a reaction to a development of economic variables (especially to the inflation and output of the economy). This behavior is described by a suitable monetary rule. In this context the monetary policy is endogenous in the model.

The foreign sector is assumed as exogenous. It influences behavior of the domestic agents. The home economy is small and open when compared to the foreign economy (rest of the world a big economy). For this reason we assume that the home economy is much smaller than the foreign one. The small open economy assumption implies that foreign variables and world aggregates are given from the perspective of the domestic economy. The exogenity of foreign variables provides a set of restrictions for the domestic economy which we use in our empirical analysis. On the other hand the big economy is not influenced by the small one. Imports of foreign goods have a negligible influence on the foreign economy. More detailed description of these conditions and their consequences is presented in Ghironi (2000b).

The model contains some rigidities to reflect the economic reality.[8] Firms set their prices in a staggered Calvo style. Another rigidity is connected to the law of one price – this law does not hold exactly and there is a law of one price gap. It influences behavior of households.

We proceed from a model of P. Liu (2005) and articles of Liu (2006), Galí and Monacelli (2002) and (2005), Lubik and Schorfheide (2005a), (2005b) and (2006).

If there are variables marked by small letters, they are logarithms of the original variables originally marked by capital letters. Variables with a superscript * hold for the foreign economy.

The model is in a gap form.

[8] A habit formation, which is in our model is sometimes included into the rigidities too. For detailed analysis see e.g. Smets and Wouters (2003).

3.1 THE REPRESENTATIVE HOUSEHOLDS

In this part, we introduce basic principles connected with the behavior of a representative household and its connection to a foreign economy. There is an optimizing problem of the representative household presented in subsection 3.1.1, an introduction of basic relationships among inflation, real exchange rate and terms of trade presented in subsection 3.1.2 and finally a theory connected to the complete international financial markets in subsection 3.1.3.

3.1.1 OPTIMIZATION

Every representative household tries to maximize its constant relative risk aversion (CRRA) utility function expressed as:

$$E_t \sum_{t=0}^{\infty} \beta^t \left(\frac{(C_t - hC_{t-1})^{1-\sigma}}{1-\sigma} - \frac{N^{1+\varphi}}{1+\varphi} \right), \tag{1}$$

for $t = 0, 1, 2, \ldots, \infty$, where E_t expresses an expected value of the utility function, β $(0 < \beta < 1)$ is a discount factor, C_t is the consumption, h $(0 < h < 1)$ is a habit formation factor, σ $(\sigma > 0)$ is the inverse elasticity of intertemporal substitution[9], N_t denotes hours of labor and φ $(\varphi > 0)$ is the inverse elasticity of labor supply[10].

The household maximizes the discounted value of expected utility expressed as a utility from consumption reduced by a disutility from amount of hours of labor. The representative household is limited by a budget constraint. An income from nominal wages and pay–off from a portfolio are used for paying for a consumption and new portfolio:

$$P_t C_t + E_t \left(\frac{D_{t+1}}{1 + r_t} \right) \leq D_t + W_t N_t, \tag{2}$$

for $t = 0, 1, 2, \ldots, \infty$, where P_t is the overall Consumer Price Index, D_t nominal pay–off on a portfolio held at $t - 1$, r_t nominal interest rate, and W_t nominal wage.

The first order conditions (FOCs) of the household's optimization

[9] The elasticity of intertemporal substitution can be expressed as $\frac{\partial U / \partial C_t}{\partial U / \partial C_{t+1}}$. For more details see Musil (2005).

[10] The elasticity of labor supply expresses the reaction of hours of labor to a change of a real wage.

problem are following:

$$(C_t - hC_{t-1})^{-\sigma} \frac{W_t}{P_t} = N_t^{\varphi} \tag{3}$$

$$\beta R_t E_t \left\{ \frac{P_t}{P_{t+1}} \left(\frac{C_{t+1} - hC_t}{C_t - hC_{t-1}} \right)^{-\sigma} \right\} = 1, \tag{4}$$

for $t = 0, 1, 2, \ldots$, where $R_t = 1 + r_t$. Equation (3) is intra–temporal consumption which balances marginal utility of consumption to marginal value of labor and equation (4) expresses inter–temporal Euler equation. Log–linearizing these two equations yields for all t:

$$w_t - p_t = \varphi n_t + \frac{\sigma}{1-h} (c_t - hc_{t-1}) \tag{5}$$

$$c_t - hc_{t-1} = E_t (c_{t+1} - hc_t) - \frac{1-h}{\sigma} (r_t - E_t \pi_{t+1}), \tag{6}$$

where $\pi_t = p_t - p_{t-1}$.

Behavior of a foreign household is similar to the behavior of the domestic household. We get the same optimality conditions. Variables with a superscript * are valid for the foreign economy.[11]

Because the economy is open, the household can consume not only domestic produced consumption goods ($C_{H,t}$) but the imported foreign consumption goods ($C_{F,t}$) as well. The total consumption every period can be expressed in the following way:

$$C_t \equiv \left((1-\alpha)^{\frac{1}{\eta}} C_{H,t}^{\frac{\eta-1}{\eta}} + \alpha^{\frac{1}{\eta}} C_{F,t}^{\frac{\eta-1}{\eta}} \right)^{\frac{\eta}{\eta-1}}, \tag{7}$$

where η ($\eta > 0$) is the elasticity of substitution between home and foreign goods and α ($0 \leq \alpha \leq 1$) is the import ratio[12]. Moreover we suppose a continuum of domestic and foreign products and aggregate domestic and foreign consumption are given by:[13]

$$C_{H,t} \equiv \left(\int_0^1 C_{H,t}(i)^{\frac{\epsilon-1}{\epsilon}} di \right)^{\frac{\epsilon}{\epsilon-1}} \quad C_{F,t} \equiv \left(\int_0^1 C_{F,t}(i)^{\frac{\epsilon-1}{\epsilon}} di \right)^{\frac{\epsilon}{\epsilon-1}} \tag{8}$$

[11] In this context we use $C^* = C^*_{F,t}, P^*_t = P^*_{F,t}$.
[12] It measures imported consumption to total households' consumption ratio. It can be understood in this case as a degree of openness.
[13] We use CES functions in this case to make an aggregate index but in some other cases as well.

for $t = 0, 1, 2, \ldots$, where ϵ ($\epsilon > 0$) is the elasticity between different types of goods for consumption in domestic economy. Without loss of generality (as it is shown in Galí and Monacelli (2005)), we can suppose the same value of ϵ for the imported goods.[14]

The representative household aims to allocate optimally its expenditure for the total consumption between domestic produced and imported consumption goods. The result of this optimizing behavior are following optimal allocation functions for all t:

$$C_{H,t} = (1-\alpha)\left(\frac{P_{H,t}}{P_t}\right)^{-\eta} C_t \qquad C_{F,t} = \alpha\left(\frac{P_{F,t}}{P_t}\right)^{-\eta} C_t, \quad (9)$$

where the price index of home produced goods and the import price index are

$$P_{H,t} \equiv \left(\int_0^1 P_{H,t}(i)^{1-\epsilon} di\right)^{\frac{1}{1-\epsilon}} \quad P_{F,t} \equiv \left(\int_0^1 P_{F,t}(i)^{1-\epsilon} di\right)^{\frac{1}{1-\epsilon}} \quad (10)$$

Then it is possible to derive the overall Consumer Price Index (CPI)

$$P_t \equiv \{(1-\alpha)P_{H,t}^{1-\eta} + \alpha P_{F,t}^{1-\eta}\}^{\frac{1}{1-\eta}} \quad (11)$$

[14] In this situation we should reformulate the budget constraint (2) with respect to the possibility of purchasing the domestic produced consumption $C_{H,t}$ and imported goods $C_{F,t}$ at time t to this form:

$$\int_0^1 \{P_{H,t}(i)C_{H,t}(i) + P_{F,t}(i)C_{F,t}(i)\}di + E_t\left(\frac{D_{t+1}}{1+r_t}\right) \leq D_t + W_t N_t.$$

The total consumption expenditure is given by $P_{H,t}C_{H,t} + P_{F,t}C_{F,t} = P_t C_t$ for $t = 0, 1, 2, \ldots$. Using this we get back the original budget constraint (2).

Moreover, if we take the foreign economy as a group of foreign economies, it is possible to write more general budget constraint in this form:

$$\int_0^1 P_{H,t}(i)C_{H,t}(i)\, di + \int_0^1 \int_0^1 P_{Fj,t}(i)C_{Fj,t}(i)\, di\, dj + E_t\left(\frac{D_{t+1}}{1+r_t}\right) \leq D_t + W_t N_t,$$

where $P_{Fj,t}(i)$ is the price of the i-th imported good from the j-th foreign country (expressed in the domestic currency) and $C_{Fj,t}(i)$ is the i-th good produced abroad in the j-th foreign economy. To simplify the equation we assume:

$$C_{F,t}(i) \equiv \left(\int_0^1 C_{Fj,t}(i)^{\frac{\epsilon-1}{\epsilon}} dj\right)^{\frac{\epsilon}{\epsilon-1}} \qquad P_{F,t}(i) \equiv \left(\int_0^1 P_{Fj,t}(i)^{1-\epsilon} dj\right)^{\frac{1}{\epsilon-1}}$$

for $t = 0, 1, 2, \ldots$ and all i. After this amendment, we get back the original version of the household's budget constraint.

The household's decision of dividing the expenditure between domestic and imported consumption is influenced by an extent of the total consumption C_t, a possibility to consume imported goods (expressed by the degree of openness of the economy α) and relative price of domestic produced goods $\frac{P_{H,t}}{P_t}$ and imported goods $\frac{P_{F,t}}{P_t}$ with the influence of a possibility to substitute (parameter η) between these two groups of goods.

If the household decide how much to spend on domestic and imported consumption, the last decision connected with the optimal allocation is to choose the i-th good within this two groups of production. The result of this optimizing behavior is the following demand function:

$$C_{H,t}(i) = \left(\frac{P_{H,t}(i)}{P_{H,t}}\right)^{-\epsilon} C_{H,t} \qquad C_{F,t}(i) = \left(\frac{P_{F,t}(i)}{P_{F,t}}\right)^{-\epsilon} C_{F,t} \quad (12)$$

for all $t = 0, 1, 2, \ldots$, where $P_{H,t}(i)$ and $P_{F,t}(i)$ are prices of the i-th domestic produced or imported goods. The explanation of these two demand functions is similar to the reasoning introduced previously.

For more detailed derivation of the FOCs (5) and (6), the optimal allocation of expenditure (9) and the demand functions (12) see Supplement 1.

3.1.2 TERMS OF TRADE

The terms of trade (international competitive price index of domestic producers) express a relationship between the aggregate price of exports and imports, or more exactly, the price of foreign goods per unit price of domestic good $S_t = \frac{P_{F,t}}{P_{H,t}}$, or in logs for all t:

$$s_t = p_{F,t} - p_{H,t}. \quad (13)$$

Then we use (13) together with $p_t \equiv (1-\alpha)p_{H,t} + \alpha p_{F,t}$[15] to obtain:

$$\begin{aligned} p_t &\equiv (1-\alpha)p_{H,t} + \alpha p_{F,t} && (14)\\ &= p_{H,t} - \alpha p_{H,t} + \alpha p_{F,t} = p_{H,t} + \alpha(p_{F,t} - p_{H,t}) \\ p_t &= p_{H,t} + \alpha s_t. && (15) \end{aligned}$$

[15] It is a linearization around the steady state of the overall CPI formula used for the optimal allocation functions: $P_t \equiv \{(1-\alpha)P_{H,t}^{1-\eta} + \alpha P_{F,t}^{1-\eta}\}^{\frac{1}{1-\eta}}$.

The first difference of the previous equation yields a connection of overall CPI inflation to the domestic inflation and the terms of trade:

$$\pi_t = \pi_{H,t} + \alpha \Delta s_t. \tag{16}$$

The equation (16) and the first difference of (14) give together:

$$\pi_t = \pi_{H,t} + \alpha \Delta s_t \qquad \pi_t \equiv (1-\alpha)\pi_{H,t} + \alpha \pi_{F,t}$$

$$\begin{aligned} \pi_{H,t} + \alpha \Delta s_t &= (1-\alpha)\pi_{H,t} + \alpha \pi_{F,t} \\ \pi_{H,t} + \alpha \Delta s_t &= \pi_{H,t} - \alpha \pi_{H,t} + \alpha \pi_{F,t} \\ \Delta s_t &= \pi_{F,t} - \pi_{H,t} \end{aligned} \tag{17}$$

The equation (17) shows that the difference between foreign and domestic inflation is proportional to the change in terms of trade, or according to (16), the difference between overall and domestic inflation is proportional to the change in terms of trade – the higher the degree of openness, the smaller the change in the terms of trade.

A nominal exchange rate is Z_t.[16] We suppose that law of one price does not hold strictly, there is incomplete pass-through especially for imports[17]. Law of one price can be expressed in the following form:

$$\Psi_t = \frac{P_t^*}{Z_t P_{F,t}} \tag{18}$$

for $t = 0, 1, 2, \ldots$

If the law of one price holds exactly ($\Psi_t = 1$), the foreign price index equals the import price index expressed in foreign currency ($Z_t P_{F,t} = P_t^*$). A law of one price gap is a difference between the foreign world price and the domestic price of imports.

Substituting the law of one price (18) in logs into the equation for terms of trade (13) yields:

$$s_t = p_{F,t} - p_{H,t} \qquad \psi_t = p_t^* - z_t - p_{F,t}$$

$$s_t = p_t^* - z_t - p_{H,t} - \psi_t \tag{19}$$

[16] The nominal exchange rate is expressed in terms of foreign currency per a domestic currency: an increase of Z_t is an appreciation of domestic currency.
[17] There are some restrictions for the law of one price to hold exactly. For more details see e.g. Mach (1998).

for $t = 0, 1, 2, \ldots$ The law of one price gap can be understood as a difference between the world price p_t^* and the domestic price expressed in foreign currency $z_t + p_{H,t}$.

The relationship between terms of trade and real exchange rate is important to formulate the competitive price index in terms of exchange rate in real conditions. The real exchange rate $Q_t \equiv \frac{Z_t P_t}{P_t^*}$ for $t = 0, 1, 2, \ldots$ in log in:

$$q_t = z_t + p_t - p_t^*. \tag{20}$$

This is combined together with equation (19):

$$q_t = z_t + p_t - p_t^* \qquad\qquad s_t = p_t^* - z_t - p_{H,t} - \psi_t$$

$$\begin{aligned} q_t &= z_t + p_t - (s_t + z_t + p_{H,t} + \psi_t) \\ &= z_t + p_t - s_t - z_t - p_{H,t} - \psi_t \\ &= p_t - p_{H,t} - s_t - \psi_t, \end{aligned}$$

now we use equation (15) together with the previous equation:

$$q_t = p_t - p_{H,t} - s_t - \psi_t \qquad\qquad p_t = p_{H,t} + \alpha s_t$$

$$\begin{aligned} q_t &= p_{H,t} + \alpha s_t - p_{H,t} - s_t - \psi_t \\ &= \alpha s_t - s_t - \psi_t \\ q_t &= -(1-\alpha)s_t - \psi_t \end{aligned}$$

and reformulate it into the following form:

$$\psi_t = -[q_t + (1-\alpha)s_t]. \tag{21}$$

The law of one price gap is inversely proportionate to the real exchange rate q_t and the degree of competitiveness for the domestic economy s_t (terms of trade).

3.1.3 INTERNATIONAL FINANCIAL MARKET

We use an assumption of complete international financial markets together with perfect capital mobility. There are two consequences – the international risk sharing and the uncovered interest parity.

Under international risk sharing, a price of similar bonds[18] must be same in the domestic and foreign economy – expressed as a rate of return in terms of nominal interest rate:

$$1 + r_t = (1 + r_t^*) E_t \left(\frac{Z_t}{Z_{t+1}} \right)$$

$$R_t = R_t^* E_t \left(\frac{Z_t}{Z_{t+1}} \right) \tag{22}$$

for all $t = 0, 1, 2, ...$, where $R_t = 1 + r_t$ and $R_t^* = 1 + r_t^*$ are gross nominal domestic and foreign interest rates respectively. Using (22), it is possible to use the intertemporal optimality condition (4) for domestic and foreign households' optimization problem (Euler equations):

$$\beta E_t \left\{ \frac{P_t}{P_{t+1}} \left(\frac{C_{t+1} - hC_t}{C_t - hC_{t-1}} \right)^{-\sigma} \right\} =$$

$$= E_t \left(\frac{Z_{t+1}}{Z_t} \right) \beta E_t \left\{ \frac{P_t^*}{P_{t+1}^*} \left(\frac{C_{t+1}^* - hC_t^*}{C_t^* - hC_{t-1}^*} \right)^{-\sigma} \right\} \tag{23}$$

for $t = 0, 1, 2, \ldots$, assuming the same habit formation parameter and discount factor for a representative domestic and foreign household.

After an amendment of the previous equation to the conditions of equilibrium it yields for all t:[19]

$$C_t - hC_{t-1} = \vartheta (C_t^* - hC_{t-1}^*) Q_t^{-\frac{1}{\sigma}},$$

where ϑ is a constant depending on initial assets positions. Log–linearizing of the equation around steady–state gives:

$$c_t - hc_{t-1} = (c_t^* - hc_{t-1}^*) - \frac{1-h}{\sigma} q_t \tag{24}$$

$$c_t - hc_{t-1} = (y_t^* - hy_{t-1}^*) - \frac{1-h}{\sigma} q_t \tag{25}$$

for $t = 0, 1, 2, \ldots$.

The condition for parallel optimizing of domestic and foreign households (25) supposes relationship $y_t^* = c_t^*$ for all t.[20] More detailed derivation of the previous equations is included in Supplement 1.

[18] We use the term of similar bonds for similarly liqiud and risky bonds.

[19] Supplement 1 describes a more detailed derivation of this equation from (22). The presented condition is labeled (75) in the Supplement.

[20] More precisely, the relationship for the foreign economy takes the form $y_t^* = c_t^* + c_{F,t}$, but the influence of domestic economy is negligible and it is possible to simplify the equation to $y_t^* = c_t^*$.

The condition (22) is connected with another important relationship – uncovered interest parity:

$$R_t = R_t^* E_t \left(\frac{Z_t}{Z_{t+1}} \right)$$

Log–linearizing around steady state yields:

$$\begin{aligned} logR_t &= logR_t^* + E_t(logZ_t - logZ_{t+1}) \\ r_t &= r_t^* + E_t(z_t - z_{t+1}) \\ r_t &= r_t^* - E_t(z_{t+1} - z_t) \\ r_t^* - r_t &= E_t \Delta z_{t+1} \end{aligned}$$ [21]

[21] Together with the modified equation (19) we get:

$$\begin{aligned} s_t &= p_t^* - p_{H,t} - z_t - \psi_t \\ z_t &= p_t^* - p_{H,t} - s_t - \psi_t \\ E_t z_{t+1} &= E_t(p_{t+1}^* - p_{H,t+1} - s_{t+1} - \psi_{t+1}) \\ E_t \Delta z_{t+1} &= E_t(\Delta p_{t+1}^* - \Delta p_{H,t+1} - \Delta s_{t+1} - \Delta \psi_{t+1}) \\ E_t \Delta z_{t+1} &= E_t(\pi_{t+1}^* - \pi_{H,t+1} - \Delta \psi_{t+1} - (s_{t+1} - s_t)) \end{aligned}$$

and combine it with $E_t \Delta z_{t+1} = r_t^* - r_t$:

$$\begin{aligned} r_t^* - r_t &= E_t(\pi_{t+1}^* - \pi_{H,t+1} - \Delta \psi_{t+1} - (s_{t+1} - s_t) \\ s_t &= E_t s_{t+1} + r_t^* - r_t - E_t(\pi_{t+1}^* - \pi_{H,t+1} - \Delta \psi_{t+1}) \\ s_t &= E_t s_{t+1} + r_t^* - E_t \pi_{t+1}^* - r_t + E_t \pi_{H,t+1} - E_t \Delta \psi_{t+1} \\ s_t &= E_t s_{t+1} + (r_t^* - E_t \pi_{t+1}^*) - (r_t - E_t \pi_{H,t+1}) - E_t \Delta \psi_{t+1} \end{aligned}$$

Then we try to rule out the term s_{t+1} by using the equation for $t+1$ and s_{t+2} ... The solution by forward iterations gives:

$$s_t = E_t \sum_{k=0}^{\infty} \left\{ (r_{t+k}^* - E_t \pi_{t+k+1}^*) - (r_{t+k} - E_t \pi_{H,t+k+1}) - \Delta \psi_{t+k+1} \right\}.$$

The result is intuitive – the terms of trade (in logs) are a function of the current and expected real interest rate differential and $lim_{t \to \infty} s_t = 0$. A possible deviation is ensured by the development of the law of one price gap. However, it is not additional equilibrium condition because it can be derived from the Euler equations (6) for the domestic and foreign economies together with risk sharing condition (24) and equation (16). The importance of this relationship is different. It shows that the structure of the model is consistent with the economic theory. It proofs that the relationship between the domestic and foreign economy measured by the current value of

Then we calculate the first difference of equation (20) for $t+1$:

$$\Delta q_t = \Delta z_t + \Delta p_t - \Delta p_t^* = \Delta z_t + \pi_t - \pi_t^*$$
$$E_t \Delta q_{t+1} = E_t \Delta z_{t+1} + E_t(\pi_{t+1} - \pi_{t+1}^*)$$
$$E_t \Delta z_{t+1} = E_t \Delta q_{t+1} - E_t(\pi_{t+1} - \pi_{t+1}^*)$$

Now these two equations are combined together:

$$E_t \Delta z_{t+1} = E_t \Delta q_{t+1} - E_t(\pi_{t+1} - \pi_{t+1}^*) \qquad E_t \Delta z_{t+1} = r_t^* - r_t$$

$$E_t \Delta q_{t+1} - E_t(\pi_{t+1} - \pi_{t+1}^*) = r_t^* - r_t$$
$$E_t \Delta q_{t+1} = r_t^* - E_t \pi_{t+1}^* - r_t + E_t \pi_{t+1}$$
$$E_t \Delta q_{t+1} = (r_t^* - E_t \pi_{t+1}^*) - (r_t - E_t \pi_{t+1}) \quad (26)$$

The equation (26) expresses the familiar relationship between the expected change in real exchange rate and the real interest rate differential.[22]

terms of trade (index of competitiveness of the domestic producers) depends on foreign exogenous interest rate and the domestic interest rate, which is influenced by the monetary policy of the central bank and the capital flows (we negligible them in this model). By changing the interest rate to hit the inflation target, the central bank influences also the competitiveness and subsequently output of the domestic producers (the real economy).

[22] The interest rate differential is calculated as the foreign real interest rate reduced by the domestic real interest rate because the indirect quotation for the exchange rate is used.

3.2 THE REPRESENTATIVE FIRMS

We introduce basic characteristics connected with the behavior of a representative firm. In the first subsection there is a description of production possibilities. Then in the subsection 3.2.2 there is a Phillips Curve and its derivation connected to the price setting behavior of a representative firm.

3.2.1 PRODUCTION

The aggregate output is described by the constant elasticity of substitution (CES) function:

$$Y_t = \left[\int_0^1 Y_t(i)^{\frac{\epsilon-1}{\epsilon}} di \right]^{\frac{\epsilon}{\epsilon-1}} \tag{27}$$

for $t = 0, 1, 2, \ldots$, where ϵ is the elasticity between different types of goods $Y_t(i)$.

There is a continuum of monopolistically competitive firms producing differentiated good $Y(i)$ using following production function:

$$Y_t(i) = A_t N_t(i), \tag{28}$$

for the i-th firm, for $t = 0, 1, 2, \ldots$, where $a_t = \log A_t$ describing the technological progress (the firm specific productivity index) by the following AR(1) process for all t:

$$a_t = \rho_a a_{t-1} + \epsilon_t^a. \tag{29}$$

The log–linear approximation of the aggregate production function is

$$y_t = a_t + n_t. \tag{30}$$

The real total costs of production can be calculated as a product of a real wage $\frac{W_t}{P_{H,t}}$ and total number of used hours of labor $N_t = \frac{Y_t}{A_t}$ for $t = 0, 1, 2, \ldots$:

$$TC_t = \frac{W_t}{P_{H,t}} \frac{Y_t}{A_t}.$$

We derive marginal costs and log–linearize them:

$$MC_t = \frac{\partial TC_t}{\partial Y_t} = \frac{\partial\left(\frac{W_t}{P_{H,t}}\frac{1}{A_t}\right)}{\partial Y_t}$$
$$MC_t = \frac{W_t}{P_{H,t}}\frac{1}{A_t}$$
$$logMC_t = log\frac{W_t}{P_{H,t}} + log\frac{1}{A_t}$$
$$mc_t = w_t - p_{H,t} - a_t \tag{31}$$

The equation (31) can be reformulated into $mc_t = w_t - p_{H,t} - a_t + \gamma$, where γ expresses a long run deviation. Very often $\gamma = log(1 - \tau)$, where τ is an employment subsidy or deduction (tax) from the government. It increases or decreases the real wage from the point of employer's view – the real wage is $\frac{W_t(1-\tau)}{P_{H,t}}$. We suppose there is no government in our model and thus $\gamma = \tau = 0$.[23]

3.2.2 PRICE SETTING BEHAVIOR

Every representative firm sets a price of its production to maximize its profit. In the domestic economy, firms set their prices in Calvo style effect. The result of this behavior are staggered prices and rigidities in the economy.

According to the Calvo price setting, every period only $1 - \theta_H$ of domestic firms is able to change the prices of production to optimize the behavior ($0 \leq \theta_H \leq 1$). The rest of the firms is not able to do this. Using the parameter in this expression $\frac{1}{1-\theta_H}$ we get an average duration of price contracts.

The aggregate domestic price level is a result of setting new prices of the fraction of firms, which are able to optimize $(1 - \theta_H)$, and the remaining fraction, which is not able to optimize prices (θ_H):

$$P_{H,t} = \left[(1-\theta_H)\overline{P}_{H,t}^{1-\rho} + \theta_H \widehat{P}_{H,t}^{1-\rho}\right]^{\frac{1}{1-\rho}} \tag{32}$$

for $t = 0, 1, 2, \ldots$, where $\overline{P}_{H,t}$ is the price level of optimizing firms and $\widehat{P}_{H,t}$ is the price level of firms that only adjust their prices by indexing.

[23] The relationship for the marginal costs holds without any deviations. There is no measurement error. An innovation to the marginal costs comes only from the technology shock.

The log–linearizing of the equation (32) yields:

$$\pi_{H,t} = (1-\theta_H)(\overline{p}_{H,t} - p_{H,t-1}) + \theta_H^2 \pi_{H,t-1}. \tag{33}$$

The firms, which are not able to optimize their prices, only adjust their prices by indexing it to the last period inflation. It can be described according to the relation:

$$\widehat{P}_{H,t} = P_{H,t-1}\left(\frac{P_{H,t-1}}{P_{H,t-2}}\right)^{\theta_H} = P_{H,t-1}(1+\Pi_{H,t-1})^{\theta_H},$$

where $(1+\Pi_{H,t-1}) = \frac{P_{H,t-1}}{P_{H,t-2}}$ for every period. The new non–optimized price of θ_H fraction of firms is created as an adjustment of the last period price $P_{H,t-1}$ with respect to the last period inflation $\Pi_{H,t-1}$. Log–linearizing the previous formula results:

$$\widehat{p}_{H,t} = p_{H,t-1} + \theta_H \pi_{H,t-1}. \tag{34}$$

If the firm is able to optimize, it sets new price $\overline{P}_{H,t}$ every period to maximize the current value of its dividends:

$$E_t \sum_{k=0}^{\infty} \theta_H^k \frac{1}{R_{t+k}} \left\{ Y_{t+k}(\overline{P}_{H,t} - MC_{t+k}^n) \right\} \tag{35}$$

subject to the current demand constraint:

$$Y_{t+k} \leq \left(\frac{\overline{P}_{H,t}}{P_{H,t+k}}\right)^{-\epsilon} (C_{H,t+k} + C_{H,t+k}^*), \tag{36}$$

where $\frac{\theta_H^k}{R_{t+k}}$ is the effective stochastic discount rate and MC_{t+k}^n are the nominal marginal costs[24]. The firm maximizes the current value of dividends with respect to the optimal price $\overline{P}_{H,t}$ expressed as the total revenue of its sales $(\overline{P}_{H,t} Y_{t+k})$ reduced by the total costs $(MC_{t+k}^n Y_{t+k})$.

The first order condition is:

$$E_t \sum_{k=0}^{\infty} \frac{\theta_H^k}{R_{t+k}} \left\{ Y_{t+k} \left(\overline{P}_{H,t} - \frac{\epsilon}{1-\epsilon} MC_{t+k}^n \right) \right\} = 0$$

[24] Detailed description and analysis of the marginal costs are contained in section 4.2.

for $t = 0, 1, 2, \ldots$. The term $\frac{\epsilon}{1-\epsilon}$ is the markup over the marginal costs in the steady–state, or equivalently the optimal markup in a flexible price economy.

The log–linear approximation of the optimal price setting strategy (FOC) in period t is following:

$$\overline{p}_{H,t} = p_{H,t-1} + E_t \sum_{k=0}^{\infty} (\beta\theta_H)^k \{\pi_{H,t+k} + (1-\beta\theta_H) mc_{t+k}\} \quad (37)$$

or similarly

$$\overline{p}_{H,t} = (1-\beta\theta_H) \sum_{k=0}^{\infty} (\beta\theta_H)^k E_t mc^n_{t+k}, \quad (38)$$

where the real marginal costs are $mc_t = mc^n_{t+k} - p_{H,t+k}$. According to (38) firms set the price as a markup over a weighted average of expected future marginal costs. In the flexible price situation ($\theta_H \rightarrow 0$), there is a common markup rule $\overline{p}_{H,t} = \mu + mc^n_t$, where $\mu = \frac{\epsilon}{1-\epsilon}$ for $t = 0, 1, 2, \ldots$

The result of the optimal price setting of firms is a rule for the development of the domestic inflation:

$$\pi_{H,t} = \beta(1-\theta_H) E_t \pi_{H,t+1} + \theta_H \pi_{H,t-1} + \lambda_H mc_t, \quad (39)$$

where $\lambda_H = \frac{(1-\beta\theta_H)(1-\theta_H)}{\theta_H}$, for $t = 0, 1, 2, \ldots$ The equation (39) is the New Keynesian Phillips Curve (NKPC)[25] – the domestic inflation dynamics is not only backward–looking but the forward–looking as well. If no firm is able to optimize the new prices ($\theta_H \rightarrow 1$), the Phillips Curve is purely backward–looking with adaptive expectations[26]. On the other hand, if all firms in the economy had a chance for reoptimizing of their prices ($\theta_H \rightarrow 0$), the domestic inflation would be forward–looking and desinflationary policy would be fully costless. The domestic inflation is always influenced by the marginal costs of firms, not only in both extreme cases.

Now we try to derive the import inflation in a similar way. We assume that the law of one price holds for all imports. But the distribution of

[25] The detailed derivation of the New Keynesian Phillips Curve is in Supplement 2.

[26] In case of $\theta_H = 1$, we know that $\lambda_H = 0$ and the Phillips Curve has following form: $\pi_{H,t} = \pi_{H,t-1}$

the goods by monopolistic firms increases their prices. Hence the law of one price for the final buyers does not hold. We use a similar Calvo price setting for domestic importers.

The fraction θ_F $(0 \leq \theta_F \leq 1)$ of importers can not optimize their prices every period. The rest of the firms $(1-\theta_F)$ sets the new price of imports as:

$$\overline{p}_{F,t} = p_{F,t-1} + E_t \sum_{k=0}^{\infty} (\beta\theta_F)^k \left\{\pi_{F,t+k} + (1-\beta\theta_F)\psi_{t+k}\right\}$$

for $t = 0, 1, 2, \ldots$ It is similar to equation (37) – the new price depends on the last period price and future path of import inflation and the law of one price gap ψ_t as well. A positive law of one price gap implies a difference between the foreign economy price and domestic import price. It is a mark–up over the import price. The law of one price gap is a factor for an incomplete import pass-through and provides an influence of the foreign economy prices to the domestic aggregate price level.

The result of this behavior is the Phillips Curve for the import inflation similar to (39) for all t:

$$\pi_{F,t} = \beta(1-\theta_F)E_t\pi_{F,t+1} + \theta_F\pi_{F,t-1} + \lambda_F\psi_t, \tag{40}$$

where $\lambda_H = \frac{(1-\beta\theta_F)(1-\theta_F)}{\theta_F}$.

The overall inflation is the first difference of a log–linear definition of CPI:

$$\pi_t = (1-\alpha)\pi_{H,t} + \alpha\pi_{F,t}. \tag{41}$$

The complete inflation dynamics of the small open economy is given by equation (41) together with (39) and (40). The firms' decisions to smooth prices make the prices sticky which gives rise to nominal rigidities. There are some costs of inflation in case of no price optimization (it is not valid for the fully flexible prices because there is no deviation of the marginal costs and the law of one price gap[27]).

[27] In this case it is possible to think about monetary policy as a tool for reducing the inflation costs by replicating the fully flexible price equilibrium.

3.3 THE CENTRAL MONETARY AUTHORITY

A domestic central monetary authority is the third agent in the model. The central bank implements monetary policy. Its basic aim is to stabilize both inflation and output[28].

The Taylor rule tells the central bank how to change the interest rate if there is an output gap or a deviation of an inflation from the target inflation. The rule is expressed e.g. by Woodford (2001). The central bank increases nominal interest rate in case of a positive output and/or inflation gap.

This behavior of the central bank in the situation of the inflation targeting regime can be approximated by the modified Taylor rule as we can see in Musil (2006). The central bank monetary policy development can be approximated by a causal relation of the modified Taylor rule (in a gap form) and the inflation targeting is obtained in the relation implicitly. It is in a development of the inflation gap (π_t), π_t is a deviation of the consumer price inflation from its target:

$$r_t = \rho_r r_{t-1} + (1 - \rho_r)(\phi_1 \pi_t + \phi_2 y_t), \tag{42}$$

for all t, where ρ_r $(0 \leq \rho_r \leq 1)$ is the degree of interest rate smoothing (backward–looking parameter for the interest rate gap), ϕ_1 and ϕ_2 $(\phi_1, \phi_2 \geq 0)$ are the relative weights on inflation gap and growth rate of output of the economy gap (output gap).

The higher the value of the degree of interest rate smoothing, the lower the influence of inflation and output on the interest rate. The extreme situation ($\rho_r \rightarrow 1$) means that the central bank is only backward–looking and sets the current value of the interest rate only according to its last value. It is not interested in a development of the inflation nor in output. On the other hand, in the opposite situation ($\rho_r \rightarrow 0$), the central bank is devoted only to the basic economic goals of specific rate of inflation and growth rate of output. The fundamental goal of the monetary authority is a stable price[29] (parameter ϕ_1 should be higher than ϕ_2).

[28] The situation connected to the inflation targeting in the Czech Republic is described e.g. in Czech National Bank (2005).
[29] Generally we speak about a price stabilization.

3.4 THE FOREIGN SECTOR

We introduce a foreign economy in the simplest way. Although it is a simplification, it allows us to establish some basic relationships between domestic and foreign economy.

The foreign sector is assumed to be exogenous to the small open economy. It is described by two equations. The first one is connected to the output of the foreign economy:

$$y_t^* = \lambda_1 y_{t-1}^* + \epsilon_t^{y^*}, \tag{43}$$

for $t = 0, 1, 2, \ldots$ The development of the foreign output y_t^* is described by AR(1) process for $0 < \lambda_1 < 1$ and the production shock $\epsilon_t^{y^*}$.

The second equation describes behavior of the foreign real interest rate:

$$r_t^* - E_t \pi_{t+1}^* = \rho_{r^*}(r_{t-1}^* - \pi_t^*) + \epsilon_t^{r^*}, \tag{44}$$

for all t, where ρ_{r^*} $(0 < \lambda_1 < 1)$ is a parameter of an AR(1) process. The short run real interest rate is expressed in terms of nominal interest rate and inflation. This expression is useful especially for the interpretation because the foreign inflation influences the domestic inflation through the prices of imported goods.[30]

[30] See the equation (41).

4 EQUILIBRIUM

To complete the model it is necessary to establish two conditions of the equilibrium. The first condition goes out from the goods market. A goods market–clearing condition expresses a basic fact, that the domestic output depends on the foreign output. It is described in section 4.1. The inflation dynamics with respect to a development of marginal costs of domestic firms is contained in section 4.2. There is a derivation of marginal costs and an introduction of a basic relations between marginal costs and variables, which influence the costs.

4.1 OUTPUT

The equilibrium on goods market for the domestic economy needs a logical condition that domestic product (Y_t) amounts to the domestic consumption ($C_{H,t}$) and foreign consumption of home produced goods ($C^*_{H,t}$).[31]

We know that according to the equation (12) there is the demand function for the i-th product and the same relationship holds for the foreign demand for the i-th domestic product for all t:

$$C_{H,t}(i) = \left(\frac{P_{H,t}(i)}{P_{H,t}}\right)^{-\epsilon} C_{H,t} \qquad C^*_{H,t}(i) = \left(\frac{P_{H,t}(i)}{P_{H,t}}\right)^{-\epsilon} C^*_{H,t} \quad (45)$$

In the calculation we also use the optimal allocation function of the household for the domestic produced consumption goods (9):

$$C_{H,t} = (1-\alpha)\left(\frac{P_{H,t}}{P_t}\right)^{-\eta} C_t.$$

Then it is necessary to find the optimal allocation function of the foreign household for the imported product. We use the previous relationship. The amount of the domestic consumption ($C_{H,t}$) from the domestic production depends on:

- the amount of the total domestic consumption C_t,
- the degree of openness $(1-\alpha)$,
- the elasticity of substitution between domestic and foreign consumption goods η,

[31] It is an export of the domestic economy (X_t). Then it is possible to write $C^*_{H,t} = X_t$.

- and the relative price of the good that is purchased to the aggregate domestic price level $\frac{P_{H,t}}{P_t}$.

In this way it is possible to derivate the foreign consumption from the foreign production ($C^*_{H,t}$) that is influenced by:

- the amount of the total consumption in the bigger economy C^*_t,
- the degree of openness of the bigger economy α,[32]
- the elasticity of substitution between domestic and foreign consumption goods η [33]
- and the relative price of domestic good that is purchased (in this case the purchased good is from the small open economy and expressed in the price of the domestic currency in the bigger economy) to the aggregate price level in the bigger economy $\frac{Z_t P_{H,t}}{P^*_t}$.

And we can write:

$$C^*_{H,t} = \alpha \left(\frac{Z_t P_{H,t}}{P^*_t} \right)^{-\eta} C^*_t \tag{46}$$

for all t.

The goods market–cleaning condition[34] holds for the i-th domestic

[32] We assume similar behavior of the economies (home and foreign) with the same degree of openness: $\alpha = \alpha^*$. If the domestic economy exports are the portion α of the total output (and the rest $(1-\alpha)$ leaves at home), the foreign economy must imports the same portion expressed by α.

[33] We suppose the ratio of the substitution between domestic and foreign consumption goods is the same for the domestic and foreign economy, because there are only two types of goods. This condition holds because there is only one foreign economy and two types of goods - one type of domestic and one type of foreign goods. The analysis must be extended in a situation of more foreign economies as the export partners for the home economy.

[34] If we allow a possibility of a group of foreign economies instead of one world economy as in the first section, the market–cleaning condition should be reformulated. It is necessary to amend it to this form:

$$Y_t(i) = C_{H,t}(i) + \int_0^1 C^*_{Hj,t}(i) dj,$$

where $C^*_{Hj,t}(i)$ denotes the j-th country demand for the i-th good produced in the home economy and it is possible to use the similar procedure outlined product and can be expressed in the following form[35]:

$$Y_t(i) = C_{H,t}(i) + C^*_{H,t}(i)$$

for $t = 0, 1, 2, \dots$ and now we plug both equations (45) to the previous formula:

$$Y_t(i) = \left(\frac{P_{H,t}(i)}{P_{H,t}}\right)^{-\epsilon} C_{H,t} + \left(\frac{P_{H,t}(i)}{P_{H,t}}\right)^{-\epsilon} C^*_{H,t}$$

and then we use the equations (9) and (46) to eliminate $C_{H,t}$ and $C^*_{H,t}$:

$$\begin{aligned}
Y_t(i) = & \left(\frac{P_{H,t}(i)}{P_{H,t}}\right)^{-\epsilon} (1-\alpha) \left(\frac{P_{H,t}}{P_t}\right)^{-\eta} C_t + \\
& + \left(\frac{P_{H,t}(i)}{P_{H,t}}\right)^{-\epsilon} \alpha \left(\frac{Z_t P_{H,t}}{P^*_t}\right)^{-\eta} C^*_t \\
Y_t(i) = & \left(\frac{P_{H,t}(i)}{P_{H,t}}\right)^{-\epsilon} \cdot \\
& \cdot \left((1-\alpha) \left(\frac{P_{H,t}}{P_t}\right)^{-\eta} C_t + \alpha \left(\frac{Z_t P_{H,t}}{P^*_t}\right)^{-\eta} C^*_t\right) \\
Y_t(i)^{\frac{\delta-1}{\delta}} = & \left(\frac{P_{H,t}(i)}{P_{H,t}}\right)^{-\epsilon\left(\frac{\delta-1}{\delta}\right)} \cdot \\
& \cdot \left((1-\alpha) \left(\frac{P_{H,t}}{P_t}\right)^{-\eta} C_t + \alpha \left(\frac{Z_t P_{H,t}}{P^*_t}\right)^{-\eta} C^*_t\right)^{\frac{\delta-1}{\delta}}
\end{aligned}$$

Substituting the equation (27) for the aggregate output $Y_t = \left[\int_0^1 Y_t(i)^{\frac{\delta-1}{\delta}} di\right]^{\frac{\delta}{\delta-1}}$

---

to derive that

$$C^*_{Hj,t}(i) = \alpha \left(\frac{P_{H,t}(i)}{P_{H,t}}\right)^{-\epsilon} \left(\frac{P_{H,t}}{Z_{j,t} P_{Fj,t}}\right)^{-\epsilon} \left(\frac{P_{Fj,t}}{P_{i,t}}\right)^{-\eta} C_{j,t}.$$

The final market–cleaning condition is the same as it is introduced.

[35] The method of solving is also outlined in Vlček (2006).

into the previous result yields:

$$
\begin{aligned}
Y_t = & \left[\int_0^1 \left(\frac{P_{H,t}(i)}{P_{H,t}}\right)^{-\epsilon\left(\frac{\delta-1}{\delta}\right)} \left((1-\alpha)\left(\frac{P_{H,t}}{P_t}\right)^{-\eta} C_t + \right.\right. \\
& \left.\left. + \, \alpha \left(\frac{Z_t P_{H,t}}{P_t^*}\right)^{-\eta} C_t^*\right)^{\frac{\delta-1}{\delta}} di\right]^{\frac{\delta}{\delta-1}} \\
Y_t^{\frac{\delta-1}{\delta}} = & \int_0^1 \left(\frac{P_{H,t}(i)}{P_{H,t}}\right)^{-\epsilon\left(\frac{\delta-1}{\delta}\right)} \left((1-\alpha)\left(\frac{P_{H,t}}{P_t}\right)^{-\eta} C_t + \right. \\
& \left. + \, \alpha \left(\frac{Z_t P_{H,t}}{P_t^*}\right)^{-\eta} C_t^*\right)^{\frac{\delta-1}{\delta}} di \\
Y_t = & \int_0^1 \left(\frac{P_{H,t}(i)}{P_{H,t}}\right)^{-\epsilon} \left((1-\alpha)\left(\frac{P_{H,t}}{P_t}\right)^{-\eta} C_t + \right. \\
& \left. + \, \alpha \left(\frac{Z_t P_{H,t}}{P_t^*}\right)^{-\eta} C_t^*\right) di \\
Y_t = & \left(\frac{1}{P_{H,t}}\right)^{-\epsilon} \left((1-\alpha)\left(\frac{P_{H,t}}{P_t}\right)^{-\eta} C_t + \alpha \left(\frac{Z_t P_{H,t}}{P_t^*}\right)^{-\eta} C_t^*\right) \cdot \\
& \cdot \int_0^1 P_{H,t}(i)^{-\epsilon} di \\
Y_t = & \left(\frac{1}{P_{H,t}}\right)^{-\epsilon} \cdot \\
& \cdot \left((1-\alpha)\left(\frac{P_{H,t}}{P_t}\right)^{-\eta} C_t + \alpha \left(\frac{Z_t P_{H,t}}{P_t^*}\right)^{-\eta} C_t^*\right) P_{H,t}^{-\epsilon} \\
Y_t = & (1-\alpha)\left(\frac{P_{H,t}}{P_t}\right)^{-\eta} C_t + \alpha \left(\frac{Z_t P_{H,t}}{P_t^*}\right)^{-\eta} C_t^* \\
Y_t = & C_{H,t} + C_{H,t}^* \qquad (47)
\end{aligned}
$$

In the last step we used optimal allocation functions (9) and (46).

The total differential of the first order condition (FOC) yields for all t: [36]

$$
Y y_t = C_H c_{H,t} + C_H^* c_{H,t}^*
$$

[36] We suppose zero net export in long run and that firms are owned by households ($C = Y$).

$$y_t = \frac{C_H}{Y} c_{H,t} + \frac{C_H^*}{Y} c_{H,t}^*$$
$$y_t = \frac{C_H}{C} c_{H,t} + \frac{C_H^*}{C} c_{H,t}^*$$
$$y_t = (1-\alpha) c_{H,t} + \alpha c_{H,t}^*. \tag{48}$$

The results are intuitive. Both equations (47) and (48) give the similar explanation. In (47), the aggregate domestic output is divided into the domestic and the foreign consumption. According to the expression (48)[37], the increase in aggregate output is divided between the increase in domestic and foreign consumption with respect to the openness of the economy (some part is consumed at home and the rest is exported). Equation (48) describes the previous equation in a growing form.

Log–linearizing optimal allocation functions for the domestic economy (9) and $p_t = p_{H,t} + \alpha s_t$ from the equation (15) gives for all t:

$$c_{H,t} = -\eta(p_{H,t} - p_t) \qquad\qquad p_{H,t} - p_t = \alpha s_t$$
$$c_{H,t} = \alpha\eta s_t + c_t. \tag{49}$$

It is evident that an improvement in terms of trade for the domestic economy (s_t or domestic competitiveness on the foreign market increases) enables to the domestic representative households to augment its consumption and substitute out the foreign produced goods for a given level of consumption. The magnitude depends on the possibility of substitution between domestic and foreign goods (η) and the degree of openness of the economy (α).

The allocation function for the foreign economy (46) is also log–linearized and simultaneously is used log–linearized version of law of one price:

$$c_{H,t}^* = -\eta(z_t + p_{H,t} - p_t^*) + c_t^* \qquad \psi_t = p_t^* - z_t - p_{F,t} \Rightarrow z_t - p_t^* = -p_{F,t} - \psi_t$$
$$c_{H,t}^* = -\eta(p_{H,t} - p_{F,t} - \psi_t) + c_t^*$$

and together with log–linearized version of terms of trade (13):

$$c_{H,t}^* = -\eta(p_{H,t} - p_{F,t} - \psi_t) + c_t^* \qquad\qquad p_{F,t} - p_{H,t} = s_t$$

[37] The equation is possible to rewrite into this form $y_t = (1-\alpha)c_{H,t} + \alpha x_t$.

$$c^*_{H,t} = -\eta(-s_t - \psi_t) + c^*_t$$
$$c^*_{H,t} = \eta(s_t + \psi_t) + c^*_t \tag{50}$$

for $t = 0, 1, 2, \ldots$

The explanation of the previous equation is similar. An increase in s_t causes higher consumption of goods produced in the small open economy for foreigners accompanied by a decrease of domestic goods for consumption.

Plugging (49) and (50) in (48) the equation has the following form:

$$y_t = (1-\alpha)[\alpha\eta s_t + c_t] + \alpha[\eta(s_t + \psi_t) + c^*_t]$$
$$y_t = \alpha\eta s_t + c_t - \alpha^2\eta s_t - \alpha c_t + \alpha\eta s_t + \alpha\eta\psi_t + \alpha c^*_t$$
$$y_t = (2-\alpha)\alpha\eta s_t + (1-\alpha)c_t + \alpha\eta\psi_t + \alpha y^*_t \tag{51}$$

for $t = 0, 1, 2, \ldots$ The equation (51) is the goods market–clearing condition for the small open economy. In case of closed economy ($\alpha = 0$) we have condition of this form $y_t = c_t$.

4.2 MARGINAL COSTS

The equation (39) is a result of behavior of domestic firms with respect to the Calvo style pricing. This domestic New Keynesian Phillips Curve shows the development of domestic inflation (inflation dynamics). We have derived this form:

$$\pi_{H,t} = \beta(1-\theta_H)E_t\pi_{H,t+1} + \theta_H\pi_{H,t-1} + \lambda_H mc_t,$$

for $t = 0, 1, 2, \ldots$

The evolution of the current inflation depends on last period inflation (by indexation of some firms) and discounted value of expected inflation for the next period (by optimizing behavior of the rest of the firms). The real marginal costs are the third important factor. They stem from the production possibilities (expressed as a CES production function) of monopolistic firms (31). A symmetric equilibrium assumes:

$$mc_t = w_t - p_{H,t} - a_t$$
$$mc_t = (w_t - p_t) + (p_t - p_{H,t}) - a_t$$

Now we employ the log–linearized FOC of household's optimizing expressed as the intratemporal consumption (5) and the adjusted formula for the terms of trade (15) to plug them into the equation for marginal costs.

$$w_t - p_t = \varphi n_t + \frac{\sigma}{1-h}(c_t - hc_{t-1}) \qquad p_t = p_{H,t} + \alpha s_t \Rightarrow p_t - p_{H,t} = \alpha s_t$$

$$mc_t = \varphi n_t + \frac{\sigma}{1-h}(c_t - hc_{t-1}) + \alpha s_t - a_t$$

The last task is to substitute out the term n_t with using log–linear version of the firms' production function (30) $y_t = a_t + n_t \;\Rightarrow\; n_t = y_t - a_t$ in this way:

$$mc_t = \varphi(y_t - a_t) + \frac{\sigma}{1-h}(c_t - hc_{t-1}) + \alpha s_t - a_t$$
$$mc_t = \frac{\sigma}{1-h}(c_t - hc_{t-1}) + \varphi y_t + \alpha s_t - (1+\varphi)a_t \qquad (52)$$

for all t.

The marginal costs are positively related to the domestic output and terms of trade and inversely related to the level of technological progress (the firm specific productivity index).

5 LINEARIZED SYSTEM

The log–linearized model consists of equations (6), (17), (21), (25), (26), (29), (39) – (44), (51) and (52). These 14 equations are rearranged and completed by exogenous domestic and foreign shocks. The system is following:

$$\psi_t = -[q_t + (1-\alpha)s_t] \tag{53}$$

$$\Delta s_t = \pi_{F,t} - \pi_{H,t} + \epsilon_t^s \tag{54}$$

$$\Delta E_t q_{t+1} = (r_t^* - E_t\pi_{t+1}^*) - (r_t - E_t\pi_{t+1}) + \epsilon_t^q \tag{55}$$

$$\pi_t = (1-\alpha)\pi_{H,t} + \alpha\pi_{F,t} \tag{56}$$

$$\pi_{F,t} = \beta(1-\theta_F)E_t\pi_{F,t+1} + \theta_F\pi_{F,t-1} + \lambda_F\psi_t + \epsilon_t^{\pi F} \tag{57}$$

$$\pi_{H,t} = \beta(1-\theta_H)E_t\pi_{H,t+1} + \theta_H\pi_{H,t-1} + \lambda_H mc_t + \epsilon_t^{\pi H} \tag{58}$$

$$mc_t = \frac{\sigma}{1-h}(c_t - hc_{t-1}) + \varphi y_t + \alpha s_t - (1+\varphi)a_t \tag{59}$$

$$a_t = \rho_a a_{t-1} + \epsilon_t^a \tag{60}$$

$$c_t - hc_{t-1} = E_t(c_{t+1} - hc_t) - \frac{1-h}{\sigma}(r_t - E_t\pi_{t+1}) \tag{61}$$

$$c_t - hc_{t-1} = y_t^* - hy_{t-1}^* - \frac{1-h}{\sigma}q_t \tag{62}$$

$$y_t = (2-\alpha)\alpha\eta s_t + (1-\alpha)c_t + \alpha\eta\psi_t + \alpha y_t^* \tag{63}$$

$$r_t = \rho_r r_{t-1} + (1-\rho_r)(\phi_1\pi_t + \phi_2 y_t) + \epsilon_t^r \tag{64}$$

$$y_t^* = \lambda_1 y_{t-1}^* + \epsilon_t^{y^*} \tag{65}$$

$$r_t^* - E_t\pi_{t+1}^* = \rho_{r^*}(r_{t-1}^* - \pi_t^*) + \epsilon_t^{r^*} \tag{66}$$

for $t = 0, 1, 2, \ldots$

The linearized model consists of 11 equations for endogenous variables and 3 equations for exogenous processes – equation (60), (65) and (66). There are six shocks: ϵ_t^s, ϵ_t^q, ϵ_t^a, ϵ_t^r, $\epsilon_t^{y^*}$ and $\epsilon_t^{r^*}$.

The description of the equations with shocks in the system is following:

- equation (53) – law of one price (LOP) gap,
- equation (54) – terms of trade with a measurement error ϵ_t^s,
- equation (55) – uncovered interest parity (UIP) with a risk premium shock ϵ_t^q,
- equation (56) – overall inflation,
- equation (57) – New Keynesian Phillips Curve (NKPC) for import inflation with a foreign inflation shock $\epsilon_t^{\pi F}$,

- equation (58) – New Keynesian Phillips Curve (NKPC) for domestic inflation with a domestic inflation shock $\epsilon_t^{\pi_H}$,
- equation (59) – firm's marginal costs,
- equation (60) – AR(1) process for a technological progress with an innovation ϵ_t^a,
- equation (61) – consumption Euler equation,
- equation (62) – international risk sharing condition,
- equation (63) – goods market–clearing condition,
- equation (64) – modified Taylor rule with a monetary shock ϵ_t^r,
- equation (65) – exogenous AR(1) process for the foreign economy output with an innovation $\epsilon_t^{y^*}$,
- equation (66) – exogenous AR(1) process for the foreign economy short–run real interest rate with a shock $\epsilon_t^{r^*}$.

The model has 14 parameters[38] and 8 other parameters representing standard deviations of shocks $(\sigma_a, \sigma_s, \sigma_q, \sigma_{\pi_H}, \sigma_{\pi_F}, \sigma_r, \sigma_{y^*}, \sigma_{r^*})$. Table 1 shows a short overview about parameters in the linearized model.

[38] Parameters in equations (57) and (58) are $\lambda_F = \frac{(1-\beta\theta_F)(1-\theta_F)}{\theta_F}$ and $\lambda_H = \frac{(1-\beta\theta_H)(1-\theta_H)}{\theta_H}$.

Table 1: Parameters of the Linearized Model

Parameter	**Equation**	**Interpretation of the Parameter**	**Restriction**
α	53, 56, 59, 63	degree of openness	$\langle 0;1\rangle$
β	57, 58	discount factor	$(0;1)$
h	59, 61, 62	habit formation parameter in consumption	$(0;1)$
σ	59, 61, 62	inverse elasticity of intertemporal substitution	$(0;\infty)$
η	63	elasticity of substitution between home and foreign goods	$(0;\infty)$
φ	59	inverse elasticity of labor supply	$(0;\infty)$
θ_H	57	fraction of non–optimizing firms	$\langle 0;1\rangle$
θ_F	58	fraction of non–optimizing importers	$\langle 0;1\rangle$
ϕ_1	64	elasticity of interest rate to inflation	$\langle 0;\infty)$
ϕ_2	64	elasticity of interest rate to output	$\langle 0;\infty)$
ρ_r	64	backward looking parameter for interest rate	$\langle 0;1\rangle$
ρ_r^*	66	foreign real interest rate inertia parameter	$(0;1)$
ρ_a	60	inertia of technology development	$(0;1)$
λ_1	65	foreign output inertia parameter	$(0;1)$

6 SOLVING THE MODEL

The log–linearized model presented by equations (53) – (66) can be rewritten into a form of a linear rational expectations system (LRE system):

$$0 = Ax_t + Bx_{t-1} + Cy_t + Dz_t \tag{67}$$

$$0 = FE_t(x_{t+1}) + Gx_t + Hx_{t-1} + \\ + \; JE_t(y_{t+1}) + Ky_t + LE_t(z_{t+1}) + Mz_t \tag{68}$$

$$E_t(z_{t+1}) = Nz_t + E_t(\xi_{t+1}) \tag{69}$$

$$E_t(\xi_{t+1}) = 0, \tag{70}$$

for $t = 0, 1, 2, \ldots$

Equations are declared in this order. In (67) there are non–expectational equations – (53), (63), (64), (54), (59), (56), (65), and (66). In (68) there are expectational equations – (55), (58), (57) and (61) together with (62)[39]. The equation (69) is for exogenous equations connected to the shocks and innovations in the model with respect to the restriction of (70).[40]

The vector x_t is the endogenous state vector, y_t is the endogenous vector of unobservable variables and z_t is the exogenous stochastic

[39] The equations (61) and (62) are combined together and used in the form

$$E_t(c_{t+1} - hc_t) = \frac{1-h}{\sigma}(r_t - E_t\pi_{t+1}) + (y_t^* - hy_{t-1}^*) - \frac{1-h}{\sigma}q_t.$$

[40] Every equations contain some ν_{t+1}, which is an iid shock, and are described by the following equations:

$$\begin{aligned}
a_{t+1} &= \rho_a a_t + \nu_{t+1}^a \\
\epsilon_{t+1}^s &= 0\ \epsilon_t^s + \nu_{t+1}^s \\
\epsilon_{t+1}^q &= 0\ \epsilon_t^q + \nu_{t+1}^q \\
\epsilon_{t+1}^{\pi_H} &= 0\ \epsilon_t^{\pi_H} + \nu_{t+1}^{\pi_H} \\
\epsilon_{t+1}^{\pi_F} &= 0\ \epsilon_t^{\pi_F} + \nu_{t+1}^{\pi_F} \\
\epsilon_{t+1}^r &= 0\ \epsilon_t^r + \nu_{t+1}^r \\
\epsilon_{t+1}^{y^*} &= 0\ \epsilon_t^{y^*} + \nu_{t+1}^{y^*} \\
\epsilon_{t+1}^{r^*} &= 0\ \epsilon_t^{r^*} + \nu_{t+1}^{r^*}
\end{aligned}$$

for all t and every ν_{t+1} with $E_t(\nu_{t+1}) = 0$.

process:

$$
\begin{aligned}
x_t &= \{y_t, q_t, r_t, \pi_t, \pi_{F,t}, s_t, c_t, r_t^*, y_t^*, \pi_{H,t}\} \\
y_t &= \{\psi_t, mc_t\} \\
z_t &= \{a_t, \epsilon_t^s, \epsilon_t^q, \epsilon_t^{\pi_H}, \epsilon_t^{\pi_F}, \epsilon_t^r, \epsilon_t^{y^*}, \epsilon_t^{r^*}\},
\end{aligned}
$$

where r_t^* expresses foreign real interest rate instead of foreign nominal interest rate as it was used so far.

The matrices of system $A_{8\times10}$, $B_{8\times10}$, $C_{8\times2}$, $D_{8\times8}$, $F_{4\times10}$, $G_{4\times10}$, $H_{4\times10}$, $J_{4\times2}$, $K_{4\times2}$, $L_{4\times8}$, $M_{4\times8}$ and $N_{8\times8}$ are following:

$$
A=\begin{bmatrix}
0 & -1 & 0 & 0 & 0 & 1-\alpha & 0 & 0 & 0 & 0 \\
-1 & 0 & 0 & 0 & 0 & (2-\alpha)\alpha\eta & 1-\alpha & 0 & \alpha & 0 \\
(1-\rho_r)\phi_2 & 0 & -1 & \frac{(1-\rho_r)}{\phi_1} & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 1 & -1 & 0 & 0 & 0 & -1 \\
\varphi & 0 & 0 & 0 & 0 & \alpha & \frac{\sigma}{1-h} & 0 & 0 & 0 \\
0 & 0 & 0 & -1 & \alpha & 0 & 0 & 0 & 0 & 1-\alpha \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0
\end{bmatrix}
$$

$$
B = \begin{bmatrix}
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & \rho_r & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & \frac{-\sigma h}{1-h} & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \lambda_1 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & \rho_{r^*} & 0 & 0
\end{bmatrix}
$$

$$
C = \begin{bmatrix}
-1 & 0 \\
\alpha\eta & 0 \\
0 & 0 \\
0 & 0 \\
0 & -1 \\
0 & 0 \\
0 & 0 \\
0 & 0
\end{bmatrix}
$$

$$D = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ -(1+\varphi) & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

$$F = \begin{bmatrix} 0 & 1 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \beta(1-\theta_H) \\ 0 & 0 & 0 & 0 & \beta(1-\theta_F) & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{1-h}{\sigma} & 0 & 0 & 1 & 0 & 0 & 0 \end{bmatrix}$$

$$G = \begin{bmatrix} 0 & -1 & 1 & 0 & 0 & 0 & 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 \\ 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 \\ 0 & \frac{1-h}{\sigma} & \frac{-(1-h)}{\sigma} & 0 & 0 & 0 & -h & 0 & -1 & 0 \end{bmatrix}$$

$$H = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \theta_H \\ 0 & 0 & 0 & 0 & \theta_F & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & h & 0 \end{bmatrix}$$

$$K = \begin{bmatrix} 0 & 0 \\ 0 & \frac{(1-\beta\theta_H)(1-\theta_H)}{\theta_H} \\ \frac{(1-\beta\theta_F)(1-\theta_F)}{\theta_F} & 0 \\ 0 & 0 \end{bmatrix}$$

$$M = \begin{bmatrix} 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$$N = \begin{bmatrix} \rho_a & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

The matrices J and L are matrices of zeros.

Using algorithm of Uhlig (1995) the system of equations (67) – (69) can be transformed to the recursive rule for the general equilibrium (GE) expressed by the following state model:

$$x_t = Px_{t-1} + Qz_t \tag{71}$$

$$y_t = Ry_{t-1} + Sz_t, \tag{72}$$

for $t = 0, 1, 2, \ldots$, where the equilibrium is stable and described by matrices P, Q, R and S.

It is possible to rewrite the structural model and using the Blanchard–Kahn setup (with using Q–Z decomposition[41]) to get the final form of a state–space representation of the model for estimation.

Finally the economic model has the following state–space representation:

$$S_{t+1} = \Gamma_1 S_t + \Gamma_2 w_{t+1} \tag{73}$$

$$Y_t = \Lambda S_t + v_t \tag{74}$$

for $t = 0, 1, 2, \ldots$, where:

$S_t = \{x_t, y_t\}$	is a vector of the states from (71) and (72)
Y_t	is a vector of observed variables
Γ_1 and Γ_2	are matrices of functions of the model's deep parameters (matrices P, Q, R and S) from the state equation (73) representing the dynamic core of the model
Λ	is a matrix expressing the relationship between observed and state variables
w_t	is a vector of state innovations: $w_t \sim N(0, \Xi)$
v_t	is a vector of measurement errors: $v_t \sim N(0, \Upsilon)$

[41] It is an approach to the computation of generalized eigenvalues. Some matrices with special features are calculated by using this method. This approach is used by Klein (2000).

7 THE BAYESIAN ESTIMATION

In this section we introduce some basic steps within the Bayesian approach for solving the model. We are especially interested in a procedure of obtaining the posterior distribution of the estimated parameters.

The Bayesian approach enables to use some model uncertainties and especially with the parameters of the model. It looks for a model with the highest posterior probability of estimated parameters instead of a classical approach based on testing whether the chosen model is the correct one.

We use the likelihood principle – all inference about the parameter vector θ is contained in the posterior distribution. According to the Bayesian formula the posterior density of the model parameter θ for the model i holds:

$$p(\theta|Y^T, i) = \frac{L(Y^T|\theta, i)p(\theta|i)}{\int L(Y^T|\theta, i)p(\theta|i)d\theta},$$

where:
$p(\theta|i)$ is the prior density
$L(Y^T|\theta, i)$ is the likelihood condition[42] (ML)

As it has been introduced we search for the model i that maximizes the posterior probability $p(\theta|Y^T, i)$.

We suppose that $\int L(Y^T|\theta, i)p(\theta|i)d\theta$ is for the particular model i constant.

The likelihood function (ML) is computed from the state–space representation expressed by the equations (73) and (74):

$$S_{t+1} = \Gamma_1 S_t + \Gamma_2 w_{t+1}$$
$$Y_t = \Lambda S_t + v_t$$

for all t and with the restrictions connected to the state innovation and measurement error vectors:

$$w_t \sim N(0, \Xi) \qquad\qquad v_t \sim N(0, \Upsilon)$$

The likelihood function for the model has the following form:

$$logL(Y^T|\Theta) = \frac{1}{2}\sum_{t-1}^{T}\left[Nlog2\pi + log|\Omega_{t|t-1}| + \sum_{t-1}^{T} v_t' \Omega_{t|t-1}^{-1} v_t\right]$$

[42] More exactly it is the likelihood condition on the observed data Y^T.

where:

$$\Theta = \{\ \Gamma_1,\ \Gamma_2,\ \Lambda,\ \Xi,\ \Upsilon\ \}$$
$$\Omega_{t|t-1} = \Lambda' \Sigma_{t|t-1} \Lambda + \Upsilon$$
$$\Sigma_{t|t-1} = \Gamma_1 \Sigma_{t-1|t-1} \Gamma_1' + \Gamma_2 \Xi \Gamma_2'$$

The likelihood function is solved by the Kalman filter algorithm for the initial state value $S_0 \sim N(\hat{S_0}, \Sigma_0)$.[43]

To calculate the posterior density we use the fact that it is summarized information contained in the likelihood $L(Y^T|\theta)$ weighted by the prior density $p(\theta)$:

$$p(\theta|Y^T) \propto L(Y^T|\theta) p(\theta)$$

The advantage of this way of calculation is that the prior density can bring inferences that are not contained in the observed data Y^T.

For the sequence of draws holds:

$$\{\theta^j\}_1^N \sim p(\theta|Y^T)$$

and then it is used the law of large numbers:

$$E_\theta(g(\theta)|Y^T) = \frac{1}{N} \sum_{j=1}^{N} g(\theta^j),$$

where $g(.)$ is a suitable function.

The sequence of posterior draws $\{\theta^j\}_1^N$ used in the law of the large numbers is obtained using Markov Chain which is generated by the Monte Carlo method (MCMC)[44]. For the Markov Chain (MC) is employed the Random Walk Metropolis Hastings algorithm.

For our estimation of the model we used the outlined Bayesian technique. The method contains only functions and procedures which are open – it is possible to check the calculations and sequence of calculation steps. It is an important advantage because it gives some possibilities to change some conditions or calculations with respect to the conditions of the Czech economy.[45]

[43] The whole procedure of solving is described in more details in Hamilton (1994).

[44] The whole process is called Markov Chain Monte Carlo method (MCMC).

[45] We can employ Dynare for the Bayesian estimation too. However no procedures within this tootbox for Matlab can be modified and must be used unchanged.

8 RESULTS OF THE ESTIMATION

In this section, we introduce data used for the estimation in subsection 8.1, then the posterior estimated parameters in subsection 8.2 and an interpretation of the parameters in subsection 8.3. Section 9 contains a detailed analysis of the characteristic of the Czech economy with respect to the estimated parameters.

8.1 DATA AND PREREQUISITIES FOR THE ESTIMATION

Quarterly data from I. Q 1995 to IV.Q 2005 was used for the estimation of the model for the conditions of the Czech economy. The model is the gap model. The structure of all equations remains similar without any change. Only all variables are in the gap form.

All data except for the terms of trade (s_t) are entered as deviations from their long run balanced growth. The overall inflation gap is a deviation from the inflation target.

The data used for estimation are described in the following way:

- y_t: *macroeconomic productivity gap* of the domestic real GDP, i.e. a deviation of the real GDP per employed person productivity from the development of the balanced output productivity,
- π_t: *overall inflation gap*, i.e. a deviation of the annualized domestic Consumer Price Index (CPI) inflation from the development of the inflation target,
- $\pi_{F,t}$: *import inflation gap*, i.e. a deviation of the annualized imported prices inflation from the development of the dynamic equilibrium,
- r_t: *nominal interest rate gap*, i.e. a deviation of the domestic one–year interbank interest rate from the development of the dynamic equilibrium,
- q_t: *real exchange rate gap*, i.e. a deviation from its dynamic equilibrium,
- s_t: *terms of trade* (Competitive Price Index) is a logarithm of the foreign CPI to a logarithm of the domestic CPI[46] ratio,
- y_t^*: *foreign output productivity gap* in Germany, i.e. a deviation of the foreign macroeconomic productivity of the real GDP per

[46] The domestic CPI is without any influence of import deflator.

employed person from the development of the balanced macroeconomic productivity,

- rr_t^*: *foreign real interest rate*[47], i.e. a deviation of the one–year interest rate from its dynamic equilibrium.

The figures of the original data are enclosed as the third supplement.

The priors reflect some basic characteristics and dynamic properties of the Czech economy. The choice of the prior distributions reflects restrictions on the parameters as it is shown in Table 1:

- Beta distribution for parameters constrained on the unit interval,
- Gamma and Normal distributions for parameters in $\Re^+$,
- Inverse Gamma distribution for the shocks.

There are 8 extra shocks which were introduced to the linearized system of equations to avoid any problems with singularity of matrices during the calculation[48].

[47] We use the foreign real interest rate although the model is derived for the foreign nominal interest rate which is adjusted by the expected foreign inflation. It is connected to the equation (44): $rr_t^* = r_t^* - E_t\pi_{t+1}^*$.

[48] The problem can arise in a situation of more observed variables than the number of stochastic shocks. Now there are 8 shocks and 8 observed variables and the problem of singularity is irrelevant.

8.2 POSTERIOR PARAMETER ESTIMATES

Given the data and the prior specification, we generate Markov Chain (MC) containing 250 000 draws. Parameters of the degree of openness (α) and time preference (discount factor β) was fixed:

$$\alpha = 0.4$$
$$\beta = 0.99$$

The posterior estimates (medians) of parameters and shocks with 95 % probability intervals are reported in Table 2. The behavior of the MC for each parameter are in Figure 1. The prior and the estimated posterior marginal densities of parameters and shocks are plotted in Figure 2 and 3. The prior marginal density is plotted in red and the estimated posterior marginal density in light blue.

Table 2: Posterior Estimates of the Parameters and Innovations

Para-meter	**Prior Mean**	**Posterior Median**	**95 % Posterior Interval**
h	0.50	0.8918	$\langle 0.8279;\ 0.9557\rangle$
σ	1.00	0.8153	$\langle 0.2477;\ 1.3830\rangle$
η	1.00	0.3767	$\langle 0.2756;\ 0.4777\rangle$
φ	1.00	1.0806	$\langle 0.2937;\ 1.8674\rangle$
θ_H	0.50	0.6397	$\langle 0.5775;\ 0.7018\rangle$
θ_F	0.50	0.4407	$\langle 0.3385;\ 0.5429\rangle$
ϕ_1	1.50	1.2701	$\langle 1.0836;\ 1.4566\rangle$
ϕ_2	0.25	0.4671	$\langle 0.2386;\ 0.6955\rangle$
ρ_r	0.50	0.6496	$\langle 0.5533;\ 0.7176\rangle$
ρ_r^*	0.70	0.6690	$\langle 0.5156;\ 0.8223\rangle$
ρ_a	0.70	0.9717	$\langle 0.9338;\ 1.0097\rangle$
λ_1	0.70	0.8020	$\langle 0.7245;\ 0.8796\rangle$
σ_a	$\langle 0;\infty\rangle$	0.8102	$\langle 0.3576;\ 1.2628\rangle$
σ_s	$\langle 0;\infty\rangle$	15.539	$\langle 12.466;\ 18.6128\rangle$
σ_q	$\langle 0;\infty\rangle$	4.7152	$\langle 2.8978;\ 6.5327\rangle$
σ_{π_H}	$\langle 0;\infty\rangle$	3.0458	$\langle 2.2152;\ 3.8763\rangle$
σ_{π_F}	$\langle 0;\infty\rangle$	6.7013	$\langle 4.0459;\ 9.3566\rangle$
σ_r	$\langle 0;\infty\rangle$	1.8975	$\langle 1.4631;\ 2.3319\rangle$
σ_{y^*}	$\langle 0;\infty\rangle$	0.3482	$\langle 0.2636;\ 0.4329\rangle$
σ_{r^*}	$\langle 0;\infty\rangle$	0.4291	$\langle 0.3282;\ 0.5299\rangle$

Figure 1: Markov Chain

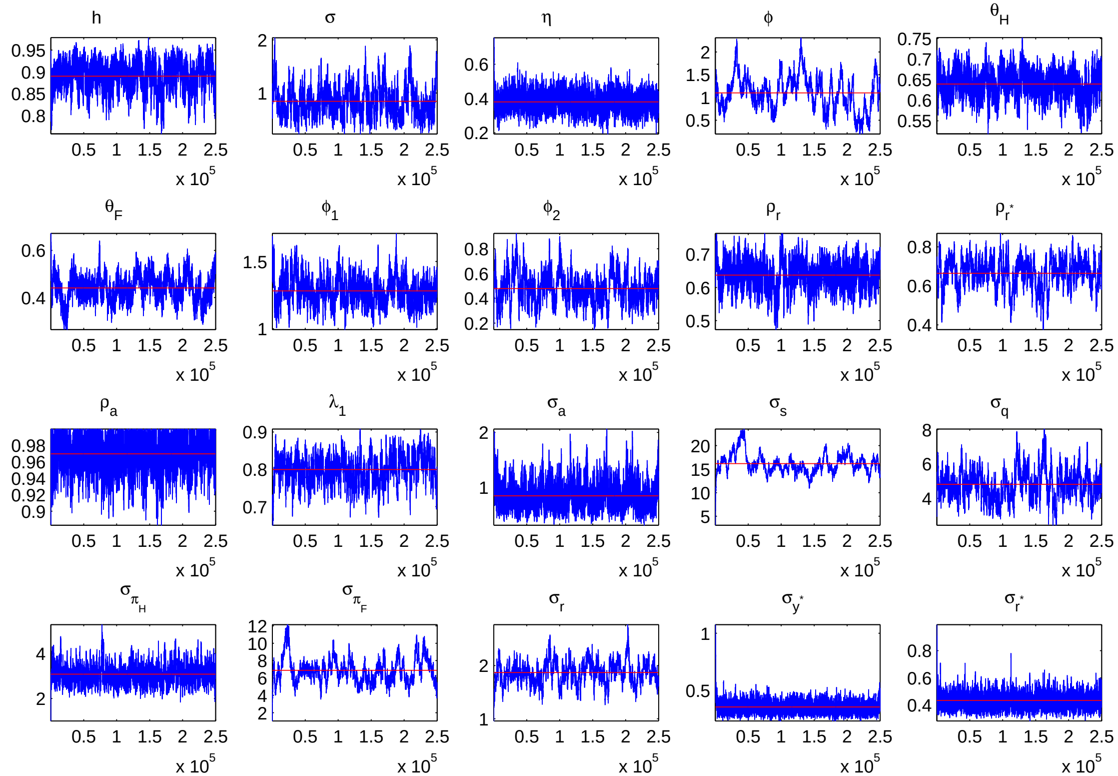


Figure 2: Posterior and Prior Marginal Density Plot of Parameters

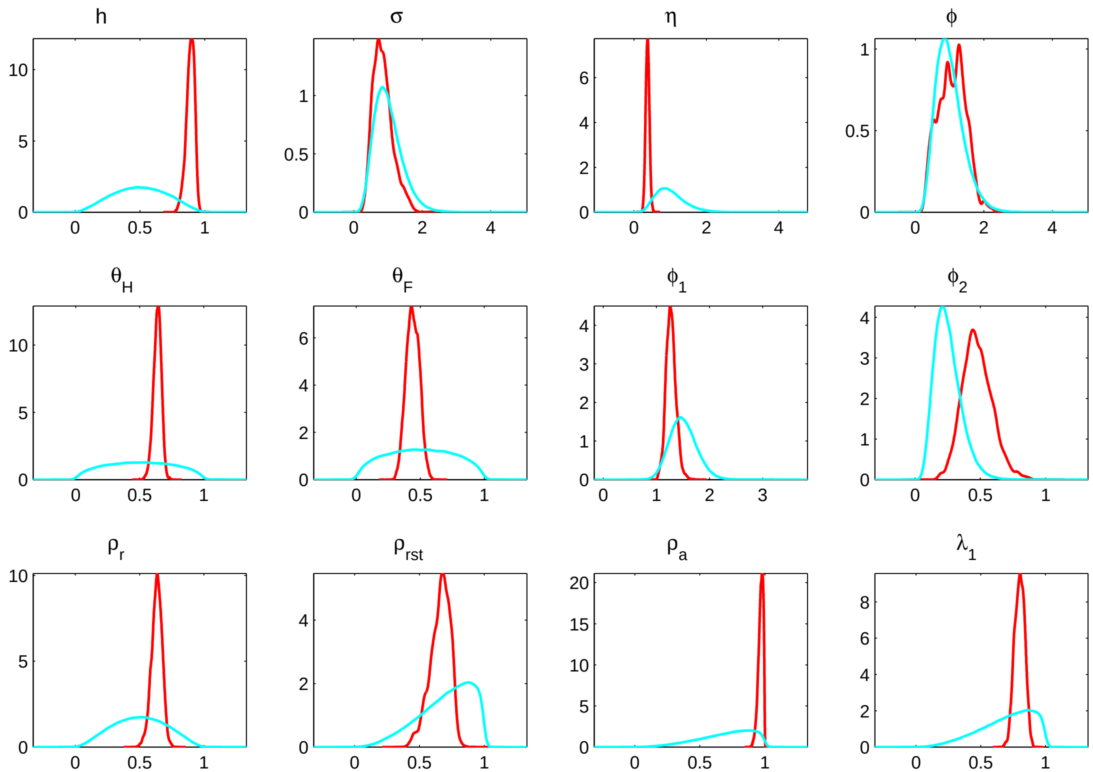
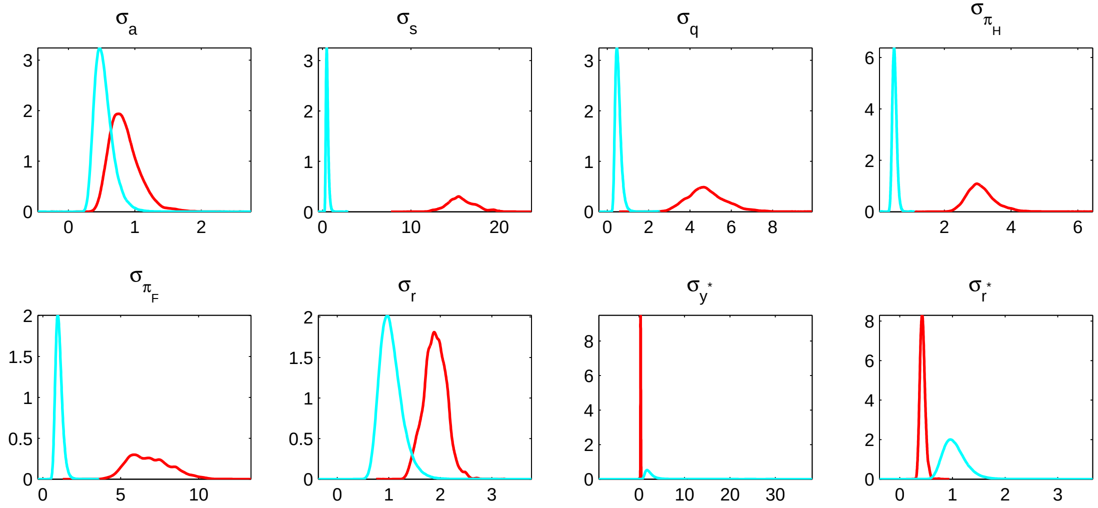


Figure 3: Posterior and Prior Marginal Density Plot of Shocks



8.3 PARAMETERS ANALYSIS

The results of the estimation in Table 2 show some basic characteristics of the Czech open economy and could be a source for the further more detailed analysis. All estimated parameters are statistically significant.

The parameter α was calibrated to the value 0.4 which expresses the degree of openness (as imported consumption to the total consumption). The value of the parameter is set to reflect the economic reality and conditions of the Czech open economy.[49]

The discount factor β was fixed at the value 0.99 – a households' rate of time preference is relatively high. The value is related to the interest rate[50]. This means that the households are very patient with respect to their future consumption. They believe that the utility from a future consumption is almost the same as the utility from this present period consumption.

The estimation a habit formation parameter for consumption implies relatively high degree of habit persistence ($h = 0.89$). The current consumption is importantly influenced by the consumption in the last period. The elasticity of intertemporal substitution in consumption is 1.22 (the inverse value of the elasticity is 0.82, parameter σ). The higher the elasticity, the less willing are households to accept deviations from their pattern of consumption behavior. According to the

[49] We used available data from 1999 to 2003. The total consumption shares 0.743 % of the Czech Gross Domestic Product according to our calculations based on the data from Czech Statistical Office (b) during the given period. This we use together with the data available from OECD (2005). According to our own calculations the total import consumption (import goods and services was in this period) is 220.1 Billion US dollars, the gross domestic product is 603.8 Billion US dollars (current prices). The calculation is following:

$$\alpha = \frac{imported\ consumption}{total\ consumption} = \frac{220.1}{0.743 \cdot 603.8} = 0.402$$

We use this value for our calibration.

[50] Usually the connection is described as $\beta = \frac{1}{1+rr^{eq}}$, where rr^{eq} is an equilibrium real interest rate. See e.g. Obstfeld and Rogoff (2002).

elasticity, the representative household is ready to postpone the current consumption to the next period with relatively very low increasing amount of the consumed goods in future.

The behavior of the household described by the values of parameters β, h, and σ seems to be consistent. The representative household has its consumption behavior and does not want to change it. The consumption depends on the last period consumption. There exist a possibility to postpone the consumption but for the household is meaningless – there is no extra consumption goods and no significant increase of the utility due to a higher future consumption.

The elasticity of substitution between domestic and foreign produced goods for consumption (parameter η) is 0.38. The value indicates a very low possibility of substitution between goods. The elasticity was very low in the first half of the 90s. Then it increased. Now the elasticity is higher, but the substitution between goods is still not easy and the consumption basket of the representative household depends still on the domestic produced goods. The exact rate for comparison is hard to compute. However, we suppose that the value lower than one is cacceptable.

The estimated elasticity of labor substitution is 0.93 (inverse elasticity $\varphi = 1.08$). It indicates slight non–elasticity of labor supply. The increase of the real wage by 1 % brings only 0.93 % increase of the labor supply. It is connected to the specific situation on the Czech labor market[51].

The probability of not changing price in a given period (we use quarterly data so in a quarter) is 0.64 for domestic producer (θ_H) and 0.44 for the importers (θ_F). In other words we can say that 64 % of domestic firms and 44 % of import firms do not reoptimize their prices every three months. The value of parameter θ can be transformed according to the relationship $\frac{1}{1-\theta}$ to an average duration. The average duration of the price contracts is almost 3 months (2.8 exactly) for the domestic producers and almost 2 months (1.8 exactly) for import firms. It is relatively short period but it approximately corresponds to the situation in the transforming Czech economy. Due to a lower stability of the Czech economy than in the developed foreign economies

[51] The elasticity is significantly influenced by the specific Czech conditions – low labor mobility, long run unemployment, a rise in the real wages is accompanied mainly by increase in labor productivity, ...

the duration of the foreign price contracts is shorter.

To complete the behavior of firms there is the parameter describing the development of the technological progress. The estimated value 0.97 for ρ_a is very high and the AR(1) process is very close to the random walk. The process converges very slowly to its steady state and each shock to this process influences its behavior very substantially. It means that every technological shock (new technologies, higher labor productivity, etc.) influences the level of the firm's output and the impact is long lasting. It corresponds to the real situation in the Czech economy (especially during its transformation) – the technological improvements had quite a long impact. The impacts of the technology shocks are not temporary but permanent shocks.

Next three parameters (ρ_r, ϕ_1 and ϕ_2) are connected to the conducting of the monetary policy by the central bank. The estimated modified Taylor rule is represented by the following relationship for all t:

$$r_t = 0.65\ r_{t-1} + (1 - 0.65)(1.27\ \pi_t + 0.47\ y_t) + \epsilon_t^r$$

The current level of the interest rate r_t is set with respect to the last period interest rate and the real situation in the economy. The value of parameter ρ_r is 0.65 which expresses a weight of a backward looking behavior in setting the interest rate. The second weight is connected to the changes in inflation and output. The ratio between inflation and output is $\frac{1.27}{0.47} = 2.7$. The central bank in the regime of the inflation targeting prefers to keep the current inflation at the level of the inflation target and it is preferred more than 2.7 times than the zero output gap. This strategy is supported further in the coefficients for the total influence of deviation of the overall inflation from the inflation target: $(1 - 0.65)\ 1.27 = 0.44$ and for the output gap: $(1 - 0.65)\ 0.47 = 0.16$. The reaction of the central bank to 1 % deviation of the inflation from the inflation target is 0.44 % change of interest rate, however 1 % deviation of output from its long run equilibrium (steady state) causes only 0.16 % change of the interest rate.

The behavior of the foreign economy is described by $\lambda_1 = 0.80$ and $\rho_{r^*} = 0.67$. The development of the foreign output is relatively high inertial and depends on its last development. It is a confirmation that the small open economy in the model is too small to influence the economy in the rest of the world. The development of the foreign real interest rate depends on its last period level. The estimated value of parameter is similar to the value for the domestic economy behavior of the interest rate – the foreign monetary policy is conducted in the same way, because the influence of the development of the inflation and output could enter the equation[52] through the foreign interest rate shock $\epsilon_t^{r^*}$.

According to the estimation of the shock parameters some relationship need not hold exactly – especially terms of trade, uncovered interest parity condition and New Keynesian Phillips Curve for domestic and import inflation.

The estimated posterior marginal densities in Figure 2 and 3 are higher and sharper than the prior marginal densities. It implies that our prior information about parameters and shocks are supported by the used data. The multiple peaks of some estimated posterior marginal density functions refer to the potential problems with traditional algorithms based on unimodal density functions and subsequently on searching the global maximum or on setting the initial conditions for estimation of the final values of parameters.

[52] The equation (66) has following form: $r_t^* - E_t\pi_{t+1}^* = \rho_{r^*}(r_{t-1}^* - \pi_t^*) + \epsilon_t^{r^*}$ for $t = 0, 1, 2, \ldots$.

9 ANALYSIS OF BEHAVIOR

In this section we provide more detailed analysis of the model behavior with using impulse responses in subsection 9.1 and then we present a forecast of the estimated model in subsection 9.2.

9.1 IMPULSE RESPONSES

To calculate the impulse responses we used 100 000 random draws of Markov Chains (MC) from the empirical posterior distribution. Figures 4 – 11 depict the impulse response functions of the economy to a unit increase in the structural and nonstructural shocks (labor productivity, import inflation, domestic inflation, interest rate, exchange rate, competitiveness, foreign output, foreign interest rate and domestic inflation). In each figure there is the median impulse response (blue line) and the 95^{th} percentile (red dotted lines) evaluated at each point of time.

Before we start the impulse responses analysis let us point out some characteristics:

- exchange rate is the nominal exchange rate expressed in terms of foreign currency per a domestic currency – an exchange rate increase is an appreciation of domestic currency (the indirect quotation),
- interest rate is the short run nominal interest rate influenced by the central bank according to the modified Taylor rule,
- import inflation is a change of prices (by imported firms) of imported goods from the foreign economy,
- law of one price gap (lop gap) is a difference between foreign price and domestic price of imports – the lop gap arises due to a positive or negative difference between the prices,[53]
- terms of trade are used as an index of the competitiveness of the domestic producers,
- foreign economy is a big economy and it cannot be influenced by the behavior of the domestic small open economy – the foreign interest rate and output are exogenous,

[53] Because we suppose that the foreign price is unchangeable the law of one price gap is a result of any change of the price of imports. In the analysis there is expressed only rate of deviation between foreign and import inflation. We depict the lop gap for every shock but do not analyze it in more details.

- we use the gap model, which means a decrease or increase of the variable in the context of is a decrease or increase from its steady state value,
- all shocks are temporary (not permanent) and the economy converges back to its steady state level.

9.1.1 LABOR PRODUCTIVITY SHOCK

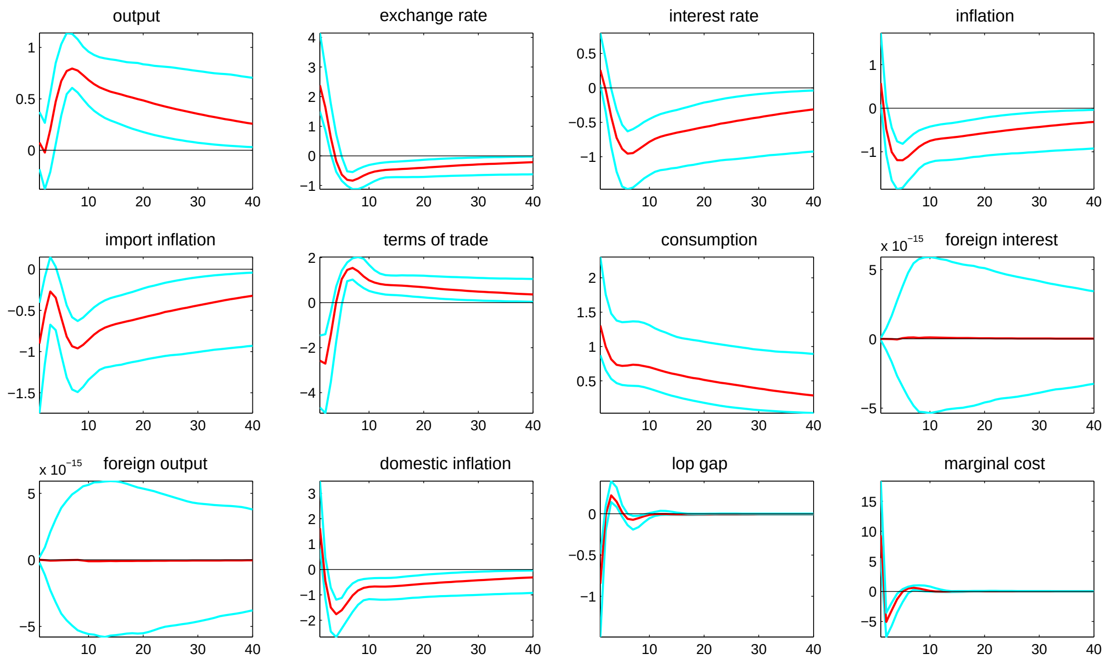
Figure 4 shows the impact of a temporary positive labor productivity shock. The shock influences the domestic producers – it decreases marginal costs very sharply (by 5 % due to lower total costs of the production), which enables to increase the production. Domestic inflation initially falls as the higher productivity helps to reduce the production costs (according to the Phillips Curve). The domestic inflation influences the overall inflation and the reaction of the central bank must be a lower interest rate (the monetary authority loosens the monetary policy to bring the overall inflation back to its target). This step has some consequences: according to the uncovered interest parity, there is an appreciation of the domestic currency. Due to higher domestic inflation a worsening of the competitiveness of the domestic producers on the foreign market (terms of trade are changed). Because the real interest rate is negative (nominal interest rate and the overall inflation decreases) the exchange rate must appreciate, which influences the prices of domestic exported goods (an improvement of the competitiveness).

The initial appreciation of the domestic currency influences the prices of the foreign imported goods to the domestic economy. The prices are lower. They makes a decrease of import inflation. Together with a drop of the domestic inflation there is a reduction of the overall inflation.

Higher productivity and an increase of output are in households' favor because they can consume more goods. Because the change is temporary, they try to smooth the consumption and spread the higher consumption to the future periods.

The labor productivity shock appears only for one period and the impact on marginal costs is very short (about 5 quarters). The total influence on the economy is important and lasts for a quite long period – more than 40 quarters for output, consumption and inflation. The long persistence can be explained by the relatively very high value

Figure 4: Impulse Response Functions from One Unit of Labor Productivity Innovation



of the estimated productivity shock parameter[54]. According to the previous description of behavior, the shock acts as a supply side shock. It is expected that this kind of shocks (although they are temporary) influences the economy for a longer period. In this respect, a change in productivity shock is important for the development of the economy.

9.1.2 IMPORT INFLATION SHOCK

Figure 5 shows the effect of a positive import inflation shock. A higher import inflation means that the price of imported goods becomes higher. It has direct impact on the overall inflation with the appropriate reaction of the central bank. If the import inflation is decreasing, the overall inflation and as well as the interest rate decrease to reach steady state values.

After a rise, there is a drop of the interest rate, that means an appreciation first and subsequently a depreciation of the exchange rate (according to the uncovered interest rate parity). The initial depreciation means higher foreign prices relative to the domestic prices and increase of the competitiveness of the domestic producers. The increased terms of trade together with a households' attempt to reduce their consumption of more expensive foreign goods and increase their consumption of domestic produced goods has positive impact on the domestic output. The short run increase of output is accompanied with a rise of the production prices (the domestic inflation must rise to hold the domestic inflation Phillips Curve). Higher prices support further growth of overall inflation.

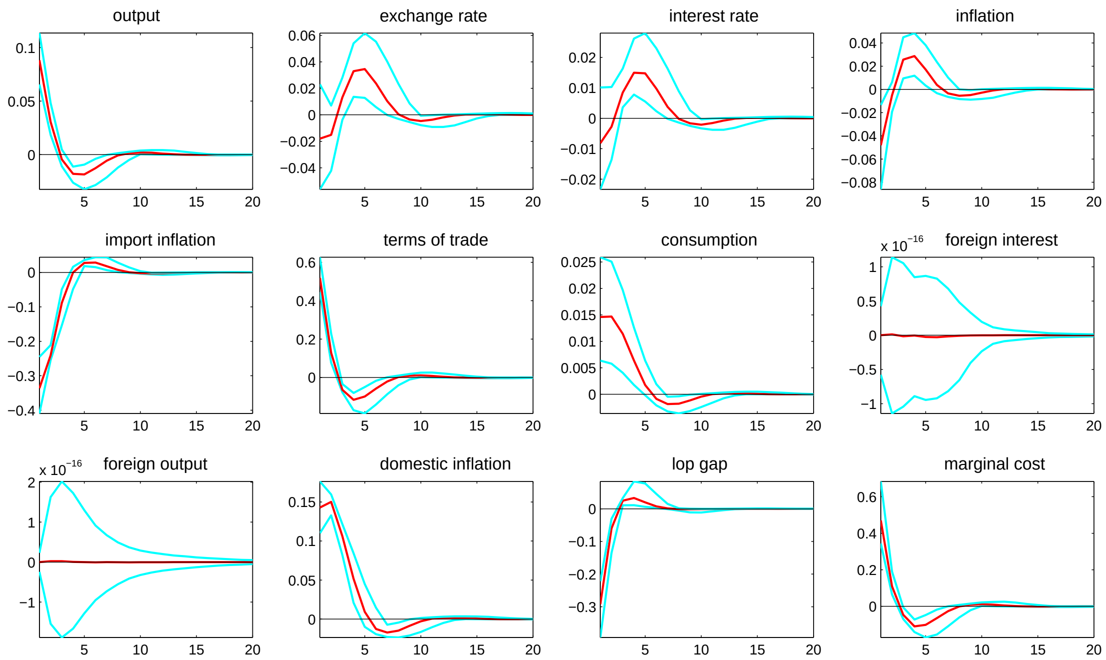
Because domestic inflation is higher and the import and overall inflation are higher as well, domestic firms must increase the prices of inputs for production, which in other words leads to higher marginal costs with subsequent negative effect on the output.

The higher level of output and the households' effort to substitute out more expensive foreign goods increases the domestic consumption too. However, the higher consumption is influenced by the intertemporal substitution. The development of the interest rate is opposite to higher

[54] The productivity shock is described by the equation (60) with estimated value $\rho_a = 0.97$ for all t:

$$a_t = 0.97\ a_{t-1} + \epsilon_t^a.$$

Figure 5: Impulse Response Functions from One Unit of Import Inflation Innovation



level of output and consumption and therefore the changes are very small.

As it emerges from this analysis, the domestic economy is small open economy influenced by the development of the foreign economy (in this situation by the import inflation). Such result is not surprising.

However, the negative influence is not very significant. The change in output (0.03 %), consumption (0.04 %), interest rate (0.13 %), and domestic inflation (0.13 %) is negligible. The significant impact is especially for the domestic producers through the marginal costs (0.6 %), overall inflation (0.3 %) and exchange rate with terms of trade (both over 0.37 %). On the other hand the duration of chnages is not long and after few periods they are decreasing. They disappear in 4 – 8 quarters and become smaller relatively very quickly.

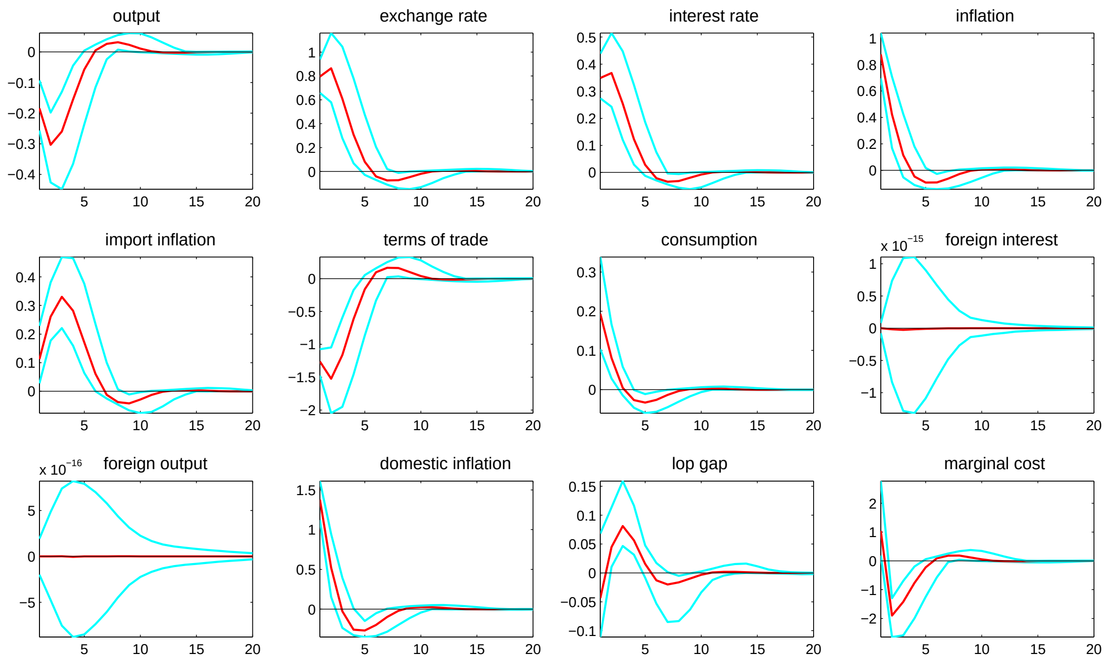
The consumer is not influenced by the import inflation shock very much. The domestic producers must react to some changes. They should increase their production (better competitiveness on the foreign market and higher domestic demand for the consumption goods) but the negative impact of higher value of costs of production acts reversely. The final situation depends on the specific conditions. In this model for the Czech economy there is a slightly positive effect. More generally it means that there are quite small threats for the economy coming from its openness.

9.1.3 DOMESTIC INFLATION SHOCK

Figure 6 shows the impact of a positive domestic inflation shock. The 1 % increase in the domestic inflation rises the overall inflation almost by 0.9 %. The domestic inflation shock is followed by the change of expectations, which increases the domestic inflation above 1.4 %.

Because the economy is small and open, the shock (relatively high jump of the domestic inflation) influences mainly the terms of trade by a decrease of the degree of competitiveness by 1.5 %. The change in import inflation is very small and the domestic inflation increases very sharply which worsens the competitive position of the domestic producers on foreign markets. This situation is supported by the reaction of the central bank. The increase of the overall inflation is accompanied by the 0.5 % increase in the interest rate. A change in the interest rate induces very strong appreciation of the domestic currency.

Figure 6: Impulse Response Functions from One Unit of Domestic Inflation Innovation



Higher interest rate forces to change the households' behavior due to intertemporal substitution. The high level of domestic inflation influences the behavior on the labor supply side too. The marginal costs of the domestic producers decrease after initial increase very sharply. The reason is simple. The vast majority of the costs are created by the real wages. If the domestic inflation increases by 1.4 % there is a sharp decrease of the real wages. Firms try to use cheaper foreign inputs for their production too. The result of this situation is a fall of marginal costs by 2 %.

The development of the output is influenced by the restrictive monetary policy, worse terms of trade connected with lower exports and very strong appreciation of the domestic currency by almost 1 %. The output decreases immediately by 0.3 % and then is coming back to its steady state.

Variables after reaching their peak achieve the steady state values not immediately but oscillate around it. It is caused by the great change in domestic and overall inflation. a change i domestic inflation influences the development of the terms of trade and exchange rate. The reaction of the central bank must be strong, which has an impact on the whole domestic economy. The development is influenced by the expectations. We suppose the result of the domestic inflation shock depends basically on the inflation expectations. If the monetary policy is trustworthy, the negative impact on the economy (measured by the domestic output gap) will be smaller and the development will be smoother (without any oscillations).

The domestic inflation shock is important for the analysis of a reaction of the monetary policy. It shows the conducting of the monetary policy as a reaction to the shock, which changes the inflation which subsequently differs from the inflation target. This impulse response seems to describe the behavior of the economy quite good.

9.1.4 NOMINAL INTEREST RATE SHOCK

Figure 7 shows the effect of a temporary positive nominal interest rate shock. It represents a monetary restriction of 1 % increase in the interest rate. Because it is a restrictive policy, the overall inflation and the output falls. The inflation decreases by 0.7 % and output by 0.35 % and then converge back to steady states. These changes appear immediately and are relatively short because they last only about

7 quarters. The policy has only instantaneous short–run impacts.

Because there is a negative impact on the output and inflation the initial 1 % interest rate shock results only as the 0.6 % change in the nominal interest rate.

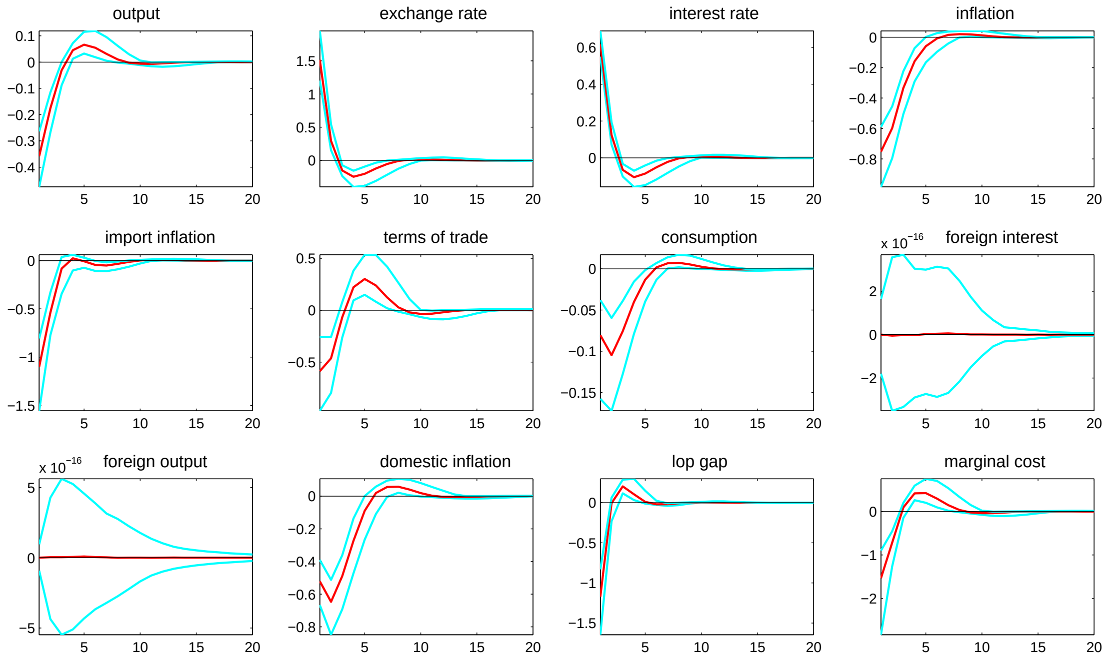
To analyze the development of the overall inflation in more details there is something specific – the probability intervals around the mean response are asymmetric. It is evident that the upper interval is closer to the median than the lower probability interval. This indicates more uncertainty about the lower limit of inflation. In this case the impact of the higher interest rate (aimed to the overall short run inflation reduction) on the overall inflation should be positive for the central bank.

Using the uncovered interest rate parity condition the higher interest rate is connected to the higher level of exchange rate. The appreciation is relatively very strong – about 1.5 %. It has impact on the producers and it is one of the transmission channels for the decrease of output. A reduction in the level of output is connected to lower production prices (domestic inflation). Moreover, the consumption is lower and the representative producer must set its prices lower to sell the whole production. Although the domestic (and also overall) inflation is lower the appreciation of domestic currency is so high that the competitive position for the domestic producers is worse. The situation measured by the terms of trade is improved in 3 quarters after some adjustment processes (the exchange rate is back at the steady state level and depreciates). The chnage in terms of trade influences the output too.

The effect of the appreciation has a positive impact on the exports of the world to the home economy. The 2 % appreciation of the domestic currency makes about 1 % decrease of import inflation. As soon as this advantage for importers dissappears, the import inflation is back to its steady state level. The higher amount of the imported consumption goods is another pressure to reduce the domestic impact.

A higher interest rate has a negative impact on the households' consumption. The future consumption becomes less attractive and the representative household tries to consume more in the present period. According to the intertemporal substitution, the higher real interest rate (initially the nominal interest rate is 0.6 %, the fall of the overall inflation is 0.65 % and the real interest rate is 1.25 %) substitutes out the future consumption into the present consumption. The change

Figure 7: Impulse Response Functions from One Unit of Interest Rate Innovation



in consumption is only about 0.1 % because under habit formation there is a smoothing of the level of consumption and the change in consumption.

The impact of the monetary restriction, represented by the interest rate increase influences the overall inflation. The reduction of the inflation depends (as it was shown in the previous situation) on the inflation expectations. Immediately after the monetary shock consumption and output falls. The impact is important because it is supported by a relatively high appreciation and a change in the terms of trade. The possible monetary shock (produced e.g. by the non–systematical monetary policy) influences the behavior of all agents very importantly and is undesirable.

9.1.5 EXCHANGE RATE SHOCK

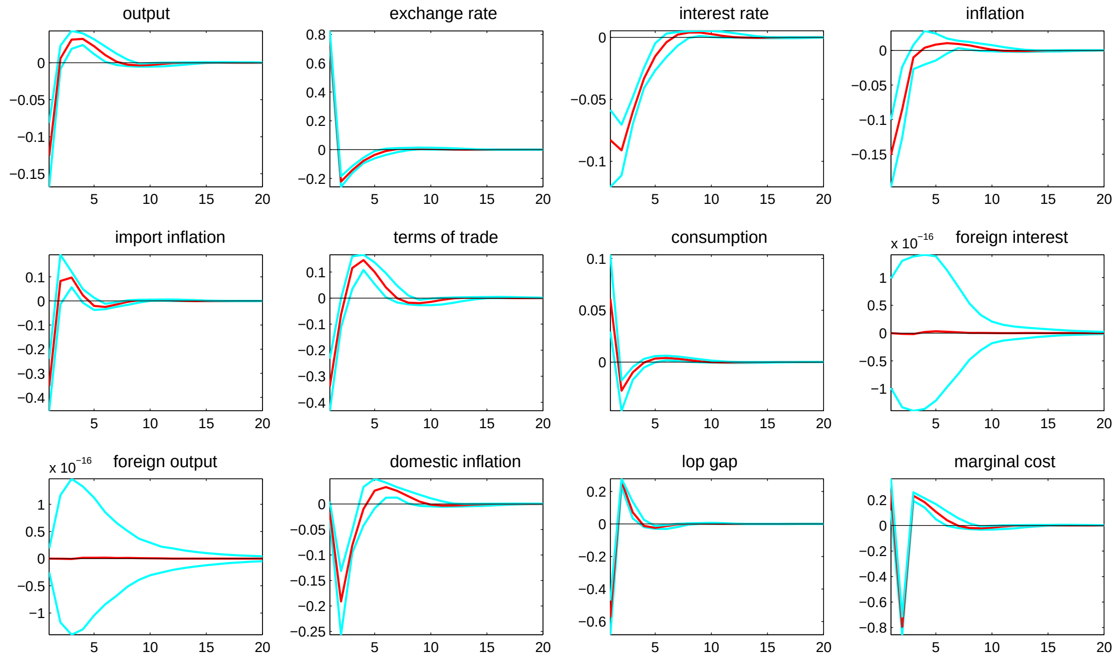
Figure 8 shows the effect of a temporary positive exchange rate shock. After this shock the exchange rate depreciates immediately. After this (only during one period) it starts to appreciate and reaches the steady state in 5 quarters. The exchange rate influences the development of the interest rate according to the uncovered interest rate parity, but the change in the interest rate is very small. The impact on the consumption through the interest rate is negligible, too.

The shock appreciates the exchange rate and it influences the production possibilities of the domestic producers. The appreciation of 0.8 % makes the domestic produced goods more expensive on the foreign markets. It reduces the domestic firms' output. The following depreciation makes the domestic goods cheaper for the foreign households and production can rise. This development influences the domestic inflation.

The strong appreciation influences the import inflation too. The imported goods become more expensive till the exchange rate starts to depreciate. The influence is about 0.35 % of the import desinflation. The similar development of the domestic and import inflation causes an initial drop in the overall inflation by 0.15 %.

Because the changes (a drop) of the import inflation are higher than in the domestic inflation, the competitiveness of the domestic producers is worsened by 0.3 % in the short run. By changing the development of the exchange rate the situation gets better for the domestic producers.

Figure 8: Impulse Response Functions from One Unit of Exchange Rate Innovation



A development of the marginal costs is crucial for the domestic firms. The marginal costs fall almost by 0.8 %. It is caused especially by the development of the terms of trade and the exchange rate. At the beginning the sharp fall of the marginal cost is influenced by the low level of the terms of trade and the strong appreciation of the domestic currency. Lower terms of trade and the appreciation make the imported inputs for the production cheaper. After further development this advantage for inputs disappears.

As it is evident the consequences of the exchange rate shock are very short lasting. After the first quarter it is almost eliminated and all variables converge quickly to the steady state. The deviations of the variable are in most cases negligible as well. To summarize the exchange rate shock should not be a shock causing important disturbances in the small open Czech economy.

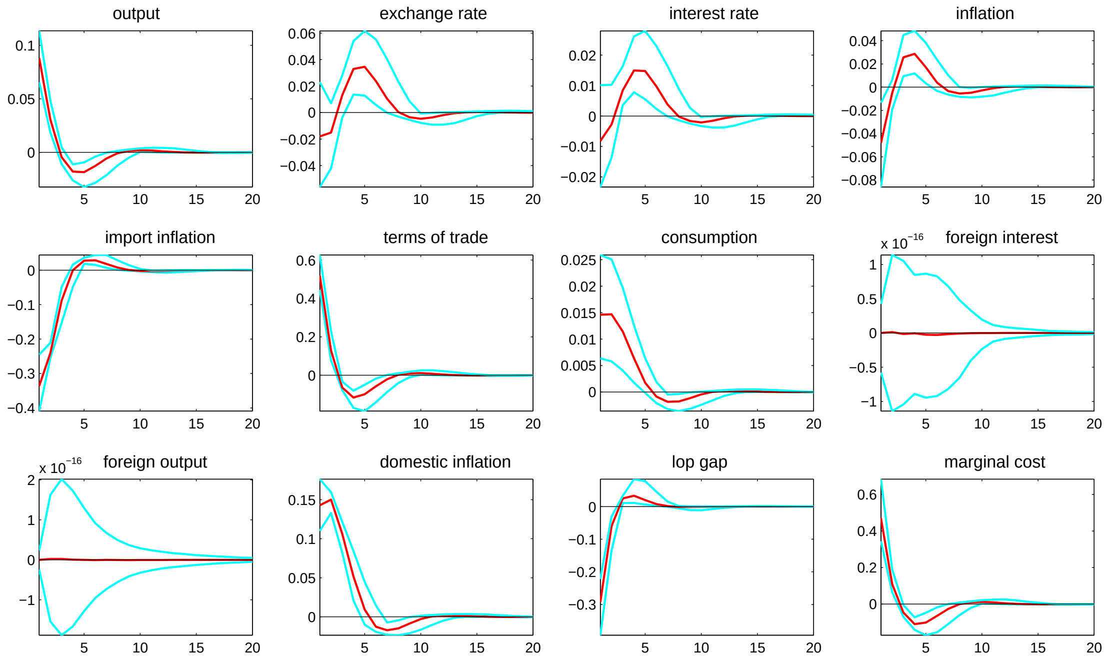
9.1.6 TERMS OF TRADE SHOCK

Figure 9 shows the effect of a temporary positive terms of trade shock. It means an increase of the degree of international competitiveness for the domestic producers. This situation motivates the domestic producers to increase their production (by 0.1 %) and export a part of their production. It is according to the Phillips Curve accompanied with the increase of the domestic inflation (by 0.15 %). The reaction to a worsening of the competitiveness is a reduction of the production and subsequently lowering of the domestic inflation. Because the marginal costs depend on the size of production, they have similar development and only the changes are bigger.

The higher production enables a potential higher consumption. However, the change in consumption is very low and negligible. It is not supported by the change in the interest rate.

The improvement of the degree of competitiveness for the domestic producers means an inverse situation for the foreign producers. Their position is worsening and foreign firms produce less goods. The price of the production is lower. The imported goods from the foreign economy must be cheaper which means lower import inflation. It is more than 0.3 % desinflation.

Figure 9: Impulse Response Functions from One Unit of Competitiveness Innovation



The mutual conditions of the terms of trade, domestic and import inflation bring negligible changes in the overall inflation, interest rate, and the exchange rate.

The terms of trade shock expresses the degree of competitiveness for the domestic producers. It is logical that its change influences especially behavior of the domestic producers (output, marginal costs and domestic inflation) and subsequently behavior of some producers oriented for exports in the foreign economy (it has an impact on the import inflation).

9.1.7 FOREIGN OUTPUT SHOCK

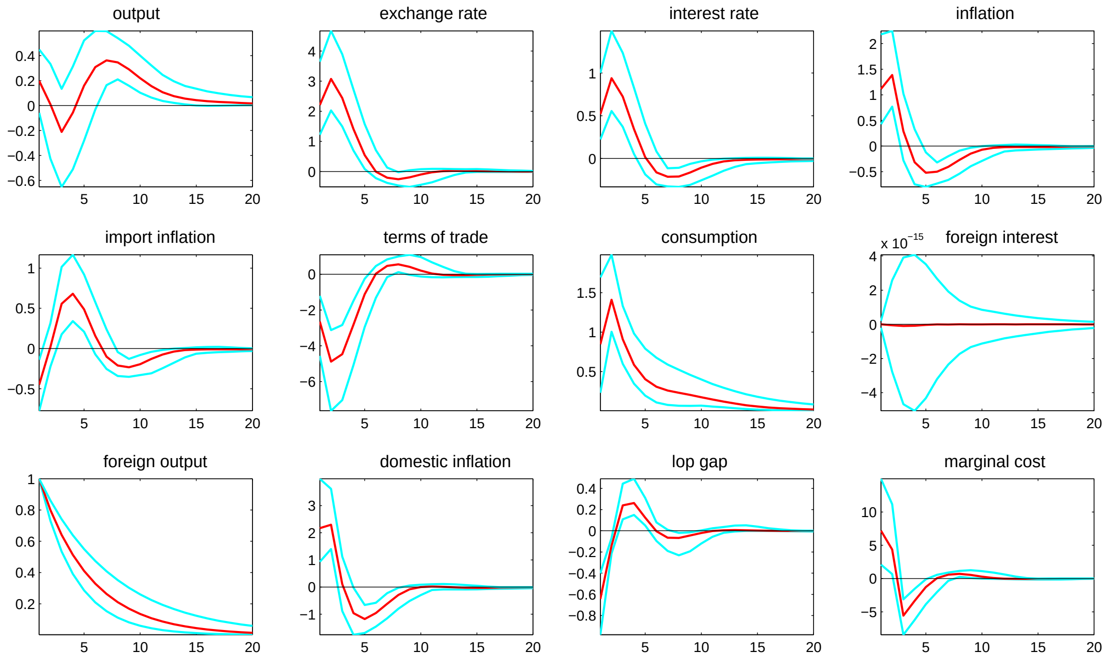
Figure 10 shows the impact of a temporary positive foreign output shock. The higher foreign production is divided between higher foreign and domestic consumption. In the small open economy, the consumption increases. Initially the foreign producers also decrease prices to make their production more attractive for the foreigners. The prices are lower by 0.5 % (measured by the import inflation). However, the higher production is connected with higher prices. The import lower prices increase. After 7 periods the import inflation is again below the steady state value (the increase of the foreign production is about 20 quarters) – the problems of foreign producers with realization pressures the prices downwards.

The higher foreign output enables to increase the domestic production by 0.2 %. However, the increase of the total world output is so high that there is no space for further increasing of production. After the initial higher production there is a break and its reduction. The development of the domestic inflation is according to the Phillips Curve (the 2 % increase). It influences the development of the marginal costs.

Higher domestic inflation influences the overall inflation which rises by almost 1.5 %. The reaction of the central monetary authority according to the modified Taylor rule is represented by the 1 % increase of the interest rate. The relatively very high interest rate with the possibility to consume more imported (and partially more domestic produced) goods results with an increase of the consumption by 1.5 %.

Higher interest rate causes the 3 % change in the exchange rate. With using the development analysis of the domestic and foreign inflation it is evident that the change in the index of competitiveness for the

Figure 10: Impulse Response Functions from One Unit of Foreign Output Innovation



domestic producers is not positive. The very strong and fast appreciation of the domestic currency and the decrease of the terms of trade are the main factors which reduces the domestic output. The domestic production is substituted out by the foreign and the domestic consumers have a better possibility to consume of imported goods.

It is evident that the foreign output shock is very important for the domestic economy. The changes in variables are in many cases higher than the initial value of the shock. On the other hand, the shock has a positive impact. The representative household consumes more goods. The behavior of the domestic producers imply higher production. The whole economy is influenced by the very strong monetary restriction (interest rate is higher by 1 %) aimed to hit the inflation target. In this respect the monetary policy reduces the potentially bigger gains (for consumers and producers) coming from this shock.

9.1.8 FOREIGN INTEREST RATE SHOCK

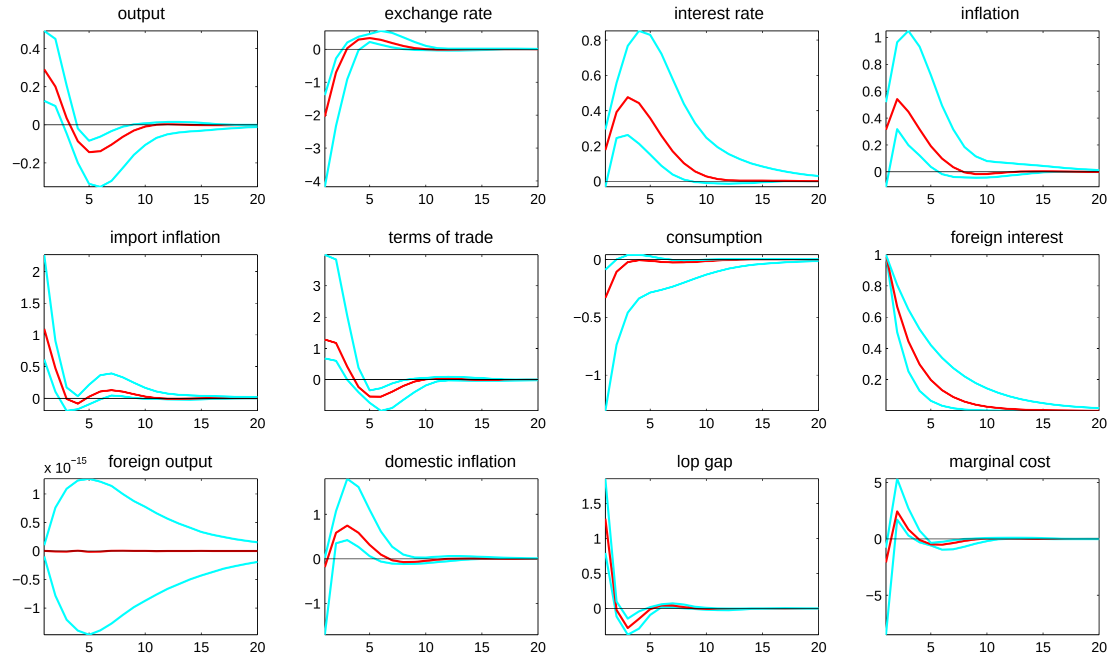
Figure 11 shows the impact of a temporary positive foreign interest rate shock. According to the uncovered interest rate parity condition the increase of the foreign interest rate by 1 % induces very strong depreciation of the domestic currency by 2 %. The exchange rate reaches its steady state when the shock disappears.

The relatively strong depreciation influences other variables. It enables to produce and export more goods to the domestic producers. It increases the domestic inflation and marginal costs of production too. The new value of the exchange rate makes also the imports more expensive for the domestic economy and the import inflation rises. Higher domestic and import inflation cause an increase of the overall inflation by about 0.55 %.

The higher overall inflation leads to a reaction of the central bank according to the modified Taylor rule. The result is an increase of the nominal interest rate by 0.5 %. It is necessary to note that the rise in the nominal interest rate is motivated by trying to equal the real domestic and foreign interest rate for the uncovered interest rate parity to hold.

The result of the foreign interest rate shock is evident. It influences importantly the overall, domestic and import inflation together with the exchange rate and domestic interest rate. The reason is simple – these variables ensure to hold the uncovered interest parity.

Figure 11: Impulse Response Functions from One Unit of Foreign Interest Rate Innovation



9.2 FORECAST

To analyze the behavior of the estimated model we introduce the forecast. Figure 12 shows the forecast of the overall inflation (π), nominal interest rate (R), real interest rate (r), output (y) and exchange rate (z) in terms of gap.

A future development of economy is simulated by an algorithm which produces the T period forecast of the state space representation of the economic model for a given (calibrated or estimated) parameter θ, present state of the economy x_0 and present state of the shock ϵ_0.

The presented forecast is a simultaneous extrapolation demonstration of the future trajectory of a model output based on a future trajectory of shocks chains simulated with respect to an estimation of means and variances on the past data.[55]

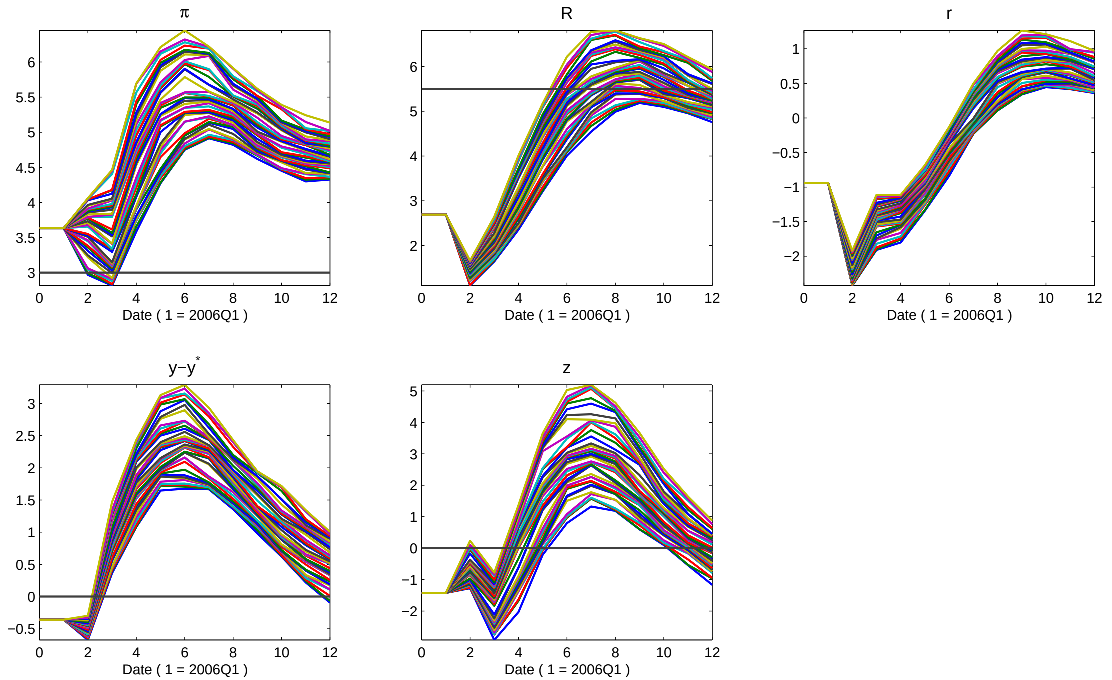
The forecast starts in the first quarter of 2006 at the estimated values of the model. The initial values for the inflation is 3.6 %, for the nominal interest rate is 2.6 %. The value for the real interest rate is -1.05 % ($r = R - \pi$), the output gap ($y - y^*$) is -0.4 % and the exchange rate is -1.4 %. In the figures there are plotted an average values of a variables for some last periods.

The forecast for the output growth is following. After two quarters there is a growth of output. The initial position for the growth is slightly below the steady state level. The economy reaches the peak of the growth in 6 quarters and we expect the value between 1.5 – 3.05 %. It reflects the fact that the Czech economy has a good background for the economic growth. The growth potential depends on the real situation and the decisions of the policy makers. However these possibilities are limited and after this period the estimated model predicts a recession of the output by about 0.4 % for a quarter.

The prediction of the inflation is not very clear. Initially it can fall or rise. It depends especially on the development of the import and domestic inflation, which enter the overall inflation. There is a possibility to reach or moreover to get below the inflation target. After the third quarter the forecast is clear – the inflation increases up to the value between 4.5 – 6.4 % in the sixth quarter. The high uncertainty about the future development is connected with the inflation expectations.

[55] It is necessary to simulate the shocks chains for a forecast interval on the base of variant scenarios of the expected shocks development for a real macroeconomic forecast.

Figure 12: Forecast of Inflation, Nominal and Real Interest Rate, Output and Exchange Rate



After this period, the inflation tends to a small decrease.

After the analysis of the forecast of output and inflation is clear that the output growth is based on the aggregate demand. The higher expenditures of the economic agents increase the output as well as the inflation. During the fall of output the inflation decreases, too.

The forecast of the nominal interest rate is consistent with the implementation of the monetary policy with respect to the modified Taylor rule. One–quarter–prediction of inflation is not clear but one–quarter–prediction of the output gap is evident – the output is still below its steady state. The central bank decreases the interest rate to support higher output (more exactly zero output gap). However, the inflation is increasing. The central bank desires to keep the overall inflation at the steady state level (the inflation target) and increases very rapidly the interest rate. It is clear that there is one quarter lag in the reaction of the central bank. To reduce the inflation in the sixth quarter the monetary authority increases the interest rate by 5 – 6.4 % with two–quarter lag. This rise in the rate is very high (from 2.6 to almost 5 – 6.4 %). This monetary restriction can be a source of the possible recession in the Czech economy.

The real interest rate forecast depends on the prediction of the nominal interest rate and the overall inflation (inflation expectations of the agents). Also a strong decrease at the beginning is interesting. The real interest rate becomes higher than zero probably after 7 quarter and is expected to further rise. From this point of view, the negative real interest rate can be one of the impulse for the higher output.

After a slight initial appreciation, there is a depreciation to the value -2.4 – -0.8 %. The exchange rate then goes back to its steady state and appreciates. The expected value of the exchange rate depends on the forecast development of the foreign economy. It is a reason for relatively very wide estimation possibilities of the appreciation. It is expected between 1 – 5.1 %. Then there is a tendency to return the exchange rate to its steady state by slow depreciation of the domestic currency. An appreciation is connected to an output growth although the appreciation makes the domestic goods for foreigners more cheaper. So the growth of the aggeragate demands is based on decisions of the domestic agents inside the economy.

The higher uncertainty about the future is expressed by the different (and wider) possibilities of the future development.

Figure 13: The Probability of Output Being 2 % Above and Below Average

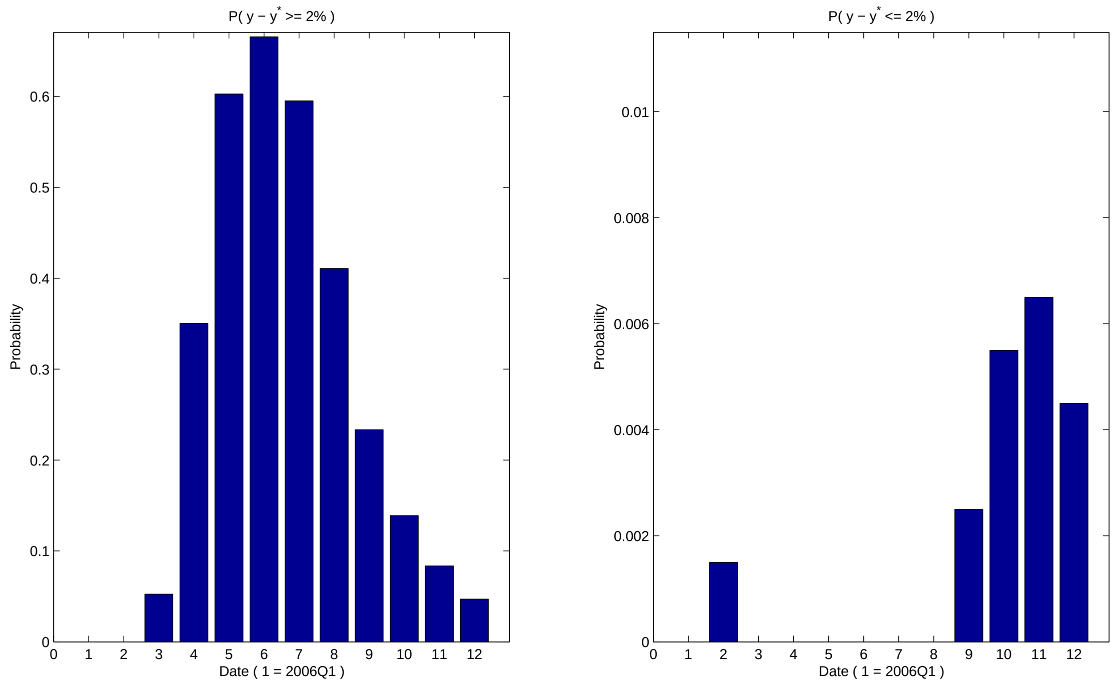


Figure 13 is devoted to the more detailed analysis of the output forecast. It plots the probability that the output gap is 2 % below or above its average. The figure indicates relatively very high probability that the output is more than 2 % above its steady state value (positive output gap) especially in prediction for 5 to 8 quarters.

10 CONCLUSION

In this paper we developed a small open DSGE model of the Czech economy. The model and the estimated parameters are used for the analysis of the behavior.

The model is New Open Economy Macroeconomics (NOEM) model. It is strictly based on the microeconomic foundations in the New Keynesian tradition. There are representative households, representative firms, a central bank, and the exogenous foreign sector. All agents optimize their behavior with respect to their constraints.

Parameters of the solved model are estimated by Bayesian method with Monte–Carlo simulation technique. This method combines prior information and the historical data. The desired result is confirmed by the fact that the posterior marginal densities of the parameters are sharper than the priors. The method gives well qualified parameters and reduces the model uncertainty.

The estimated parameters reflect the basic structural characteristics of the Czech economy. The behavior of the representative household is influenced by the higher level of the habit persistence in consumption, low elasticity of substitution between domestic and foreign consumption goods, and relatively low elasticity of the labor supply.

The average duration of the price contracts is 3 quarters for domestic producers and 2 quarters for importers. The domestic producers are importantly influenced by the high persistence of the used technologies.

A description of the monetary policy by the modified Taylor rule together with the estimated parameters provides a suitable way of an analysis of the Czech National Bank's behavior.

The impulse response functions offer a plausible way of explanation of the dynamic behavior of the economy. It describes the transmission mechanism of the monetary policy. The economy is influenced by the rest of the world – especially the world recession has a negative impact. The implementation of the monetary policy in this framework is more complicated, too. However, the result of the policy depends on expectations, which can make the implementation easier.

The quantified model gives a suitable approximation of the behavior of the Czech economy with respect to the results. It is relatively simple because there are no capital and investment rigidities, no government sector, no labor market and rigidities, no two sector of production (an intermediate–goods and finished–goods producing firms), only exogenous sector and no complete financial market. Despite these weaknesses, the model is able to describe basic dynamic behavior of the Czech economy and can be used e.g. for an improving of the implementation of the monetary policy.

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SUPPLEMENT 1: CALCULATIONS OF THE HOUSEHOLD'S OPTIMIZING BEHAVIOR

In this section we outline some calculations, which are connected to the behavior of a representative household. We are interested especially in the first order conditions of the optimizing behavior, a derivation of the demand functions, calculations connected to the overall Consumer Price Index and in the internal risk sharing condition.

THE FIRST ORDER CONDITIONS

A representative household maximizes its utility function:

$$E_t \sum_{t=0}^{\infty} \beta^t U(C_t, N_t) = E_t \sum_{t=0}^{\infty} \beta^t \left(\frac{(C_t - hC_{t-1})^{1-\sigma}}{1-\sigma} - \frac{N_t^{1+\varphi}}{1+\varphi} \right)$$

subject to a budget constraint (2):

$$P_t C_t + E_t \left(\frac{D_{t+1}}{1+r_t} \right) \leq D_t + W_t N_t$$

First we calculate the FOC for intratemporal consumption. The Lagrangian function has following form (a Lagrangian multiplier is Λ_t):

$$\begin{aligned} L(C_t, N_t, \Lambda_t) = E_t \sum_{t=0}^{\infty} \beta^t \left(\frac{(C_t - hC_{t-1})^{1-\sigma}}{1-\sigma} - \frac{N^{1+\varphi}}{1+\varphi} \right) - \\ - \sum_{t=0}^{\infty} \Lambda_t \beta^t \left(P_t C_t + E_t \left(\frac{D_{t+1}}{R_t} \right) - D_t - W_t N_t \right) \end{aligned}$$

and derivatives of the Lagrangian are:

$$\begin{aligned} \frac{\partial L(C_t, N_t, \Lambda_t)}{\partial C_t} = \beta^t (1-\sigma) \frac{(C_t - hC_{t-1})^{1-\sigma-1}}{1-\sigma} - \Lambda_t \beta^t P_t = 0 \\ \frac{(C_t - hC_{t-1})^{\sigma}}{P_t} = \Lambda_t \\ \frac{\partial L(C_t, N_t, \Lambda_t)}{\partial C_t} = \beta^t (1+\varphi) \frac{-N^{1+\varphi-1}}{1+\varphi} - \Lambda_t \beta^t (-W_t) = 0 \\ \frac{N^{\varphi}}{W_t} = \Lambda_t \end{aligned}$$

The FOC is following:

$$(C_t - hC_{t-1})^{-\sigma}\frac{W_t}{P_t} = N_t^{\varphi},$$

which is the equation (3). Log–linearizing of the equation (as it is introduced e.g. in Malley (2004)) yields:

$$\begin{aligned}
-\sigma\log(C_t - hC_{t-1}) + \log W_t - \log P_t &= \varphi \log N_t \\
-\sigma\frac{\Delta C_t - h\Delta C_{t-1}}{c - hc} + w_t - p_t &= \varphi n_t \\
-\sigma\frac{cc_t - hcc_{t-1}}{c(1-h)} - \varphi n_t &= -w_t + p_t \\
\sigma\frac{c(c_t - hc_{t-1})}{c(1-h)} + \varphi n_t &= w_t - p_t \\
\frac{\sigma}{1-h}(c_t - hc_{t-1}) + \varphi n_t &= w_t - p_t
\end{aligned}$$

The last equation corresponds to the equation (5).

The Euler (interetemporal) equation can be solved by the Bellman equation. To the household's optimizing problem we form a value function $w(D_t)$:

$$\begin{aligned}
w(D_t) &= E_t\sum_{t=0}^{\infty}\beta^t U(C_t, N_t) \\
&= \beta^0 U(C_t, N_t) + E_t\sum_{t=1}^{\infty}\beta^t U(C_t, N_t) \\
&= U(C_t, N_t) + \beta E_t w(D_{t+1})
\end{aligned}$$

The Bellman equation for this problem is:

$$w(D_t) = \max_{C_t}\ U(C_t, N_t) + \beta E_t w(D_{t+1})$$

or equivalently for $max\ U(C_t, N_t) = U(\tilde{C}_t, \tilde{N}_t)$:

$$w(D_t) = U(\tilde{C}_t, \tilde{N}_t) + \beta E_t w(D_{t+1})$$

We use the budget constraint in the form of $D_{t+1} = R_t(D_t + W_t N_t - P_t C_t)$ and plug it into the Bellman equation:

$$w(D_t) = U(\tilde{C}_t, \tilde{N}_t) + \beta E_t w(R_t(D_t + W_t N_t - P_t C_t))$$

The calculation of a condition for an optimality is:

$$\begin{aligned}\frac{\partial w(D_t)}{\partial C_t} &= \frac{\partial U(\tilde{C}_t, \tilde{N}_t)}{\partial C_t} + \frac{\partial \beta E_t w(D_{t+1})}{\partial D_{t+1}} \\ &= \frac{\partial U(\tilde{C}_t, \tilde{N}_t)}{\partial C_t} - \beta E_t \frac{\partial w(D_{t+1})}{\partial D_{t+1}} R_t P_t = 0\end{aligned}$$

So we can rewrite the optimality condition into the following form:

$$\frac{\partial U(\tilde{C}_t, \tilde{N}_t)}{\partial C_t} = \beta E_t \left[\frac{\partial w(D_{t+1})}{\partial D_{t+1}} R_t P_t\right]$$

From the result is clear that optimal value of $\tilde{C}_t$ is a function of D_t and it is possible to reformulate the function $w(.)$ in the Bellman equation:

$$w(D_t) = U(\tilde{C}_t(D_t), \tilde{N}_t) + \beta E_t w(D_{t+1})$$

A derivative of $w(D_t)$ with respect to D_t:

$$\frac{\partial w(D_t)}{\partial D_t} = \frac{\partial U(\tilde{C}_t, \tilde{N}_t)}{\partial C_t} \frac{\partial \tilde{C}_t}{\partial D_t} + \beta E_t \left[\frac{w(D_{t+1})}{\partial D_{t+1}} \left(1 - P_t \frac{\partial \tilde{C}_t}{\partial D_t}\right) R_t\right]$$

then we plug the optimality condition into the previous equation and get:

$$\begin{aligned}\frac{\partial w(D_t)}{\partial D_t} &= \beta E_t \left[\frac{\partial w(D_{t+1})}{\partial D_{t+1}} R_t P_t\right] \frac{\partial \tilde{C}_t}{\partial D_t} + \\ &+ \ \beta E_t \left[\frac{w(D_{t+1})}{\partial D_{t+1}} \left(1 - P_t \frac{\partial \tilde{C}_t}{\partial D_t}\right) R_t\right] \\ &= \beta E_t \left[\frac{\partial w(D_{t+1})}{\partial D_{t+1}} R_t\right] \left(P_t \frac{\partial \tilde{C}_t}{\partial D_t} + 1 - P_t \frac{\partial \tilde{C}_t}{\partial D_t}\right) \\ &= \beta E_t \frac{\partial w(D_{t+1})}{\partial D_{t+1}} R_t\end{aligned}$$

If we combine the previous equation together with the optimality condition we have:

$$\frac{\partial U(\tilde{C}_t, \tilde{N}_t)}{\partial C_t} = \beta E_t \left[\frac{\partial w(D_{t+1})}{\partial D_{t+1}} R_t P_t\right] \quad \beta E_t \left[\frac{\partial w(D_{t+1})}{\partial D_{t+1}} R_t\right] = \frac{\partial w(D_t)}{\partial D_t}$$

$$\frac{\partial U(\tilde{C}_t, \tilde{N}_t)}{\partial C_t} = \frac{\partial w(D_t)}{\partial D_t} P_t$$

and for $t+1$:

$$\frac{\partial U(\tilde{C}_{t+1}, \tilde{N}_{t+1})}{\partial C_{t+1}} = \frac{\partial w(D_{t+1})}{\partial D_{t+1}} P_{t+1}$$

Now we can plug it back to the optimality condition and use the utility function to derive following:

$$\begin{aligned}
\frac{\partial U(\tilde{C}_t, \tilde{N}_t)}{\partial C_t} &= \beta E_t \left[\frac{\partial w(D_{t+1})}{\partial D_{t+1}} \frac{R_t P_t}{P_{t+1}} \right] \\
\frac{\partial \left(\frac{(C_t - hC_{t-1})^{1-\sigma}}{1-\sigma} - \frac{N_t^{1+\varphi}}{1+\varphi} \right)}{\partial C_t} &= \beta E_t \left[\frac{\partial \left(\frac{(C_{t+1} - hC_t)^{1-\sigma}}{1-\sigma} - \frac{N_{t+1}^{1+\varphi}}{1+\varphi} \right)}{\partial D_{t+1}} \frac{R_t P_t}{P_{t+1}} \right] \\
(1-\sigma) \frac{(C_t - hC_{t-1})^{1-\sigma-1}}{1-\sigma} &= \beta E_t (1-\sigma) \frac{(C_{t+1} - hC_t)^{1-\sigma-1}}{1-\sigma} \frac{R_t P_t}{P_{t+1}} \\
(C_t - hC_{t-1})^{-\sigma} &= \beta R_t E_t (C_{t+1} - hC_t)^{-\sigma} \frac{P_t}{P_{t+1}} \\
1 &= \beta R_t E_t \left\{ \frac{P_t}{P_{t+1}} \left(\frac{C_{t+1} - hC_t}{C_t - hC_{t-1}} \right)^{-\sigma} \right\}
\end{aligned}$$

The equation is the Euler intertemporal equation (4). The log–linearizing of the equation around the steady state is following:

$$\begin{aligned}
\log 1 = \; & \log \beta + \log R_t + E_t[\log P_t - \log P_{t+1} - \\
- \; & \sigma \log(C_{t+1} - hC_t) + \sigma \log(C_t - hC_{t-1})] \\
0 = \; & 0 + r_t + E_t[p_t - p_{t+1} - \\
- \; & \sigma \log(C_{t+1} - hC_t) + \sigma \log(C_t - hC_{t-1})] \\
0 = \; & 0 + r_t + E_t \left[-\pi_{t+1} - \sigma \frac{c_{t+1} - hc_t}{1-h} + \sigma \frac{c_t - hc_{t-1}}{1-h} \right] \\
0 = \; & E_t \left[(r_t - \pi_{t+1}) - \frac{\sigma}{1-h}(c_{t+1} - hc_t) + \frac{\sigma}{1-h}(c_t - hc_{t-1}) \right] \\
0 = \; & E_t \left[\frac{1-h}{\sigma}(r_t - \pi_{t+1}) - (c_{t+1} - hc_t) + (c_t - hc_{t-1}) \right] \\
c_t - hc_{t-1} = \; & E_t(c_{t+1} - hc_t) - \frac{1-h}{\sigma} E_t(r_t - \pi_{t+1})
\end{aligned}$$

ALLOCATION FUNCTIONS OF EXPENDITURES

A representative household decides about optimal allocation of expenditures between domestic and foreign goods[56]. It maximizes the total consumption expressed by the equation (7):

$$C_t \equiv \left((1-\alpha)^{\frac{1}{\eta}} C_{H,t}^{\frac{\eta-1}{\eta}} + \alpha^{\frac{1}{\eta}} C_{F,t}^{\frac{\eta-1}{\eta}}\right)^{\frac{\eta}{\eta-1}}$$

subject to its expenditure constraint:

$$P_t C_t = P_{H,t} C_{H,t} + P_{F,t} C_{F,t}$$

The Lagrangian and its partial derivatives take the following forms (with a multiplier Λ_t):

$$\begin{aligned} L_t(C_{H,t}, C_{F,t}, \Lambda_t) = & \left((1-\alpha)^{\frac{1}{\eta}} C_{H,t}^{\frac{\eta-1}{\eta}} + \alpha^{\frac{1}{\eta}} C_{F,t}^{\frac{\eta-1}{\eta}}\right)^{\frac{\eta}{\eta-1}} + \\ & + \Lambda_t (P_t C_t - P_{H,t} C_{H,t} - P_{F,t} C_{F,t}) \\ \frac{\partial L_t}{\partial C_{H,t}} = & \frac{\eta}{\eta-1} \left((1-\alpha)^{\frac{1}{\eta}} C_{H,t}^{\frac{\eta-1}{\eta}} + \alpha^{\frac{1}{\eta}} C_{F,t}^{\frac{\eta-1}{\eta}}\right)^{\frac{\eta}{\eta-1}-1} \cdot \\ & \cdot (1-\alpha)^{\frac{1}{\eta}} \frac{\eta-1}{\eta} C_{H,t}^{\frac{\eta-1}{\eta}-1} - \Lambda_t P_{H,t} \\ \frac{\partial L_t}{\partial C_{F,t}} = & \frac{\eta}{\eta-1} \left((1-\alpha)^{\frac{1}{\eta}} C_{H,t}^{\frac{\eta-1}{\eta}} + \alpha^{\frac{1}{\eta}} C_{F,t}^{\frac{\eta-1}{\eta}}\right)^{\frac{\eta}{\eta-1}-1} \cdot \\ & \cdot \alpha^{\frac{1}{\eta}} \frac{\eta-1}{\eta} C_{F,t}^{\frac{\eta-1}{\eta}-1} - \Lambda_t P_{F,t} \end{aligned}$$

Both derivatives equal zero:

$$\begin{aligned} 0 = & \frac{\eta}{\eta-1} \left((1-\alpha)^{\frac{1}{\eta}} C_{H,t}^{\frac{\eta-1}{\eta}} + \alpha^{\frac{1}{\eta}} C_{F,t}^{\frac{\eta-1}{\eta}}\right)^{\frac{1}{\eta-1}} (1-\alpha)^{\frac{1}{\eta}} \frac{\eta-1}{\eta} C_{H,t}^{-\frac{1}{\eta}} - \\ & - \Lambda_t P_{H,t} \\ \Lambda_t = & \frac{1}{P_{H,t}} \frac{\eta}{\eta-1} \left((1-\alpha)^{\frac{1}{\eta}} C_{H,t}^{\frac{\eta-1}{\eta}} + \alpha^{\frac{1}{\eta}} C_{F,t}^{\frac{\eta-1}{\eta}}\right)^{\frac{1}{\eta-1}} (1-\alpha)^{\frac{1}{\eta}} \frac{\eta-1}{\eta} C_{H,t}^{-\frac{1}{\eta}} \end{aligned}$$

[56] The optimizing problem can be solved as a dual problem too. The representative household tries to minimize the total expenditure $P_t C_t = P_{H,t} C_{H,t} + P_{F,t} C_{F,t}$ subject to a possible consumption $C_t \equiv \left((1-\alpha)^{\frac{1}{\eta}} C_{H,t}^{\frac{\eta-1}{\eta}} + \alpha^{\frac{1}{\eta}} C_{F,t}^{\frac{\eta-1}{\eta}}\right)^{\frac{\eta}{\eta-1}}$.

and

$$
\begin{aligned}
0 = & \frac{\eta}{\eta-1}\left((1-\alpha)^{\frac{1}{\eta}} C_{H,t}^{\frac{\eta-1}{\eta}} + \alpha^{\frac{1}{\eta}} C_{F,t}^{\frac{\eta-1}{\eta}}\right)^{\frac{1}{\eta-1}} \alpha^{\frac{1}{\eta}} \frac{\eta-1}{\eta} C_{F,t}^{-\frac{1}{\eta}} - \\
& - \Lambda_t P_{F,t} \\
\Lambda_t = & \frac{1}{P_{F,t}} \frac{\eta}{\eta-1}\left((1-\alpha)^{\frac{1}{\eta}} C_{H,t}^{\frac{\eta-1}{\eta}} + \alpha^{\frac{1}{\eta}} C_{F,t}^{\frac{\eta-1}{\eta}}\right)^{\frac{1}{\eta-1}} \alpha^{\frac{1}{\eta}} \frac{\eta-1}{\eta} C_{F,t}^{-\frac{1}{\eta}}
\end{aligned}
$$

Then we equal both formulas for Λ_t, which yields:

$$
\begin{aligned}
\alpha^{\frac{1}{\eta}} C_{F,t}^{-\frac{1}{\eta}} \frac{1}{P_{F,t}} &= (1-\alpha)^{\frac{1}{\eta}} C_{H,t}^{-\frac{1}{\eta}} \frac{1}{P_{H,t}} \\
\frac{P_{H,t}}{P_{F,t}} &= \frac{\alpha^{-\frac{1}{\eta}} C_{H,t}^{-\frac{1}{\eta}}}{(1-\alpha)^{-\frac{1}{\eta}} C_{F,t}^{-\frac{1}{\eta}}} \\
\left(\frac{P_{H,t}}{P_{F,t}}\right)^{-\eta} &= \frac{\alpha C_{H,t}}{(1-\alpha) C_{F,t}}
\end{aligned}
$$

Now from the previous equation we formulate demand functions with using $P_t C_t = P_{H,t} C_{H,t} + P_{F,t} C_{F,t}$:

- for domestic goods:

$$
\left(\frac{P_{H,t}}{P_{F,t}}\right)^{-\eta} = \frac{\alpha C_{H,t}}{(1-\alpha) C_{F,t}} \qquad C_{F,t} = \frac{P_t C_t - P_{H,t} C_{H,t}}{P_{F,t}}
$$

$$
\begin{aligned}
\left(\frac{P_{H,t}}{P_{F,t}}\right)^{-\eta} &= \frac{\alpha C_{H,t}}{(1-\alpha)\frac{P_t C_t - P_{H,t} C_{H,t}}{P_{F,t}}} \\
\left(\frac{P_{H,t}}{P_{F,t}}\right)^{-\eta} &= \frac{\alpha P_{F,t} C_{H,t}}{(1-\alpha) P_t C_t - P_{H,t} C_{H,t}} \\
\left(\frac{P_{H,t}}{P_{F,t}}\right)^{-\eta} (1-\alpha) P_t C_t &= C_{H,t}\left[\alpha P_{F,t} + \left(\frac{P_{H,t}}{P_{F,t}}\right)^{-\eta} (1-\alpha) P_{H,t}\right] \\
C_{H,t} &= \frac{\left(\frac{P_{H,t}}{P_{F,t}}\right)^{-\eta} (1-\alpha) P_t C_t}{\alpha P_{F,t} + \left(\frac{P_{H,t}}{P_{F,t}}\right)^{-\eta} (1-\alpha) P_{H,t}}
\end{aligned}
$$

$$C_{H,t} = \frac{\left(\frac{P_{H,t}}{P_{F,t}}\right)^{-\eta}(1-\alpha)\frac{P_tC_t}{P_{F,t}}}{\alpha + \left(\frac{P_{H,t}}{P_{F,t}}\right)^{-\eta}\frac{P_{H,t}}{P_{F,t}}(1-\alpha)}$$

$$C_{H,t} = \frac{(1-\alpha)\frac{P_tC_t}{P_{F,t}}\left(\frac{P_{H,t}}{P_{F,t}}\right)^{-\eta}}{\alpha + (1-\alpha)\left(\frac{P_{H,t}}{P_{F,t}}\right)^{1-\eta}}$$

- for foreign goods:

$$\left(\frac{P_{H,t}}{P_{F,t}}\right)^{-\eta} = \frac{\alpha C_{H,t}}{(1-\alpha)C_{F,t}} \qquad C_{H,t} = \frac{P_tC_t - P_{F,t}C_{F,t}}{P_{H,t}}$$

$$\left(\frac{P_{H,t}}{P_{F,t}}\right)^{-\eta} = \frac{\alpha\frac{P_tC_t - P_{F,t}C_{F,t}}{P_{H,t}}}{(1-\alpha)C_{F,t}}$$

$$\left(\frac{P_{H,t}}{P_{F,t}}\right)^{-\eta} = \frac{\alpha(P_tC_t - P_{F,t}C_{F,t})}{(1-\alpha)P_{H,t}C_{F,t}}$$

$$\left(\frac{P_{H,t}}{P_{F,t}}\right)^{-\eta}(1-\alpha)P_{H,t}C_{F,t} = \alpha(P_tC_t - P_{F,t}C_{F,t})$$

$$\left[\left(\frac{P_{H,t}}{P_{F,t}}\right)^{-\eta}(1-\alpha)P_{H,t} + \alpha P_{F,t}\right]C_{F,t} = \alpha P_tC_t$$

$$C_{F,t} = \frac{\alpha P_tC_t}{\left(\frac{P_{H,t}}{P_{F,t}}\right)^{-\eta}(1-\alpha)P_{H,t} + \alpha P_{F,t}}$$

$$C_{F,t} = \frac{\alpha\frac{P_tC_t}{P_{F,t}}}{\alpha + (1-\alpha)\left(\frac{P_{H,t}}{P_{F,t}}\right)^{1-\eta}}$$

Now we use the relationship for the overall CPI (11):

$$P_t \equiv \{(1-\alpha)P_{H,t}^{1-\eta} + \alpha P_{F,t}^{1-\eta}\}^{\frac{1}{1-\eta}}$$

$$P_t^{1-\eta} = (1-\alpha)P_{H,t}^{1-\eta} + \alpha P_{F,t}^{1-\eta}$$

$$\left(\frac{P_t}{P_{F,t}}\right)^{1-\eta} = (1-\alpha)\left(\frac{P_{H,t}}{P_{F,t}}\right)^{1-\eta} + \alpha$$

$$(1-\alpha)\left(\frac{P_{H,t}}{P_{F,t}}\right)^{1-\eta} = \left(\frac{P_t}{P_{F,t}}\right)^{1-\eta} - \alpha$$

$$\left(\frac{P_{H,t}}{P_{F,t}}\right)^{1-\eta} = \frac{1}{(1-\alpha)}\left[\left(\frac{P_t}{P_{F,t}}\right)^{1-\eta} - \alpha\right]$$

We substitute the previous equation back to the demand functions:

- for domestic consumption goods:

$$
\begin{aligned}
C_{H,t} &= \frac{(1-\alpha)\frac{P_tC_t}{P_{F,t}}\left(\frac{P_{H,t}}{P_{F,t}}\right)^{-\eta}}{\alpha+(1-\alpha)\left(\frac{P_{H,t}}{P_{F,t}}\right)^{1-\eta}} \\
&= \frac{(1-\alpha)\frac{P_tC_t}{P_{F,t}}\left(\frac{P_{H,t}}{P_{F,t}}\right)^{-\eta}}{\alpha+(1-\alpha)\frac{1}{(1-\alpha)}\left[\left(\frac{P_t}{P_{F,t}}\right)^{1-\eta}-\alpha\right]} \\
&= (1-\alpha)\frac{P_tC_t}{P_{F,t}}\left(\frac{P_{H,t}}{P_{F,t}}\right)^{-\eta}\left(\frac{P_{F,t}}{P_t}\right)^{1-\eta} \\
&= (1-\alpha)\frac{P_tC_t}{P_{F,t}}\left(\frac{P_{H,t}}{P_{F,t}}\right)^{-\eta}\left(\frac{P_{F,t}}{P_t}\right)^{-\eta}\frac{P_{F,t}}{P_t} \\
&= (1-\alpha)\left(\frac{P_{H,t}}{P_t}\right)^{-\eta}C_t
\end{aligned}
$$

- for foreign consumption goods:

$$
\begin{aligned}
C_{F,t} &= \frac{\alpha\frac{P_tC_t}{P_{F,t}}}{\alpha+(1-\alpha)\left(\frac{P_{H,t}}{P_{F,t}}\right)^{1-\eta}} \\
&= \frac{\alpha\frac{P_tC_t}{P_{F,t}}}{\alpha+(1-\alpha)\frac{1}{(1-\alpha)}\left[\left(\frac{P_t}{P_{F,t}}\right)^{1-\eta}-\alpha\right]} \\
&= \frac{\alpha\frac{P_tC_t}{P_{F,t}}}{\left(\frac{P_t}{P_{F,t}}\right)^{1-\eta}} \\
&= \alpha\frac{P_tC_t}{P_{F,t}}\left(\frac{P_{F,t}}{P_t}\right)^{1-\eta} \\
&= \alpha\frac{P_tC_t}{P_{F,t}}\left(\frac{P_{F,t}}{P_t}\right)^{-\eta}\left(\frac{P_{F,t}}{P_t}\right) \\
&= \alpha\left(\frac{P_{F,t}}{P_t}\right)^{-\eta}C_t
\end{aligned}
$$

Both optimal allocation functions of expenditures between domestic and imported goods correspond to the relationships (9).

DEMAND FUNCTIONS

In this subsection we derive the demand function for the i-th domestic consumption good. The whole procedure is similar for the demand function for the foreign goods.

A representative household optimizes its behavior. It tries to minimize its expenditure for consumption of the domestic goods:

$$\int_0^1 P_{H,t}(i)C_{H,t}(i)di$$

subject to its constraint expressed as:

$$C_{H,t} \equiv \left(\int_0^1 C_{H,t}(i)^{\frac{\epsilon-1}{\epsilon}}di\right)^{\frac{\epsilon}{\epsilon-1}}$$

The Lagrangian function is in the following form (Λ_t is a Lagrangian multiplier):

$$\begin{aligned} L_t(C_{H,t}(i),\Lambda_t) = & \int_0^1 P_{H,t}(i)C_{H,t}(i)di- \\ & -\Lambda_t\left\{\left(\int_0^1 C_{H,t}(i)^{\frac{\epsilon-1}{\epsilon}}di\right)^{\frac{\epsilon}{\epsilon-1}} - C_{H,t}\right\} \end{aligned}$$

and then

$$\begin{aligned} \frac{\partial L_t(C_{H,t}(i),\Lambda)}{\partial C_{H,t}(i)} &= P_{H,t}(i) - \Lambda_t\left\{\frac{\epsilon}{\epsilon-1}\left(\int_0^1 C_{H,t}(i)^{\frac{\epsilon-1}{\epsilon}}di\right)^{\frac{\epsilon}{\epsilon-1}-1}\cdot\right. \\ &\qquad \left.\cdot\frac{\epsilon-1}{\epsilon}C_{H,t}(i)^{\frac{\epsilon-1}{\epsilon}-1}\right\} \\ &= P_{H,t}(i) - \Lambda_t\left(\int_0^1 C_{H,t}(i)^{\frac{\epsilon-1}{\epsilon}}di\right)^{\frac{1}{\epsilon-1}} C_{H,t}(i)^{-\frac{1}{\epsilon}} \end{aligned}$$

The first order condition (FOC) equals zero and subsequently is multiplied by $C_{H,t}(i)$:

$$\begin{aligned} 0 &= P_{H,t}(i) - \Lambda_t\left(\int_0^1 C_{H,t}(i)^{\frac{\epsilon-1}{\epsilon}}di\right)^{\frac{1}{\epsilon-1}} C_{H,t}(i)^{-\frac{1}{\epsilon}} \\ P_{H,t}(i) &= \Lambda_t\left(\int_0^1 C_{H,t}(i)^{\frac{\epsilon-1}{\epsilon}}di\right)^{\frac{1}{\epsilon-1}} C_{H,t}(i)^{-\frac{1}{\epsilon}} \\ P_{H,t}(i)C_{H,t}(i) &= \Lambda_t\left(\int_0^1 C_{H,t}(i)^{\frac{\epsilon-1}{\epsilon}}di\right)^{\frac{1}{\epsilon-1}} C_{H,t}(i)^{-\frac{1}{\epsilon}}C_{H,t}(i) \end{aligned}$$

$$P_{H,t}(i)C_{H,t}(i) = \Lambda_t \left(\int_0^1 C_{H,t}(i)^{\frac{\epsilon-1}{\epsilon}} di \right)^{\frac{1}{\epsilon-1}} C_{H,t}(i)^{\frac{\epsilon-1}{\epsilon}}$$

Then both sides of last equation are integrated and the constraint $C_{H,t} \equiv \left(\int_0^1 C_{H,t}(i)^{\frac{\epsilon-1}{\epsilon}} di \right)^{\frac{\epsilon}{\epsilon-1}}$ is used:

$$\begin{aligned}
\int_0^1 P_{H,t}(i)C_{H,t}(i)di &= \int_0^1 \Lambda_t \left(\int_0^1 C_{H,t}(i)^{\frac{\epsilon-1}{\epsilon}} di \right)^{\frac{1}{\epsilon-1}} C_{H,t}(i)^{\frac{\epsilon-1}{\epsilon}} di \\
\int_0^1 P_{H,t}(i)C_{H,t}(i)di &= \int_0^1 \Lambda_t \left(\int_0^1 C_{H,t}(i)^{\frac{\epsilon-1}{\epsilon}} di \right)^{\frac{1}{\epsilon-1}} C_{H,t}(i)^{\frac{\epsilon-1}{\epsilon}} di \\
\int_0^1 P_{H,t}(i)C_{H,t}(i)di &= \int_0^1 \Lambda_t C_{H,t}^{\frac{1}{\epsilon}} C_{H,t}(i)^{\frac{\epsilon-1}{\epsilon}} di \\
P_{H,t}C_{H,t} &= \Lambda_t C_{H,t}^{\frac{1}{\epsilon}} \int_0^1 C_{H,t}(i)^{\frac{\epsilon-1}{\epsilon}} di \\
P_{H,t}C_{H,t} &= \Lambda_t C_{H,t}^{\frac{1}{\epsilon}} C_{H,t}^{\frac{\epsilon-1}{\epsilon}} \\
P_{H,t}C_{H,t} &= \Lambda_t C_{H,t} \\
P_{H,t} &= \Lambda_t
\end{aligned}$$

The multiplier is identical to the domestic price index ($\Lambda_t = P_{H,t}$). We plug it back to the FOC:

$$\begin{aligned}
P_{H,t}(i)C_{H,t}(i) &= \Lambda_t \left(\int_0^1 C_{H,t}(i)^{\frac{\epsilon-1}{\epsilon}} di \right)^{\frac{1}{\epsilon-1}} C_{H,t}(i)^{\frac{\epsilon-1}{\epsilon}} \\
P_{H,t}(i)C_{H,t}(i) &= P_{H,t} \left(\int_0^1 C_{H,t}(i)^{\frac{\epsilon-1}{\epsilon}} di \right)^{\frac{1}{\epsilon-1}} C_{H,t}(i)^{\frac{\epsilon-1}{\epsilon}} \\
P_{H,t}(i)C_{H,t}(i) &= P_{H,t} C_{H,t}^{\frac{1}{\epsilon}} C_{H,t}(i)^{\frac{\epsilon-1}{\epsilon}} \\
\frac{P_{H,t}(i)}{P_{H,t}} C_{H,t}^{-\frac{1}{\epsilon}} &= C_{H,t}(i)^{\frac{\epsilon-1}{\epsilon}} C_{H,t}(i)^{-1} \\
\frac{P_{H,t}(i)}{P_{H,t}} C_{H,t}^{-\frac{1}{\epsilon}} &= C_{H,t}(i)^{-\frac{1}{\epsilon}} \\
\left(\frac{P_{H,t}(i)}{P_{H,t}} \right)^{-\epsilon} C_{H,t} &= C_{H,t}(i)
\end{aligned}$$

The equation is the representative household's demand function for domestic produced consumption goods. We have got the same relationship as the equation (12).

OVERALL CONSUMER PRICE INDEX (CPI)

We have known that the representative household has following optimal allocation functions (9):

$$C_{H,t} = (1-\alpha)\left(\frac{P_{H,t}}{P_t}\right)^{-\eta} C_t \qquad C_{F,t} = \alpha\left(\frac{P_{F,t}}{P_t}\right)^{-\eta} C_t$$

and the total consumption of the household (7) consists of the domestic and foreign produced goods described according to this relationship:

$$C_t \equiv \left\{(1-\alpha)^{\frac{1}{\eta}} C_{H,t}^{\frac{\eta-1}{\eta}} + \alpha^{\frac{1}{\eta}} C_{F,t}^{\frac{\eta-1}{\eta}}\right\}^{\frac{\eta}{\eta-1}}$$

We combine these 3 equation together to yield the overall Consumer Price Index (CPI) – equation (11):

$$\begin{aligned}
C_t \equiv & \left\{(1-\alpha)^{\frac{1}{\eta}} C_{H,t}^{\frac{\eta-1}{\eta}} + \alpha^{\frac{1}{\eta}} C_{F,t}^{\frac{\eta-1}{\eta}}\right\}^{\frac{\eta}{\eta-1}} \\
= & \left\{(1-\alpha)^{\frac{1}{\eta}} \left[(1-\alpha)\left(\frac{P_{H,t}}{P_t}\right)^{-\eta} C_t\right]^{\frac{\eta-1}{\eta}} + \right. \\
& \left. +\alpha^{\frac{1}{\eta}} \left[\alpha\left(\frac{P_{F,t}}{P_t}\right)^{-\eta} C_t\right]^{\frac{\eta-1}{\eta}}\right\}^{\frac{\eta}{\eta-1}} \\
= & \left\{(1-\alpha)^{\frac{1}{\eta}} (1-\alpha)^{\frac{\eta-1}{\eta}} \left(\frac{P_{H,t}}{P_t}\right)^{-\eta\frac{\eta-1}{\eta}} C_t^{\frac{\eta-1}{\eta}} + \right. \\
& \left. +\alpha^{\frac{1}{\eta}} \alpha^{\frac{\eta-1}{\eta}} \left(\frac{P_{F,t}}{P_t}\right)^{-\eta\frac{\eta-1}{\eta}} C_t^{\frac{\eta-1}{\eta}}\right\}^{\frac{\eta}{\eta-1}} \\
C_t^{\frac{\eta-1}{\eta}} \equiv & (1-\alpha)^{\frac{1}{\eta}+\frac{\eta-1}{\eta}} \left(\frac{P_{H,t}}{P_t}\right)^{-(\eta-1)} C_t^{\frac{\eta-1}{\eta}} + \\
& + \alpha^{\frac{1}{\eta}+\frac{\eta-1}{\eta}} \left(\frac{P_{F,t}}{P_t}\right)^{-(\eta-1)} C_t^{\frac{\eta-1}{\eta}} \\
= & (1-\alpha)^{\frac{\eta}{\eta}} \left(\frac{P_{H,t}}{P_t}\right)^{1-\eta} C_t^{\frac{\eta-1}{\eta}} + \alpha^{\frac{\eta}{\eta}} \left(\frac{P_{F,t}}{P_t}\right)^{1-\eta} C_t^{\frac{\eta-1}{\eta}}
\end{aligned}$$

$$
\begin{aligned}
1 &\equiv (1-\alpha)\left(\frac{P_{H,t}}{P_t}\right)^{1-\eta} + \alpha\left(\frac{P_{F,t}}{P_t}\right)^{1-\eta} \\
{P_t}^{1-\eta} &\equiv (1-\alpha)P_{H,t}^{1-\eta} + \alpha P_{F,t}^{1-\eta} \\
P_t &\equiv \left\{(1-\alpha)P_{H,t}^{1-\eta} + \alpha P_{F,t}^{1-\eta}\right\}^{\frac{1}{1-\eta}}
\end{aligned}
$$

The previous equation is used in a linearized form. It can be rearranged:

$$
\begin{aligned}
P_t &\equiv \left\{(1-\alpha)P_{H,t}^{1-\eta} + \alpha P_{F,t}^{1-\eta}\right\}^{\frac{1}{1-\eta}} \\
{P_t}^{1-\eta} &\equiv (1-\alpha)P_{H,t}^{1-\eta} + \alpha P_{F,t}^{1-\eta} \\
\frac{{P_t}^{1-\eta}}{{P_{H,t}}^{1-\eta}} &\equiv (1-\alpha) + \alpha\frac{P_{F,t}^{1-\eta}}{P_{H,t}^{1-\eta}} \\
\left(\frac{P_t}{P_{H,t}}\right)^{1-\eta} &\equiv (1-\alpha) + \alpha\left(\frac{P_{F,t}}{P_{H,t}}\right)^{1-\eta}
\end{aligned}
$$

and with using the Taylor approximation[57] it is possible to write:

$$
\begin{aligned}
\left(\frac{P}{P_H}\right)^{1-\eta} e^{(1-\eta)(p_t-p_{H,t})} &\equiv (1-\alpha) + \alpha\left(\frac{P_F}{P_H}\right)^{1-\eta}\cdot \\
&\quad \cdot e^{(1-\eta)(p_{F,t}-p_{H,t})} \\
\left(\frac{P}{P_H}\right)^{1-\eta}\left[1+(1-\eta)(p_t-p_{H,t})\right] &\equiv (1-\alpha) + \alpha\left(\frac{P_F}{P_H}\right)^{1-\eta}\cdot \\
&\quad \cdot \left[1+(1-\eta)(p_{F,t}-p_{H,t})\right] \\
\left(\frac{P}{P_H}\right)^{1-\eta} + & \\
+\left(\frac{P}{P_H}\right)^{1-\eta}(1-\eta)(p_t-p_{H,t}) &\equiv (1-\alpha) + \alpha\left(\frac{P_F}{P_H}\right)^{1-\eta} + \\
&\quad + \alpha\left(\frac{P_F}{P_H}\right)^{1-\eta}(1-\eta)\cdot \\
&\quad \cdot (p_{F,t}-p_{H,t})
\end{aligned}
$$

Because $\left(\frac{P_t}{P_{H,t}}\right)^{1-\eta} \equiv (1-\alpha) + \alpha\left(\frac{P_{F,t}}{P_{H,t}}\right)^{1-\eta}$ as it can be seen in

[57] For more details see e.g. Malley (2004).

the former calculation[58], in steady state holds $\left(\frac{P}{P_H}\right)^{1-\eta} \equiv (1-\alpha) + \alpha\left(\frac{P_F}{P_H}\right)^{1-\eta}$ too. Subsequently it is possible to use it for the calculation and continue.

$$\begin{aligned}
\left(\frac{P}{P_H}\right)^{1-\eta}(1-\eta)(p_t - p_{H,t}) &\equiv \alpha\left(\frac{P_F}{P_H}\right)^{1-\eta}(1-\eta)(p_{F,t} - p_{H,t}) \\
P^{1-\eta}(p_t - p_{H,t}) &\equiv \alpha P_F^{1-\eta}(p_{F,t} - p_{H,t}) \\
(p_t - p_{H,t}) &\equiv \alpha(p_{F,t} - p_{H,t}) \\
p_t &\equiv \alpha p_{F,t} - \alpha p_{H,t} + p_{H,t} \\
&= (1-\alpha)p_{H,t} + \alpha p_{F,t}
\end{aligned}$$

The last equation is a linearized version of the overall CPI (14) used for the derivation of a connection between terms of trade and inflation.

During the calculation we used an assumption that in the steady state holds $P = P_H = P_F$, or equivalently we assume the condition $\pi = \pi_H = \pi_F$. In the steady state the development of the overall, domestic and foreign inflation is the same. It is logically consistent with (7).

INTERNATIONAL RISK SHARING CONDITION

Conditions connected to a complete international markets and perfect mobility can be expressed as in equation (22):

$$R_t^* E_t\left(\frac{Z_t}{Z_{t+1}}\right) = R_t$$

together with the domestic and foreign Euler equation (4):

$$\beta R_t E_t\left\{\frac{P_t}{P_{t+1}}\left(\frac{C_{t+1} - hC_t}{C_t - hC_{t-1}}\right)^{-\sigma}\right\} = 1$$

$$R_t = \frac{1}{\beta} E_t\left\{\frac{P_{t+1}}{P_t}\left(\frac{C_t - hC_{t-1}}{C_{t+1} - hC_t}\right)^{-\sigma}\right\}$$

[58] The relationship can be rewritten into the form of $\frac{P_t}{P_{H,t}} = \left[(1-\alpha) + \alpha s_t^{1-\eta}\right]^{\frac{1}{1-\eta}}$ with knowing that the terms of trade are $S_t = \frac{P_{F,t}}{P_{H,t}}$. After a linearizing this formula around the steady state it yields the equation (21): $\psi_t = -[q_t + (1-\alpha)s_t]$.

and

$$\beta R_t^* E_t \left\{ \frac{P_t^*}{P_{t+1}^*} \left(\frac{C_{t+1}^* - hC_t^*}{C_t^* - hC_{t-1}^*} \right)^{-\sigma} \right\} = 1$$

$$R_t^* = \frac{1}{\beta} E_t \left\{ \frac{P_{t+1}^*}{P_t^*} \left(\frac{C_t^* - hC_{t-1}^*}{C_{t+1}^* - hC_t^*} \right)^{-\sigma} \right\}$$

We plug both equations in (22) to substitute out the R_t and R_t^*. After this step we get:

$$E_t \left(\frac{Z_t}{Z_{t+1}} \right) \frac{1}{\beta} E_t \left\{ \frac{P_{t+1}^*}{P_t^*} \left(\frac{C_t^* - hC_{t-1}^*}{C_{t+1}^* - hC_t^*} \right)^{-\sigma} \right\} = \frac{1}{\beta} E_t \left\{ \frac{P_{t+1}}{P_t} \cdot \right.$$
$$\left. \cdot \left(\frac{C_t - hC_{t-1}}{C_{t+1} - hC_t} \right)^{-\sigma} \right\}$$
$$E_t \left(\frac{Z_{t+1}}{Z_t} \right) \beta E_t \left\{ \frac{P_t^*}{P_{t+1}^*} \left(\frac{C_{t+1}^* - hC_t^*}{C_t^* - hC_{t-1}^*} \right)^{-\sigma} \right\} = \beta E_t \left\{ \frac{P_t}{P_{t+1}} \cdot \right.$$
$$\left. \cdot \left(\frac{C_{t+1} - hC_t}{C_t - hC_{t-1}} \right)^{-\sigma} \right\}$$

The last equation corresponds to equation (23).

Assuming the same habit formation parameter for consumption h and the same rate of time preference β:

$$E_t \left\{ \frac{Z_{t+1}}{Z_t} \frac{P_t^*}{P_{t+1}^*} \left(\frac{C_{t+1}^* - hC_t^*}{C_t^* - hC_{t-1}^*} \right)^{-\sigma} \right\} = E_t \left\{ \frac{P_t}{P_{t+1}} \left(\frac{C_{t+1} - hC_t}{C_t - hC_{t-1}} \right)^{-\sigma} \right\}$$
$$\frac{P_t^*}{Z_t P_t} \frac{(C_t - hC_{t-1})^{-\sigma}}{(C_t^* - hC_{t-1}^*)^{-\sigma}} = E_t \left\{ \frac{P_{t+1}^*}{Z_{t+1} P_{t+1}} \frac{(C_{t+1} - hC_t)^{-\sigma}}{(C_{t+1}^* - hC_t^*)^{-\sigma}} \right\}$$

Then we use the relationship for the real exchange rate $Q_t \equiv \frac{Z_t P_t}{P_t^*}$ and continue:

$$\frac{1}{Q_t} \frac{(C_t - hC_{t-1})^{-\sigma}}{(C_t^* - hC_{t-1}^*)^{-\sigma}} = E_t \left\{ \frac{1}{Q_{t+1}} \left(\frac{C_{t+1} - hC_t}{C_{t+1}^* - hC_t^*} \right)^{-\sigma} \right\}$$
$$(C_t - hC_{t-1})^{-\sigma} = E_t \left\{ \frac{1}{Q_{t+1}} \left(\frac{C_{t+1} - hC_t}{C_{t+1}^* - hC_t^*} \right)^{-\sigma} \right\} \cdot$$
$$\cdot (C_t^* - hC_{t-1}^*)^{-\sigma} Q_t$$

$$C_t - hC_{t-1} = E_t \left\{ \frac{1}{Q_{t+1}} \left(\frac{C_{t+1} - hC_t}{C^*_{t+1} - hC^*_t} \right)^{-\sigma} \right\}^{-\frac{1}{\sigma}} \cdot$$

$$\cdot (C^*_t - hC^*_{t-1}) Q_t^{-\frac{1}{\sigma}}$$

$$C_t - hC_{t-1} = E_t \left\{ Q_{t+1}^{\frac{1}{\sigma}} \frac{C_{t+1} - hC_t}{C^*_{t+1} - hC^*_t} \right\} (C^*_t - hC^*_{t-1}) Q_t^{-\frac{1}{\sigma}}$$

$$C_t - hC_{t-1} = \vartheta (C^*_t - hC^*_{t-1}) Q_t^{-\frac{1}{\sigma}} \tag{75}$$

This equation expresses the equilibrium condition. A constant ϑ is depending on initial conditions regarding relative net assets positions. In this case $\vartheta = E_t \left(Q_{t+1}^{\frac{1}{\sigma}} \frac{C_{t+1} - hC_t}{C^*_{t+1} - hC^*_t} \right)$. It indicates that the expected development of the consumptions (a change of domestic consumption to foreign with respect to the real exchange rate) influences the current domestic consumption. The expected development is importantly influenced by initial assets holding expressed as a constant ratio of future consumption in equilibrium.

Log–linearizing the equation around the steady state gives:

$$\log(C_t - hC_{t-1}) = \log \left\{ \vartheta (C^*_t - hC^*_{t-1}) Q_t^{-\frac{1}{\sigma}} \right\}$$

$$\log(C_t - hC_{t-1}) = \log \vartheta + \log(C^*_t - hC^*_{t-1}) + \log(Q_t^{-\frac{1}{\sigma}})$$

$$\log(C_t - hC_{t-1}) = 0 + \log(C^*_t - hC^*_{t-1}) - \frac{1}{\sigma} \log(Q_t)$$

$$\frac{\Delta C_t - h\Delta C_{t-1}}{c - hc} = \frac{\Delta C^*_t - h\Delta C^*_{t-1}}{c^* - hc^*} - \frac{1}{\sigma} q_t$$

$$\frac{cc_t - hcc_{t-1}}{c - hc} = \frac{c^* c^*_t - hc^* c^*_{t-1}}{c^* - hc^*} - \frac{1}{\sigma} q_t$$

$$\frac{c(c_t - hc_{t-1})}{c(1-h)} = \frac{c^*(c^*_t - hc^*_{t-1})}{c^*(1-h)} - \frac{1}{\sigma} q_t$$

$$\frac{c_t - hc_{t-1}}{1-h} = \frac{c^*_t - hc^*_{t-1}}{1-h} - \frac{1}{\sigma} q_t$$

$$c_t - hc_{t-1} = (c^*_t - hc^*_{t-1}) - \frac{1-h}{\sigma} q_t$$

And with using the relationship $y^*_t = c^*_t$ we get equation (25):

$$c_t - hc_{t-1} = (y^*_t - hy^*_{t-1}) - \frac{1-h}{\sigma} q_t$$

SUPPLEMENT 2: CALCULATIONS OF THE NEW KEYNESIAN PHILLIPS CURVE

In this section we derive the New Keynesian Phillips Curve in the Calvo style. We introduce some calculations which are connected to this derivation.

PHILLIPS CURVE

According to the Calvo style every firm resets its price with the probability $1-\theta$ each period. The time is independent of the time since the last adjustment. The rest of the firms keeps the price adjusted by the indexation to the last period inflation. In the Calvo price setting holds $P_{H,t+k}(j) = \overline{P}_{H,t}(j)$ with probability θ^k for $k = 0, 1, 2, \ldots$, where $\overline{P}_{H,t}$ is the new price set by a firm j adjusting it in period t. Because we suppose that all firms choose $the\ best$ new price, which is the same one for all firms, we drop the subscript j.

Every firm sets its new price $\overline{P}_{H,t}$ in period t to maximize the discounted value of all future profits to reach the most effective behavior during its optimizing. For the j-th firms we can write:

$$E_t \sum_{k=0}^{\infty} \theta_H^k \frac{1}{R_{t+k}} \left\{ Y_{t+k}(j)(\overline{P}_{H,t}(j) - MC_{t+k}^n) \right\}$$

The chosen firm's price is the same we can rewrite the previous function into the following form:

$$E_t \sum_{k=0}^{\infty} \theta_H^k \frac{1}{R_{t+k}} \left\{ Y_{t+k}(\overline{P}_{H,t} - MC_{t+k}^n) \right\}$$

Every firm tries to maximize it by setting the new price $\overline{P}_{H,t}$ subject to the sequence of demand constraints (expressed as the demand constraints):

$$Y_{t+k} \leq \left(\frac{\overline{P}_{H,t}}{P_{H,t+k}} \right)^{-\epsilon} (C_{H,t+k} + C_{H,t+k}^*),$$

where MC_t^n are nominal marginal costs and the demand constraint for the i-th good is

$$Y_{t+k}^d(i) \equiv \left(\frac{\overline{P}_{H,t}}{P_{H,t+k}} \right)^{-\epsilon} Y_{t+k} = \left(\frac{\overline{P}_{H,t}}{P_{H,t+k}} \right)^{-\epsilon} (C_{H,t+k} + C_{H,t+k}^*)$$

The first order condition (FOC) of the optimizing behavior is calculated with using the Lagrangian function.

$$max \;\; E_t \sum_{k=0}^{\infty} \theta_H^k \frac{1}{R_{t+k}} \left\{ Y_{t+k}^d (\overline{P}_{H,t} - MC_{t+k}^n) \right\}$$

$$s.t. \qquad\qquad Y_{t+k}^d(i) = \left(\frac{\overline{P}_{H,t}}{P_{H,t+k}} \right)^{-\epsilon} Y_{t+k}$$

The Lagrangian has the following form:

$$L(\overline{P}_{H,t}) = E_t \sum_{k=0}^{\infty} \theta_H^k \frac{1}{R_{t+k}} \left\{ \left(\frac{\overline{P}_{H,t}}{P_{H,t+k}} \right)^{-\epsilon} Y_{t+k} (\overline{P}_{H,t} - MC_{t+k}^n) \right\}$$

$$L(\overline{P}_{H,t}) = E_t \sum_{k=0}^{\infty} \theta_H^k \frac{1}{R_{t+k}} \left\{ \left(\frac{\overline{P}_{H,t}}{P_{H,t+k}} \right)^{-\epsilon} Y_{t+k} \overline{P}_{H,t} - \right.$$

$$\left. - \left(\frac{\overline{P}_{H,t}}{P_{H,t+k}} \right)^{-\epsilon} Y_{t+k} MC_{t+k}^n \right\}$$

$$L(\overline{P}_{H,t}) = \sum_{k=0}^{\infty} \theta_H^k E_t \frac{1}{R_{t+k}} \left\{ \left(\frac{\overline{P}_{H,t}}{P_{H,t+k}} \right)^{-\epsilon+1} Y_{t+k} P_{H,t+k} - \right.$$

$$\left. - \left(\frac{\overline{P}_{H,t}}{P_{H,t+k}} \right)^{-\epsilon} Y_{t+k} MC_{t+k}^n \right\}$$

and the calculation:

$$\frac{\partial L(\overline{P}_{H,t})}{\partial \overline{P}_{H,t}} = \sum_{k=0}^{\infty} \theta_H^k E_t \frac{1}{R_{t+k}} \left\{ (-\epsilon + 1) \left(\frac{\overline{P}_{H,t}}{P_{H,t+k}} \right)^{-\epsilon} \frac{1}{P_{H,t+k}} Y_{t+k} \cdot \right.$$

$$\left. \cdot P_{H,t+k} - (-\epsilon) \left(\frac{\overline{P}_{H,t}}{P_{H,t+k}} \right)^{-\epsilon-1} \frac{1}{P_{H,t+k}} Y_{t+k} MC_{t+k}^n \right\}$$

$$= \sum_{k=0}^{\infty} \theta_H^k E_t \frac{1}{R_{t+k}} \left\{ (-\epsilon + 1) \left(\frac{\overline{P}_{H,t}}{P_{H,t+k}} \right)^{-\epsilon} Y_{t+k} + \right.$$

$$\left. + \epsilon \left(\frac{\overline{P}_{H,t}}{P_{H,t+k}} \right)^{-\epsilon} \left(\frac{\overline{P}_{H,t}}{P_{H,t+k}} \right)^{-1} \frac{1}{P_{H,t+k}} Y_{t+k} MC_{t+k}^n \right\}$$

$$= \sum_{k=0}^{\infty} \theta_H^k E_t \frac{1}{R_{t+k}} \left\{ (-\epsilon + 1) Y_{t+k}^d(i) + \right.$$

$$\left. + \epsilon \left(\frac{\overline{P}_{H,t}}{P_{H,t+k}} \right)^{-\epsilon} \frac{P_{H,t+k}}{\overline{P}_{H,t}} \frac{1}{P_{H,t+k}} Y_{t+k} MC_{t+k}^n \right\}$$

$$
\begin{aligned}
&= \sum_{k=0}^{\infty} \theta_H^k E_t \frac{1}{R_{t+k}} \Big\{ (-\epsilon+1) Y_{t+k}^d(i) \; + \\
&\quad + \epsilon \left(\frac{\overline{P}_{H,t}}{P_{H,t+k}} \right)^{-\epsilon} Y_{t+k} \frac{1}{\overline{P}_{H,t}} MC_{t+k}^n \Bigg\} \\
&= \sum_{k=0}^{\infty} \theta_H^k E_t \frac{1}{R_{t+k}} \Big\{ (-\epsilon+1) Y_{t+k}^d(i) \; + \\
&\quad + \epsilon Y_{t+k}^d(i) \frac{1}{\overline{P}_{H,t}} MC_{t+k}^n \Bigg\} \\
&= \sum_{k=0}^{\infty} E_t \frac{\theta_H^k}{R_{t+k}} \left\{ Y_{t+k}^d(i) \left((-\epsilon+1) + \epsilon \frac{1}{\overline{P}_{H,t}} MC_{t+k}^n \right) \right\}
\end{aligned}
$$

The derivative of the Lagrangian function equals zero:

$$
\begin{aligned}
0 &= E_t \sum_{k=0}^{\infty} \frac{\theta_H^k}{R_{t+k}} \left\{ Y_{t+k}^d(i) \left((-\epsilon+1) + \epsilon \frac{1}{\overline{P}_{H,t}} MC_{t+k}^n \right) \right\} \\
0 &= E_t \sum_{k=0}^{\infty} \frac{\theta_H^k}{R_{t+k}} \left\{ Y_{t+k}^d(i) \left(\overline{P}_{H,t} - \frac{\epsilon}{\epsilon-1} MC_{t+k}^n \right) \right\}
\end{aligned}
$$

The last equation is the first order condition for the firm's optimizing problem.

We use the Euler equation (4):

$$
\begin{aligned}
\beta R_t E_t \left\{ \frac{P_t}{P_{t+1}} \left(\frac{C_{t+1} - hC_t}{C_t - hC_{t-1}} \right)^{-\sigma} \right\} &= 1 \\
\beta E_t \left\{ \frac{P_t}{P_{t+1}} \left(\frac{C_{t+1} - hC_t}{C_t - hC_{t-1}} \right)^{-\sigma} \right\} &= \frac{1}{R_t}
\end{aligned}
$$

and for $t+k$:

$$
\begin{aligned}
\frac{1}{R_{t+k}} &= \beta^k E_t \left\{ \frac{P_t}{P_{t+k}} \left(\frac{C_{t+k} - hC_{t+k-1}}{C_t - hC_{t-1}} \right)^{-\sigma} \right\} \\
\frac{1}{R_{t+k}} &= \beta^k E_t \left\{ \frac{P_t}{P_{t+k}} \left(\frac{\tilde{C}_{t+k}}{\tilde{C}_t} \right)^{-\sigma} \right\},
\end{aligned}
$$

where $\tilde{C}_t = C_t - hC_{t-1}$.

Now we rule out R_{t+k} from the FOC:

$$0 = E_t \sum_{k=0}^{\infty} \theta_H^k \beta^k \left\{ \frac{P_t}{P_{t+k}} \left(\frac{\tilde{C}_{t+k}}{\tilde{C}_t} \right)^{-\sigma} \right\} \left\{ Y_{t+k}^d(i) \left(\overline{P}_{H,t} - \right.\right.$$

$$\left.\left. - \frac{\epsilon}{\epsilon-1} MC_{t+k}^n \right) \right\}$$

$$0 = E_t \sum_{k=0}^{\infty} (\beta\theta_H)^k \frac{P_{t+k}^{-1}}{P_t^{-1}} \frac{\tilde{C}_{t+k}^{-\sigma}}{\tilde{C}_t^{-\sigma}} \left\{ Y_{t+k}^d(i) \left(\overline{P}_{H,t} - \right.\right.$$

$$\left.\left. - \frac{\epsilon}{\epsilon-1} MC_{t+k}^n \right) \right\}$$

$$0 = \frac{1}{P_t \tilde{C}_t^{\sigma}} E_t \sum_{k=0}^{\infty} (\beta\theta_H)^k P_{t+k}^{-1} \tilde{C}_{t+k}^{-\sigma} \left\{ Y_{t+k}^d(i) \left(\overline{P}_{H,t} - \right.\right.$$

$$\left.\left. - \frac{\epsilon}{\epsilon-1} MC_{t+k}^n \right) \right\}$$

$$0 = E_t \sum_{k=0}^{\infty} (\beta\theta_H)^k \left\{ P_{t+k}^{-1} \tilde{C}_{t+k}^{-\sigma} Y_{t+k}^d(i) \left(\overline{P}_{H,t} - \right.\right.$$

$$\left.\left. - \frac{\epsilon}{\epsilon-1} MC_{t+k}^n \right) \right\}$$

$$0 = E_t \sum_{k=0}^{\infty} (\beta\theta_H)^k \left\{ \tilde{C}_{t+k}^{-\sigma} Y_{t+k}^d(i) \frac{P_{H,t-1}}{P_{t+k}} \left(\frac{\overline{P}_{H,t}}{P_{H,t-1}} - \right.\right.$$

$$\left.\left. - \frac{\epsilon}{\epsilon-1} \frac{MC_{t+k}^n}{P_{H,t-1}} \right) \right\}$$

$$0 = E_t \sum_{k=0}^{\infty} (\beta\theta_H)^k \left\{ \tilde{C}_{t+k}^{-\sigma} Y_{t+k}^d(i) \frac{P_{H,t-1}}{P_{t+k}} \left(\frac{\overline{P}_{H,t}}{P_{H,t-1}} - \right.\right.$$

$$\left.\left. - \frac{\epsilon}{\epsilon-1} \frac{P_{H,t+k}}{P_{H,t-1}} MC_{t+k} \right) \right\}$$

$$0 = E_t \sum_{k=0}^{\infty} (\beta\theta_H)^k \left\{ \tilde{C}_{t+k}^{-\sigma} Y_{t+k}^d(i) \frac{P_{H,t-1}}{P_{t+k}} \left(\frac{\overline{P}_{H,t}}{P_{H,t-1}} - \right.\right.$$

$$\left.\left. - \frac{\epsilon}{\epsilon-1} \Pi_{t-1,t+k}^H MC_{t+k} \right) \right\}$$

where the real marginal costs are $MC_t = \frac{MC_t^n}{P_{H,t}}$ and $\Pi_{t-1,t+k}^H = \frac{P_{H,t+k}}{P_{H,t-1}}$.

Log–linearizing the previous equation around the steady state (zero inflation perfect foresight steady state) gives:

$$
\begin{aligned}
\overline{p}_{H,t} = p_{H,t-1} + \sum_{k=0}^{\infty}(\beta\theta_H)^k E_t(\pi_{H,t+k}) \; + \\
+ \;(1-\beta\theta_H)\sum_{k=0}^{\infty}(\beta\theta_H)^k E_t\left(\log\frac{\epsilon}{\epsilon-1}MC_{t+k}\right)
\end{aligned}
$$

for $mc_t \equiv \log\frac{\epsilon}{\epsilon-1}MC_t$, where the steady state value is $mc = \log\frac{\epsilon}{\epsilon-1}$. We rewrite the equation into the form of (37) and then continue:

$$
\begin{aligned}
\overline{p}_{H,t} &= p_{H,t-1} + \sum_{k=0}^{\infty}(\beta\theta_H)^k E_t\left[\pi_{H,t+k} + (1-\beta\theta_H)mc_t\right] \\
&= p_{H,t-1} + \left[\pi_{H,t} + (1-\beta\theta_H)mc_t\right] + \\
&\quad + \sum_{k=1}^{\infty}(\beta\theta_H)^k E_t\left[\pi_{H,t+k} + (1-\beta\theta_H)mc_{t+k}\right] \\
&= p_{H,t-1} + \left[\pi_{H,t} + (1-\beta\theta_H)mc_t\right] + \\
&\quad + (\beta\theta_H)\left\{\sum_{k=0}^{\infty}(\beta\theta_H)^k E_t\left[\pi_{H,t+k+1} + (1-\beta\theta_H)mc_{t+k+1}\right]\right\}
\end{aligned}
$$

Then we use the log–linearized condition expressed for $t+1$:

$$
\begin{aligned}
\overline{p}_{H,t} &= p_{H,t-1} + \left[\pi_{H,t} + (1-\beta\theta_H)mc_t\right] \; + \\
&\quad + \;(\beta\theta_H)E_t\left\{\overline{p}_{H,t+1} - p_{H,t}\right\} \\
\overline{p}_{H,t} - p_{H,t-1} &= \pi_{H,t} + (1-\beta\theta_H)mc_t + (\beta\theta_H)E_t\pi_{H,t+1}
\end{aligned}
$$

We use the last equation together with the log–linearized version of the domestic price level (33):

$$
\begin{aligned}
\pi_{H,t} &= (1-\theta_H)(\overline{p}_{H,t} - p_{H,t-1}) + \theta_H^2\pi_{H,t-1} \\
\pi_{H,t} &= (1-\theta_H)\left[\pi_{H,t} + (1-\beta\theta_H)mc_t \; + \right. \\
&\quad \left. + \;(\beta\theta_H)E_t\pi_{H,t+1}\right] + \theta_H^2\pi_{H,t-1} \\
\pi_{H,t} &= (1-\theta_H)\pi_{H,t} + (1-\theta_H)(1-\beta\theta_H)mc_t \; + \\
&\quad + (1-\theta_H)(\beta\theta_H)E_t\pi_{H,t+1} + \theta_H^2\pi_{H,t-1} \\
\pi_{H,t} - (1-\theta_H)\pi_{H,t} &= (1-\theta_H)(1-\beta\theta_H)mc_t \; + \\
&\quad + \;(1-\theta_H)(\beta\theta_H)E_t\pi_{H,t+1} + \theta_H^2\pi_{H,t-1} \\
\theta_H\pi_{H,t} &= (1-\theta_H)(\beta\theta_H)E_t\pi_{H,t+1} + \theta_H^2\pi_{H,t-1} \; + \\
&\quad + \;(1-\theta_H)(1-\beta\theta_H)mc_t
\end{aligned}
$$

$$\pi_{H,t} = \beta(1-\theta_H)E_t\pi_{H,t+1} + \theta_H\pi_{H,t-1} + $$
$$+ \frac{(1-\theta_H)(1-\beta\theta_H)}{\beta}mc_t$$
$$\pi_{H,t} = \beta(1-\theta_H)E_t\pi_{H,t+1} + \theta_H\pi_{H,t-1} + \lambda_H mc_t$$

The equation is the New Keynesian Phillips Curve (39).

ALTERNATIVE EXPRESSION FOR THE FOC

It is possible to calculate further with using the adjusted the equation (37):

$$\overline{p}_{H,t} - p_{H,t-1} = \pi_{H,t} + (1-\beta\theta_H)mc_t + (\beta\theta_H)E_t\pi_{H,t+1}$$
$$\overline{p}_{H,t} - p_{H,t-1} = \beta\theta_H E_t\pi_{H,t+1} + \pi_{H,t} + (1-\beta\theta_H)mc_t$$
$$\overline{p}_{H,t} - p_{H,t-1} = \beta\theta_H E_t(\overline{p}_{H,t+1} - p_{H,t}) + p_{H,t} - p_{H,t-1} + $$
$$+ (1-\beta\theta_H)mc_t$$
$$\overline{p}_{H,t} = \beta\theta_H E_t\overline{p}_{H,t+1} - (\beta\theta_H)p_{H,t} + p_{H,t} + $$
$$+ (1-\beta\theta_H)(mc_t^n - p_{H,t})$$
$$\overline{p}_{H,t} = \beta\theta_H E_t\overline{p}_{H,t+1} + (1-\beta\theta_H)p_{H,t} + (1-\beta\theta_H)mc_t^n$$
$$- (1-\beta\theta_H)p_{H,t}$$
$$\overline{p}_{H,t} = \beta\theta_H E_t\overline{p}_{H,t+1} + (1-\beta\theta_H)mc_t^n$$
$$\overline{p}_{H,t+1} = \beta\theta_H E_t\overline{p}_{H,t+2} + (1-\beta\theta_H)mc_{t+1}^n$$
$$\overline{p}_{H,t+2} = \beta\theta_H E_t\overline{p}_{H,t+3} + \ \ldots$$
$$\vdots$$
$$\overline{p}_{H,t} = \beta\theta_H\left\{\beta\theta_H\left[\beta\theta_H E_t\overline{p}_{H,t+3} + (1-\beta\theta_H)mc_{t+2}^n\right] + \right.$$
$$\left. + (1-\beta\theta_H)mc_{t+1}^n\right\} + (1-\beta\theta_H)mc_t^n$$
$$\vdots$$
$$\overline{p}_{H,t} = (\beta\theta_H)^\infty\ \overline{p}_{H,t+\infty} + (1-\beta\theta_H)\sum_{k=0}^{\infty} mc_{t+k}^n$$
$$\overline{p}_{H,t} = (1-\beta\theta_H)\sum_{k=0}^{\infty} mc_{t+k}^n$$

The equation corresponds to the equation (38). During the calculation we used $lim_{n\rightarrow\infty}(\beta\theta_H)^n = 0$ because $\beta \in (0;1)$ and $\theta_H \in \langle 0;1\rangle$.

AGGREGATE DOMESTIC PRICE LEVEL

We use a log–linearized version of the aggregate domestic price level for the calculation of the Phillips Curve. We outline the process of a derivation in this subsection.

The aggregate domestic price level (32) is expressed in the following form:

$$\begin{aligned}
P_{H,t} &= \left\{(1-\theta_H)\overline{P}_{H,t}^{1-\rho} + \theta_H \widehat{P}_{H,t}^{1-\rho}\right\}^{\frac{1}{1-\rho}} \\
P_{H,t} &= \left\{(1-\theta_H)\overline{P}_{H,t}^{1-\rho} + \theta_H \left[P_{H,t-1}\left(\frac{P_{H,t-1}}{P_{H,t-2}}\right)^{\theta_H}\right]^{1-\rho}\right\}^{\frac{1}{1-\rho}} \\
P_{H,t}^{1-\rho} &= (1-\theta_H)\overline{P}_{H,t}^{1-\rho} + \theta_H \left[P_{H,t-1}\left(\frac{P_{H,t-1}}{P_{H,t-2}}\right)^{\theta_H}\right]^{1-\rho} \\
-(1-\theta_H)\overline{P}_{H,t}^{1-\rho} &= -P_{H,t}^{1-\rho} + \theta_H \left[P_{H,t-1}\left(\frac{P_{H,t-1}}{P_{H,t-2}}\right)^{\theta_H}\right]^{1-\rho} \\
(1-\theta_H)\overline{P}_{H,t}^{1-\rho} &= P_{H,t}^{1-\rho} - \theta_H \left[P_{H,t-1}\left(\frac{P_{H,t-1}}{P_{H,t-2}}\right)^{\theta_H}\right]^{1-\rho}
\end{aligned}$$

and the log–linearizing:

$$\begin{aligned}
\log\left\{(1-\theta_H)\overline{P}_{H,t}^{1-\rho}\right\} &= \log\left\{P_{H,t}^{1-\rho} - \theta_H \left[P_{H,t-1}\left(\frac{P_{H,t-1}}{P_{H,t-2}}\right)^{\theta_H}\right]^{1-\rho}\right\} \\
\log(1-\theta_H) + \log\overline{P}_{H,t}^{1-\rho} &= \frac{\left\{\Delta P_{H,t}^{1-\rho} - \theta_H \Delta\left[P_{H,t-1}\left(\frac{P_{H,t-1}}{P_{H,t-2}}\right)^{\theta_H}\right]^{1-\rho}\right\}}{P_H^{1-\rho} - \theta_H\left[P_H\left(\frac{P_H}{P_H}\right)^{\theta_H}\right]^{1-\rho}}
\end{aligned}$$

$$(1-\rho)\log \overline{P}_{H,t} = \frac{\Delta P_{H,t}^{1-\rho}\frac{P_H^{1-\rho}}{P_H^{1-\rho}} - \theta_H \Delta \left[P_{H,t-1}\frac{P_H}{P_H}\left(\frac{P_{H,t-1}}{P_{H,t-2}}\right)^{\theta_H} \frac{\left(\frac{P_H}{P_H}\right)^{\theta_H}}{\left(\frac{P_H}{P_H}\right)^{\theta_H}} \right]^{1-\rho}}{P_H^{1-\rho} - \theta_H \left[P_H \left(\frac{P_H}{P_H}\right)^{\theta_H} \right]^{1-\rho}}$$

$$(1-\rho)\overline{p}_{H,t} = \frac{P_H^{1-\rho}\left\{ \frac{\Delta P_{H,t}^{1-\rho}}{P_H^{1-\rho}} - \theta_H \Delta \left[\frac{P_{H,t-1}}{P_H} \left(\frac{P_{H,t-1}\left(\frac{P_H}{P_H}\right)}{P_{H,t-2}\left(\frac{P_H}{P_H}\right)} \right)^{\theta_H} \right]^{1-\rho} \right\}}{P_H^{1-\rho}\left\{ 1 - \theta_H \left[\left(\frac{P_H}{P_H}\right)^{\theta_H} \right]^{1-\rho} \right\}}$$

$$(1-\rho)\overline{p}_{H,t} = \frac{(1-\rho)p_{H,t} - \theta_H \Delta \left[\frac{P_{H,t-1}}{P_H} \left(\frac{\frac{P_{H,t-1}}{P_H}P_H}{\frac{P_{H,t-2}}{P_H}P_H} \right)^{\theta_H} \right]^{1-\rho}}{1-\theta_H \left[(1)^{\theta_H} \right]^{1-\rho}}$$

$$(1-\rho)\overline{p}_{H,t} = \frac{(1-\rho)p_{H,t} - \theta_H \Delta \left[\frac{P_{H,t-1}}{P_H} \left(\frac{\frac{P_{H,t-1}}{P_H}}{\frac{P_{H,t-2}}{P_H}} \right)^{\theta_H} \right]^{1-\rho}}{1-\theta_H}$$

$$\begin{aligned}
(1-\theta_H)(1-\rho)\overline{p}_{H,t} &= (1-\rho)p_{H,t} - \theta_H(1-\rho)\left[p_{H,t-1} + \theta_H\left(p_{H,t-1} - p_{H,t-2}\right)\right] \\
(1-\theta_H)\overline{p}_{H,t} &= p_{H,t} - \theta_H\left[p_{H,t-1} + \theta_H \pi_{H,t-1}\right] \\
(1-\theta_H)\overline{p}_{H,t} &= p_{H,t} - p_{H,t-1} + p_{H,t-1} - \theta_H p_{H,t-1} - \theta_H^2 \pi_{H,t-1} \\
(1-\theta_H)\overline{p}_{H,t} &= \pi_{H,t} + (1-\theta_H)p_{H,t-1} - \theta_H^2 \pi_{H,t-1}
\end{aligned}$$

After rearranging we get equation (33):

$$\pi_{H,t} = (1-\theta_H)(\overline{p}_{H,t} - p_{H,t-1}) + \theta_H^2 \pi_{H,t-1}$$

THE ORIGINAL DATA

Following figures contain original data used for the solving of the model. There are original data (Figure 14), data used for the estimation of the model (Figure 15) and data of macroeconomic productivity (Figure 16)

Figure 14: Original Natural Data

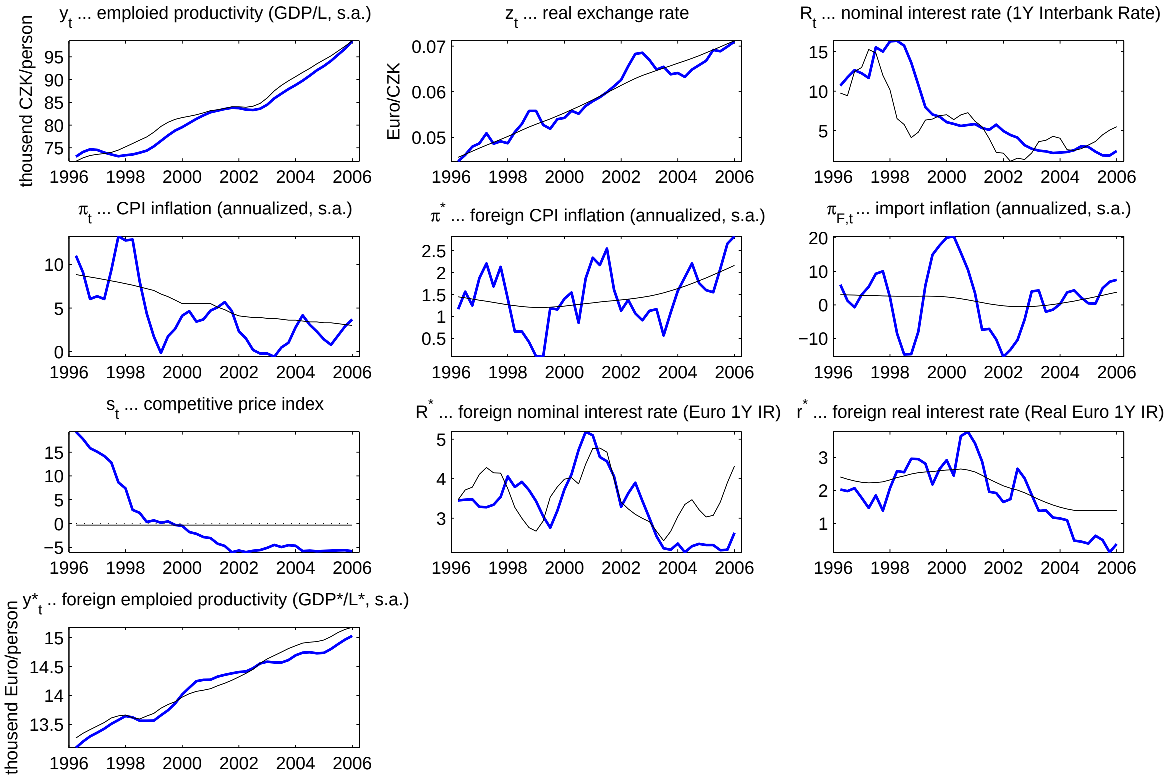


Figure 15: Data to the Czech Open Economy DSGE-based New Keynesian Model

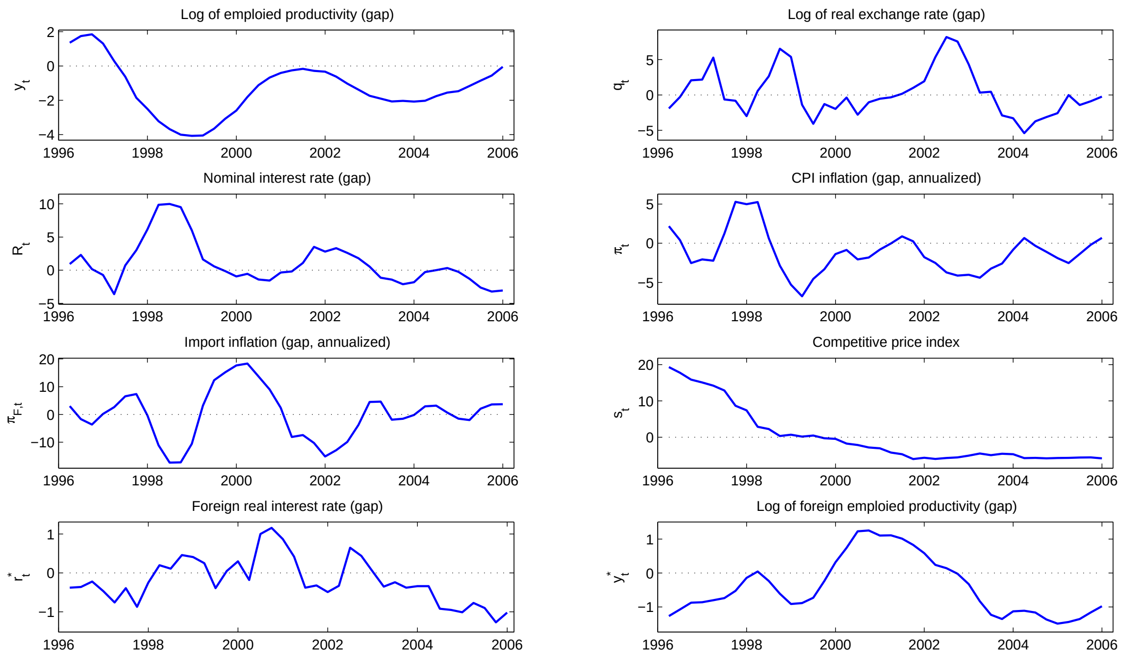
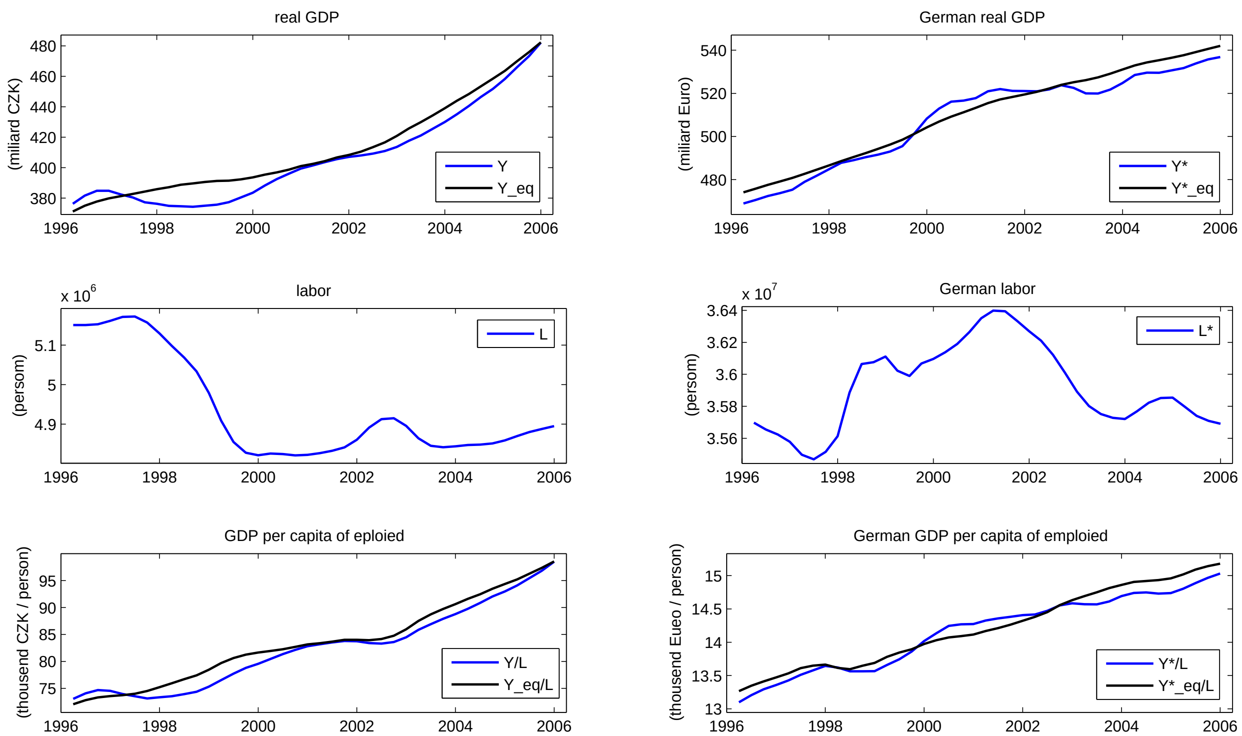


Figure 16: Data of Macroeconomic Productivity



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