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Monetary Policy and Stability of Czech Economy: Optimal Commitment Policy in NOEM DSGE Framework

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Abstract:

This article estimates the dynamic behavior of the Czech economy and preferences of the Czech National Bank. New Keynesian DSGE small open economy model developed by J. Gali and T. Monacelli with optimal commitment monetary policy is considered. The article uses the solution for optimal commitment policy proposed by R. Dennis. Estimates of the model parameters are obtained by Bayesian estimation technique with use of the Metropolis-Hastings algorithm and the Kalman filter. The diagnostics proposed by R. Brooks and A. Gelman and J. Geweke are carried out to examine the convergence of the Markov chain. The bahavior of the Czech economy is strongly dependent on production technology and the foreign economy development. Latter is the result of high openness of the Czech economy. Next, we found out that the Czech National Bank pays little attention to output stabilization in comparison to its concern over inflation targeting. The CNB attaches the highest importance to inflation stabilization. This result is in accordance with proclaimed monetary policy of the Czech National Bank.

Abstrakt:

V tomto článku je odhadnuto dynamické chování české ekonomiky a preference České národní banky. K odhadu je použit Novokeynasiánský DSGE model malé otevřené ekonomiky odvozen J. Galím a T. Monacellim zahrnující optimální monetární politiku se závazkem. V článku je použito řešení optimální monetární politiky navržené R. Dennisem. Bayesiánský odhad modelu je proveden užitím Metropolis-Hastingsova algoritmu a Kalmanova filtru. Konvergence vygenerovaného Markovského řetezce je vyšetřena pomocí diagnostik navržených J. Gewekem a S. Brooksem spolu s A. Gelmanem. Chování české ekonomiky je silně závislé na vývoji produkční technologie a vývoji v zahraničí. Silná závislost na zahraničí je důsledkem velké otevřenosti české ekonomiky. Dále jsme zjistili, že Česká národní banka věnuje jen malou pozornost stabilizaci produkce v porovnání se stabilizací inflace. Největší důležitost přikládá Česká národní banka stabilizaci inflace. Tento výsledek je v souladu s prohlášenou monetární politikou České národní banky.

Recenzoval: Ing. Petr Harasimovič

1 Introduction

In this article we focuse on estimation of the Czech economy structural characteristics and dynamics. We research the responses of the Czech economy to different shocks and the stabilization policy of the Czech National Bank (CNB).

We use Gali and Monacelli's New Keynesian concept of the small open economy developed in [12]. Two types of frictions are incorporated in this concept. These are households' habit in consumption and monopolistic competition in the sector of domestic producers and importers together with sticky prices à la Calvo. This concept was considered in the case of Czech economy by Musil and Vašíček in [22], but they incorporated Taylor-type rule in the model whereas in this article the central bank is treated as an optimizing agent.

We incorporated optimal commitment monetary policy to the model as Dennis proposed in [7]. This approach has two appealing features. The first one is that it allows us to estimate central bank's preferences. In case of Taylor-type rule specification, the central bank's preferences and sensitivity of macroeconomic variables to the nominal interest rate are mixed up in the parameters of the Taylor rule. The second feature is that the parameters representing the central bank's preferences are more "deep". It means they are more robust to structural changes.

Similar estimates in case of the central bank of Canada, New Zealand, and Australia and optimal discretionary policy were done by Kam, Lees and Liu in [17]. The comparison of model structures with Taylor rule and optimal monetary policy rule in terms of data-fit capability is undertaken by Dennis in [9], but Dennis considered only discretionary policy.

2 The Model

This section introduces the New Keynesian DSGE model of a small open economy with optimal commitment monetary policy. It briefly describes the behaviour of particular agents in the economy in terms of dynamic optimization. The first order conditions (FOCs) of such optimization problems are given here. Four types of agents in home economy are considered. They are households, producers, importers, and a monetary authority. Some important variables connecting home economy with foreign economy are also introduced in this part of the text.

The section begins with household's decision problem, then introduces the variables and equations which established a relationship between home and foreign economy. The section proceeds with producer's and importer's decision problem and a formulation of a goods market-clearing condition. Finally, the behaviour of monetary authority is formulated.

2.1 The Households

The model assumes that there exists a continuum of identical infinitely living households in home economy. The households consume goods and supply the producers with labour. The model supposes perfect competition on labour market, households and firms are therefore not able to influence the wage. The model assumes time-separable utility function with period utility given as

$$U(C_t, C_{t-1}, N_t) = \frac{(C_t - hC_{t-1})^{1-\sigma}}{1-\sigma} - \frac{N_t^{1+\phi}}{1+\phi}, \tag{1}$$

where C_t is household's consumption in period t, N_t denotes labour hours in period t, $h \in (0, 1)$ is measure of habit persistence in consumption, $\sigma > 0$ is inverse elasticity of intertemporal substitution and $\phi > 0$ is inverse elasticity of labour supply.

The household maximizes the discounted expected utility

$$E_t \sum_{t=0}^{\infty} \beta^t \left(\frac{(C_t - hC_{t-1})^{1-\sigma}}{1-\sigma} - \frac{N_t^{1+\phi}}{1+\phi} \right) \tag{2}$$

subject to its budget constraints

$$P_t C_t + E_t Q_{t,t+1} B_{t+1} \leq B_t + W_t N_t \qquad t = 0, 1, \ldots, \tag{3}$$

where P_t is the overall consumer price index, B_t is the nominal value of risk-free internationally traded bond held at the end of period $t-1$ and $E_t Q_{t,t+1}$ is stochastic discount factor. The relation between stochastic discount factor and nominal interest rate r_t is

$$E_t Q_{t,t+1} = \frac{1}{1+r_t}. \tag{4}$$

The FOCs of the households optimization problems are

$$\beta R_t E_t \left\{ \frac{P_t}{P_{t+1}} \left(\frac{C_{t+1} - hC_t}{C_t - hC_{t-1}} \right)^{-\sigma} \right\} = 1, \tag{5}$$

$$(C_t - hC_{t-1})^{-\sigma} \frac{W_t}{P_t} = N_t^{\phi}, \tag{6}$$

where $R_t = 1 + r_t$. The equation (5) is the intertemporal Euler equation. This equation states the relation between discounted ratio of marginal utility in successive time periods and real interest rate. The equation (6) connects the marginal utility of consumption and marginal disutility of labour to real wage $\frac{W_t}{P_t}$. In other words, the equation (6) states that real wage equals the marginal rate of substitution between leisure and consumption.

At the end of this subsection, it is necessary to mention the equations for overall consumption and price indices. They are

$$C_t = \left((1-\alpha)^{\frac{1}{\eta}} C_{H,t}^{\frac{\eta-1}{\eta}} + \alpha^{\frac{1}{\eta}} C_{F,t}^{\frac{\eta-1}{\eta}} \right)^{\frac{\eta}{\eta-1}}, \tag{7}$$

$$P_t = \left((1-\alpha) P_{H,t}^{1-\eta} + \alpha P_{F,t}^{1-\eta} \right)^{\frac{1}{1-\eta}}, \tag{8}$$

where $C_{H,t}$ is a consumption index of home-produced goods, $C_{F,t}$ is a consumption index of imported goods, $P_{H,t}$ is a price index of goods produced in the home economy, $P_{F,t}$ is a price index of imported goods, $\alpha \in [0,1]$ is the degree of openness of the home economy and $\eta > 0$ is the elasticity of substitution between home and foreign goods.

2.2 The Foreign Economy and Connection between Economies

The model includes foreign economy represented by three AR(1) processes. They are

$$\pi_t^* = a_1 \pi_{t-1}^* + \varepsilon_t^{\pi^*}, \tag{9}$$

$$y_t^* = b_2 y_{t-1}^* + \varepsilon_t^{y^*}, \tag{10}$$

$$r_t^* = c_3 r_{t-1}^* + \varepsilon_t^{r^*}, \tag{11}$$

where π_t^* is gap of foreign inflation rate, y_t^* is foreign output gap, r_t^* is gap of foreign nominal interest rate and $\varepsilon_t^j \sim N(0, \sigma_j^2)$ for $j = \pi^*$, y^*, and r^*.

Home and foreign economies are connected by international financial and production markets. Definitions of several variables are needed first in order to handle this topic. The first of them are terms of trade defined as

$$S_t = \frac{P_{F,t}}{P_{H,t}}, \tag{12}$$

which measures competitive strength of imports to domestic production. After log-linearization and differentiation the equation

$$\Delta s_t = \pi_{F,t} - \pi_{H,t}, \tag{13}$$

is obtained, where $\pi_{F,t}$ is inflation of imports and $\pi_{H,t}$ is inflation of domestic production. Another variables which turn out to be useful are law of one price gap

$$\Psi_t = \frac{Z_t P_t^*}{P_{F,t}}, \tag{14}$$

which is the marginal cost of importers to marginal revenue of importers ratio (P_t^* is foreign price index in period t), and real exchange rate

$$\mathcal{Q}_t = \frac{Z_t P_t^*}{P_t}. \tag{15}$$

It is not difficult to derive the relation among logarithm of terms of trade s_t, logarithm of law of one price gap ψ_t and logarithm of real exchange rate q_t, which takes the form

$$\psi_t = q_t - (1-\alpha)s_t. \tag{16}$$

With the use of definitions and relations above, it is easier to derive implications of international financial and product markets. The former is going to be discussed in the rest of this subsection, while the latter in subsequent subsections.

The model incorporates following three assumptions:

1. the structure of the foreign economy is the same as the home economy with the same structural parameters. More precisely, the foreign economy is a limiting case of home economy as $\alpha \to 0$.
2. complete international financial markets
3. perfect mobility of financial capital

As the consequence of these three assumptions, the uncovered interest parity condition

$$R_t E_t \frac{Z_t}{Z_{t+1}} = R_t^*, \tag{17}$$

must hold. The log-linearized condition could be rewritten in terms of real exchange rate gap as

$$E_t(q_{t+1} - q_t) = (r_t - E_t\pi_{t+1}) - (r_t^* - E_t\pi_{t+1}^*). \tag{18}$$

Another consequence of the assumptions above is equality (through equality of stochastic discount factors) of home and foreign Euler equations (5), both expressed in currency of the small economy

$$\begin{aligned} \beta E_t\left\{\frac{P_t}{P_{t+1}}\left(\frac{C_{t+1}-hC_t}{C_t-hC_{t-1}}\right)^{-\sigma}\right\} &= E_t Q_{t,t+1} = \\ = \beta E_t\left\{\frac{P_t^* Z_t}{P_{t+1}^* Z_{t+1}}\left(\frac{C_{t+1}^*-hC_t^*}{C_t^*-hC_{t-1}^*}\right)^{-\sigma}\right\}&. \end{aligned} \tag{19}$$

If there are no preference shocks, the following relation between domestic and foreign consumption must hold

$$C_t - hC_{t-1} = \nu^*(C_t^* - hC_{t-1}^*)\mathcal{Q}_t^{\frac{1}{\sigma}}, \tag{20}$$

where ν^* is some constant. Log-linear approximation of equation (20) is[1]

$$c_t - hc_{t-1} = y_t^* - hy_{t-1}^* + \frac{1-h}{\sigma} q_t. \tag{21}$$

Equation (21) is called international risk sharing condition.

2.3 The Producers

The production part of the home economy consists of a continuum of firms producing goods. Each producer hires labour on perfectly competitive labour market and produces differentiated goods according to the production function

$$Y_t(i) = A_t N_t(i), \tag{22}$$

where $Y_t(i)$ is the production of the i-th producer, $N_t(i)$ is the amount of hired work, and A_t is technology shock following AR(1) process in logs:

$$\log(A_t) = a_t = \rho_a a_{t-1} + \varepsilon_t^a, \tag{23}$$

where $\rho_a \in (0,1)$ and $\varepsilon_t^a \sim N(0, \sigma_a^2)$.

In this section, the first nominal rigidity is incorporated in the model structure. It is done by assuming monopolistic competition among producers and introducing restrictions on producer's ability to change prices. To do so, Calvo-style price setting behaviour is followed. This means that in each period $\theta_H \in [0,1]$ portion of producers is unable to reoptimize their prices. Such producers just change their prices according to the portion of latest (domestic goods) inflation. The producers, who can reoptimize, set their new prices in order to maximize the stochastic discounted sum of expected future profits subject to demand constraint. Hence, their optimization problem is

$$\begin{aligned}\max_{P_{H,t}(i)} E_t \sum_{s=0}^{\infty} Q_{t,t+s} \theta_H^s Y_{H,t+s}(i) \Bigg[P_{H,t}(i) \left(\frac{P_{H,t+s-1}}{P_{H,t-1}} \right)^{\delta_H} - \\ - P_{H,t+s} MC_{H,t+s} \exp(\varepsilon_{t+s}^H) \Bigg],\end{aligned} \tag{24a}$$

$$Y_{H,t+s}(i) = \left[\frac{P_{H,t}(i)}{P_{H,t+s}} \left(\frac{P_{H,t+s-1}}{P_{H,t-1}} \right)^{\delta_H} \right]^{-\varepsilon} (C_{H,t+s} + C_{H,t+s}^*), \tag{24b}$$

where (24b) is the demand constraint of the i-th firm, $\varepsilon > 1$ denotes the elasticity of substitution among produced goods, $MC_{H,t}$ are the real marginal costs in period t

$$MC_{H,t} = \frac{W_t}{A_t P_{H,t}}, \tag{25}$$

[1] the equality $y_t^* = c_t^*$ is used as consequence of the first assumption

$\varepsilon_t^H \sim N(0, \sigma_H^2)$ is the independent cost-push shock, and $\delta_H \in [0, 1]$ is degree of inflation indexation. The log-linearized FOC of this problem is the well-known Phillips curve for the domestic goods inflation

$$\pi_{H,t} = \beta(E_t \pi_{H,t+1} - \delta_H \pi_{H,t}) + \delta_H \pi_{H,t-1} + \lambda_H (mc_{H,t} + \varepsilon_t^H), \tag{26}$$

where the gap of marginal costs from the steady-state is

$$mc_{H,t} = \phi y_t - (1 + \phi) a_t + \alpha s_t + \frac{\sigma}{1-h}(y_t^* - h y_{t-1}^*) + q_t, \tag{27}$$

and $\lambda_H = (1 - \beta\theta_H)(1 - \theta_H)\theta_H^{-1}$.

2.4 The Importers

This subsection introduces importers to the model. Because the basic idea (hence problem formulation and solution too) is the same as in the case of the producers, this subsection is brief.

The model assumes monopolistic competition among importers and staggered prices à la Calvo. If the importer can reoptimize his price, he sets the price to maximize stochastic discounted sum of expected future profits subject to demand constraint. The optimization problem is the same as (24) except that subscript H is replaced with F, $C_{H,t+s} + C_{H,t+s}^*$ in (24b) is replaced with $C_{F,t+s}$ and real marginal costs of importers are

$$MC_{F,t+s} = \frac{Z_{t+s} P_{t+s}^*}{P_{F,t+s}}. \tag{28}$$

The Phillips curve of the imported goods inflation can be derived in the same way as the Phillips curve for the domestic goods inflation in the Subsection 2.3. The equation of the Phillips curve of the imported goods inflation is

$$\pi_{F,t} = \beta E_t(\pi_{F,t+1} - \delta_F \pi_{F,t}) + \delta_F \pi_{F,t-1} + \lambda_F(\psi_{F,t} + \varepsilon_t^F), \tag{29}$$

with $mc_{F,t} = \psi_{F,t}$, $\lambda_H = (1 - \beta\theta_F)(1 - \theta_F)\theta_F^{-1}$ and independent cost-push shock $\varepsilon_t^F \sim N(0, \sigma_F^2)$.

2.5 The Goods-Market Clearing Condition

The last part of the model structure before proceeding to monetary policy specification is a formalized assumption of market clearing. The condition states that production of the i-th product is equal to the domestic consumption together with export of this product:

$$Y_t(i) = C_{H,t}(i) + C_{H,t}^*(i) \tag{30}$$

With some subsequent computation, integration with respect to i and log-linearization, the final form of the goods-market clearing condition is derived as

$$y_t = (1 - \alpha) c_t + \alpha y_t^* + \alpha\eta s_t + \alpha\eta q_t, \tag{31}$$

2.6 The Monetary Authority

This subsection introduces the monetary authority to the model. It presents the behaviour of the monetary authority as optimal commitment policy. Under the commitment policy, the monetary authority exploits the agents' expectations at the initial period, optimizes, and commits to never do it again.

The model assumes the one period loss function of the monetary authority in a form:

$$L(\tilde{\pi}_t, y_t, \Delta r_t) = \frac{1}{2}[\tilde{\pi}_t^2 + \mu_y y_t^2 + \mu_r (\Delta r_t)^2], \tag{32}$$

where $\tilde{\pi}_t = \sum_{i=0}^{3} \pi_{t-i}/4$ is quarterlized gap of annual inflation and $\Delta r_t = r_t - r_{t-1} + \varepsilon_t^r$, where $\varepsilon_t^r \sim WN(0, \sigma_r^2)$, is targeted change in short-term interest rate. The monetary shock ε_t^r represents central bank's imperfect ability to control nominal interest rate. The parameters μ_y, $\mu_r \in [0, \infty)$ are weights on output stabilization and interest rate smoothing in central bank's decision-making, respectively. Both parameters are expressed relatively to the weight on inflation stabilization.

The algorithm for solving optimal commitment adopted in this article is the one developed by Richard Dennis in [7]. The remainder of the subsection briefly describes the solution proposed by R. Dennis. The interested reader is referred to working paper cited above.

The monetary authority minimizes its loss function

$$\begin{aligned} Loss(t_0, \infty) &= E_{t_0} \sum_{t=t_0}^{\infty} \beta^{t-t_0} 2L(\tilde{\pi}_t, y_t, \Delta r_t) = \\ &= E_{t_0} \sum_{t=t_0}^{\infty} \beta^{t-t_0} [z_t' W z_t + x_t' Q x_t], \end{aligned} \tag{33}$$

subject to the model constraints derived in previous subsections[2], which can be rewritten in the form

$$A_0 z_t = A_1 z_{t-1} + A_2 E_t z_{t+1} + A_3 x_t + A_4 E_t x_{t+1} + A_5 v_t, \tag{34}$$

where $v_t \sim N(0, \Omega)$ is vector of innovations, z_t is the vector of endogenous variables and x_t is the vector of policy instruments, matrices W, Q are positive semi-definite. The concrete vectors z_t, x_t and matrices W, Q within this model are presented in Appendix B. The FOCs of the optimization problem

[2] Appendix A summarizes these constraints.

(33)–(34) can be cast in the form

$$\begin{pmatrix} 0 & A_0 & -A_3 \\ A_0' & W & 0 \\ -A_3' & 0 & Q \end{pmatrix} \begin{pmatrix} \lambda_t \\ z_t \\ x_t \end{pmatrix} = \begin{pmatrix} 0 & A_1 & 0 \\ \beta^{-1} A_2' & 0 & 0 \\ \beta^{-1} A_4' & 0 & 0 \end{pmatrix} \begin{pmatrix} \lambda_{t-1} \\ z_{t-1} \\ x_{t-1} \end{pmatrix} + \\ + \begin{pmatrix} 0 & A_2 & A_4 \\ \beta A_1' & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} E_t \begin{pmatrix} \lambda_{t+1} \\ z_{t+1} \\ x_{t+1} \end{pmatrix} + \begin{pmatrix} A_5 \\ 0 \\ 0 \end{pmatrix} (v_t), \tag{35}$$

where λ_t are Lagrange multipliers. The system (35) is linear difference system with rational expectations. Number of methods to solve this system are available in the literature. The method presented in [2] is employed in this case.

3 The Estimation Technique

This section gives a brief description of the estimation algorithm and the techniques used within the algorithm. The section describes basic ideas of Bayesian estimation, Metropolis-Hastings algorithm and Kalman filter.

3.1 Bayesian Estimation[3]

Following the results of the Section 2, it is possible to write the model in the state-space form:

$$x_{t+1} = A(\theta)x_t + B(\theta)v_{t+1}, \tag{36a}$$

$$y_t = C(\theta)x_t + D(\theta)w_t, \tag{36b}$$

where $\theta \in \Theta$ is a vector of unknown parameters, x_t is state vector, $v_{t+1} \sim N(\mathbf{0}, \mathbf{I})$ is vector of disturbances, y_t is measurement vector in period t and $w_t \sim N(\mathbf{0}, \mathbf{I})$ is vector of measurement errors. The task to be done is to estimate the mean of the parameter vector θ based on the observations $y_1, y_2, \ldots, y_T$. In other words, we would like to compute $\int\limits_{\theta\in\Theta} \theta p(\theta|\Upsilon)\mathrm{d}\theta$, where $p(\theta|\Upsilon)$ is the posterior probability density of θ conditional on set of observations $\Upsilon = (y_1', y_2', \ldots, y_T')'$. Regardless of the integral computation[4], it is important to compute the posterior density $p(\theta|\Upsilon)$. Using the Bayes law, the following equation for the posterior density holds

$$p(\theta|\Upsilon) = \frac{p(\Upsilon|\theta)p(\theta)}{\int\limits_{\tilde{\theta}\in\Theta} p(\Upsilon|\tilde{\theta})p(\tilde{\theta})\mathrm{d}\tilde{\theta}}, \tag{37}$$

[3] Detailed description of Bayesian estimation can be found in [14].

[4] Subsection 3.3 pays attention to solution of this problem.

where $p(\Upsilon|\theta)$ is the data density (it is probability of observed data being generated by the model with parameter vector θ) which can be computed with Kalman filter[5], $p(\theta)$ is the prior density of parameter vector θ. The prior density is chosen with respect to parameters' restrictions, econometrician's personal beliefs, and former estimations of the parameters in literature. The prior density should include relevant information which is not contained in the data set Υ. The denominator in (37) is unfortunately unknown and hardly computable, but it is independent of θ. Hence, taken Υ as given, it is possible to write

$$\pi(\theta) := p(\theta|\Upsilon) = \frac{p(\Upsilon|\theta)p(\theta)}{\int\limits_{\tilde{\theta}\in\Theta} p(\Upsilon|\tilde{\theta})p(\tilde{\theta})\mathrm{d}\tilde{\theta}} = Kg(\theta), \tag{38}$$

where $g(\theta) = p(\Upsilon|\theta)p(\theta)$ can be computed and $K = \frac{1}{\int\limits_{\tilde{\theta}\in\Theta} p(\Upsilon|\tilde{\theta})p(\tilde{\theta})\mathrm{d}\tilde{\theta}}$ is constant.

3.2 Kalman Filter[6]

This subsection shows how to compute the data density $p(\Upsilon|\theta)$ (from the previous section) with Kalman filter. Suppose the model is written in the state-space form (36). If θ is given, the model can be rewritten as

$$x_{t+1} = Ax_t + v_{t+1}, \tag{39a}$$

$$y_t = Cx_t + w_t, \tag{39b}$$

where $v_{t+1} \sim N(\mathbf{0}, BB')$ and $w_t \sim N(\mathbf{0}, DD')$. The shocks v_t and w_τ are assumed to be uncorrelated, which means that

$$E(v_t w'_\tau) = 0 \tag{40}$$

for all t and τ and

$$E(v_t v'_\tau) = 0\,, \text{ for } t \neq \tau\,, \tag{41}$$

$$E(w_t w'_\tau) = 0\,, \text{ for } t \neq \tau\,. \tag{42}$$

If matrices A, B, C, D and vectors of observations y_t for $t = 1, 2, \ldots, T$ are known, the optimal linear least squares estimates of the state vector x_t on the basis of data observed through date t as

$$\hat{x}_{t|t} \equiv \hat{E}(x_t|\Upsilon_t), \tag{43}$$

where

$$\Upsilon_t \equiv (y'_1, y'_2, \ldots, y'_t)' \tag{44}$$

[5] This task is discussed in Subsection 3.2.

[6] Detailed description of Kalman filter can be found in [3], [15] and [21].

can be computed with the Kalman filter. $\hat{E}(x_t|\Upsilon_t)$ is linear projection of x_t on Υ_t and a constant. The Kalman filter is a recursive algorithm, which computes these projections as diagram (45) shows.

$$\hat{x}_{1|0} \rightarrow \hat{x}_{1|1} \rightarrow \hat{x}_{2|1} \rightarrow \hat{x}_{2|2} \rightarrow \cdots \rightarrow \hat{x}_{T|T-1} \rightarrow \hat{x}_{T|T}. \tag{45}$$

At the beginning of the algorithm, the forecast of x_1 based on no information is computed as

$$\hat{x}_{1|0} = E(x_1), \tag{46}$$

therefore mean squared error (MSE) of this forecast is

$$P_{1|0} = E\left\{[(x_1 - E(x_1)][x_1 - E(x_1)]'\right\}, \tag{47}$$

where $P_{t+1|t} \equiv E[(x_{t+1} - \hat{x}_{t+1|t})(x_{t+1} - \hat{x}_{t+1|t})']$.

The Kalman filter consists of two alternating steps. The first is called prediction step and consists of computing forecast of state vector x_{t+1} based on information Υ_t

$$\hat{x}_{t+1|t} = \hat{E}(x_{t+1}|\Upsilon_t) = A\hat{x}_{t|t}, \tag{48}$$

and its MSE $P_{t+1|t}$

$$P_{t+1|t} = E[(x_{t+1} - \hat{x}_{t+1|t})(x_{t+1} - \hat{x}_{t+1|t})'] = AP_{t|t}A' + BB'. \tag{49}$$

The second step is called filtration step and consists of updating the forecast $\hat{x}_{t+1|t}$ on the basis of the new observation y_{t+1}

$$\hat{x}_{t+1|t+1} = \hat{x}_{t+1|t} + P_{t+1|t}C'(CP_{t+1|t}C' + DD')^{-1}(y_{t+1} - C\hat{x}_{t+1|t}), \tag{50}$$

and MSE of the forecast $P_{t+1|t}$

$$P_{t+1|t+1} = P_{t+1|t} - P_{t+1|t}C'(CP_{t+1|t}C' + DD')^{-1}CP_{t+1|t}. \tag{51}$$

If the disturbances v_t, w_t and initial state x_1 are Gaussian, then y_t conditional on Υ_{t-1} is Gaussian with mean $C\hat{x}_{t|t-1}$ and variance $CP_{t|t-1}C' + DD'$. This is important result of the Kalman filter, which allows us to compute data density as

$$p(\Upsilon|\theta) = \prod_{i=1}^{T} f_{Y_i|\Upsilon_{i-1}}(y_i|\Upsilon_{i-1}). \tag{52}$$

3.3 Metropolis-Hastings Algorithm

This subsection contains description of the Metropolis-Hastings algorithm, which is the last self-contained part of the estimation theory used in this article. The algorithm was originally proposed in [20] and generalized in [16]. For detailed description of the algorithm together with sufficient conditions reader is referred to [10], [14] and [25]. The problem Metropolis-Hastings algorithm solves is the computation of the integral

$$E(f) = \int\limits_{\theta \in \Theta} f(\theta)\pi(\theta)\mathrm{d}\theta, \tag{53}$$

where $f(\theta)$ is known function and $\pi(\theta)$ is the probability density which is not exactly known, but has the form

$$\pi(\theta) = Kg(\theta), \tag{54}$$

where $g(\theta)$ can be computed but constant K can not.

The main idea[7] of the Metropolis-Hastings algorithm is to construct the Markov chain X^N of length N, which has the probability density $\pi(\theta)$ as a stationary measure, and following equality holds almost sure

$$\lim_{N \to \infty} \frac{1}{N} \sum_{i=1}^{N} f(X_i^N) = \int\limits_{\theta \in \Theta} f(\theta)\pi(\theta)\mathrm{d}\theta. \tag{55}$$

The construction of the Markov chain X^N is done as follows. Let's define an arbitrary Markov transition density $q(x, y)$ and acceptance function $a(x, y)$ which satisfies $0 \le a(x, y) \le 1$ for all $x, y \in \Theta$. Given the first state of the Markov chain $\theta^{(1)}$, the $(m + 1)$-th state for $m = 1, 2, 3, \ldots, N - 1$ is obtained in the two following steps. Choose a candidate θ for the $(m + 1)$-th state using the transition density $q(x, y)$. Accept the state θ and set $\theta^{(m+1)} = \theta$ with probability $a(\theta^{(m)}, \theta)$ and refuse the candidate and set $\theta^{(m+1)} = \theta^{(m)}$ otherwise.

Hastings in [16] proved, that the constructed Markov chain X^N has the probability density $\pi(\theta)$ as a stationary measure if

$$a(x, y) = \min\left\{1; \frac{\pi(y)q(y, x)}{\pi(x)q(x, y)}\right\} = \min\left\{1; \frac{g(y)q(y, x)}{g(x)q(x, y)}\right\}. \tag{56}$$

In case of this article, $f(\theta) = \theta$ and $\pi(\theta)$ is given in (38) with $g(\theta) = p(\Upsilon|\theta)p(\theta)$, where $p(\Upsilon|\theta)$ can be computed by Kalman filter and $p(\theta)$ is known prior density.

The chosen transition density is

$$q(x, y) = \varphi(x - y), \tag{57}$$

where φ is probability density of multidimensional normal distribution with zero mean and diagonal covariance matrix Σ. Because the transition density is symmetric around zero, the algorithm is called Random Walk Metropolis-Hastings algorithm, and corresponding acceptance function simplifies to

$$a(x, y) = \min\left\{1, \frac{g(y)}{g(x)}\right\}. \tag{58}$$

[7] In fact it is the idea of class of methods called Markov chain Monte Carlo, whose the Metropolis-Hastings algorithm is member of.

3.4 Estimation Algorithm

This subsection concludes the section about estimation technique by describing the estimation algorithm. The parameter vector to be estimated consists of the private sector deep parameters $\{\alpha,\, h,\, \sigma,\, \phi,\, \eta,\, \delta_H,\, \delta_F,\, \theta_H,\, \theta_F\}$, central bank preference parameters $\{\mu_y,\, \mu_r\}$, and exogenous processes parameters $\{a_1,\, b_2,\, c_3,\, \rho_a,\, \sigma_H,\, \sigma_F,\, \sigma_a,\, \sigma_q,\, \sigma_s,\, \sigma_{\pi^*},\, \sigma_{y^*},\, \sigma_{r^*},\, \sigma_r\}$. The parameter β is fixed rather then estimated. The prior densities of parameters are reported in Table 1. We generate two Markov chains of length $N = 1,000,000$ in order to carry out the convergence diagnostics.

At the beginning of the estimation, the initial value $\theta^{(1)}$ of the parameter vector is chosen. Each draw of a m-th state of the Markov chain for $n = 2, 3, \ldots, N$ is done in six steps

- First step generates new parameter vector θ as realization of random walk:
$$\theta = \theta^{(n-1)} + W,$$
where $W \sim N(0, \Sigma)$.

- Second step computes the prior probability $p(\theta)$.

- Third step solves the linear rational expectation system of equations (35) to get state equation. It constructs observation equation. The state-space representation of the model is the result of this step.

- Fourth step computes the model likelihood $p(\Upsilon|\theta)$[8] using Kalman filter.

- Fifth step computes the acceptance probability
$$a(\theta^{(n-1)}, \theta) = \min\left\{1; \frac{p(\Upsilon|\theta)p(\theta)}{p(\Upsilon|\theta^{(n-1)})p(\theta^{(n-1)})}\right\}.$$

- Sixth and final step sets $\theta^{(n)} = \theta$ with probability $a(\theta^{(n-1)}, \theta)$, else it sets $\theta^{(n)} = \theta^{(n-1)}$.

After the simulation of the Markov chain, we have removed the first half of the chain to get rid of initial condition effect. The estimate of expected value of the parameter vector θ is computed as

$$\hat{\theta} = \frac{2}{N} \sum_{i=\frac{N}{2}+1}^{N} \theta^{(i)}. \qquad (59)$$

[8] Υ is the vector of observed data

4 Estimation Results

This section briefly describes the data used in the estimation and then focuses on the results of the estimation and their economic interpretation.

4.1 Data

The data are obtained from CNB, CZSO, EABCN and DSI[9]. We used quarterly data from 1Q1996 to 4Q2007. We did not annualize the data. The used measurements and corresponding model variables are

$\pi_{F,t}$ – Deviation of seasonally adjusted import prices q-o-q inflation from a long run trend. The trend is computed by the HP filter[10].

π_t – Inflation gap is computed as

$$\pi_t = PIE_t - PIETAR_t/4,$$

where PIE is seasonally adjusted CPI q-o-q inflation and PIETAR is y-o-y inflation target.

y_t – Real GDP gap. The gap is obtained from the CNB.

i_t – Gap of 3-month PRIBOR from its long run trend. The trend is computed by the HP filter.

q_t – Real exchange rate CZK/EUR gap. The gap is obtained from the CNB.

s_t – Logarithm of the terms of trade is computed as

$$\log\left(\frac{IPI_t}{EPI_t}\right),$$

where IPI is price index of imported goods and EPI is price index of exported goods. s_t is the deviation from the long run trend. The trend is computed by the HP filter.

π_t^* – Foreign inflation gap is computed as gap of seasonally adjusted q-o-q EMU CPI inflation from the estimated trend. The trend is computed by the HP filter.

y_t^* – EMU real GDP gap. The gap is obtained from the CNB.

i_t^* – Gap of 3-month EURIBOR from its long run trend. The trend is computed by the HP filter.

The data are depicted in Figure 1.

[9] Czech National Bank, Czech Statistical Office, Euro Area Business Cycle Network and Data Service & Information.

[10] Hodrick-Prescott filter.

4.2 Parameter Estimates

In the estimation, we calibrated only parameter β which is the discount factor. The parameter is set to a calibrated value 0.99. This value is common in the literature (see [12], [17], and [22] for Czech economy case). Results of the estimation are reported in Table 2.

We created two Markov chains and carried out three convergence diagnostics. The first one is NSE (numerical standard error) of the posterior mean. Value close to zero is desirable. The second one is the chi-square test of posterior means equality between chains. p-value of the test is computed. Both test are proposed by J. Geweke and can be find in [13]. The last diagnostics done in this article is computation of the potential scale reduction factor (PSRF) which measures convergence of the Markov chains to the stationary distribution. Value close to one is desirable. Detailed description of PSRF is carried out in [4]. The results of diagnostics are reported in Table 3. All three diagnostics for each parameter are close to desired value (NSE and PSRF) or high enough (p-value is higher then 0.05). It is therefore possible to assume that the length of the Markov chain is sufficient to achieve a stationary distribution.

The posterior mean of the parameter α (degree of openness) is 0.78. This means that imports constitute 78 per cent of domestic consumption. The parameter is also the share of inflation of imports in overall domestic inflation. The estimate of the parameter is relatively high and reflects high degree of openness of the Czech economy.[11]

The estimates of the following four parameters represent preferences of households. The first one is the degree of habit in consumption (parameter h). The posterior mean of this parameter is 0.89. Value close to 0.9 is common in the literature. Musil and Vašíček in [22] estimated the same value of this parameter. The interpretation of this value is as follows. In order to achieve the same utility from consumption as in the preceding period, the growth of the consumption has to be as high as 89 per cent of the growth in the last period excluding an effect of long-run growth in consumption. The posterior mean of the parameter σ is 0.53. The parameter represents the inverse elasticity of intertemporal substitution in consumption, thus the elasticity is approximately 1.9. The estimate of the inverse elasticity of labour supply (parameter ϕ) is 2.18. The elasticity is therefore approximately 0.46. The value can be interpreted as the percentage change in labour supply caused by the one per cent change in the real wage. Such low value could reflect specifics of the Czech economy and labour market development.[12] However it is worth mentioning that the posterior density of this parameter is very similar to the prior density indicating possible lack of information on the value of the parameter in the data. The last parameter

[11] [22] calibrated α within almost the same model structure. They set α to value 0.4.

[12] Mainly growth in labour productivity and rigidities on labour market.

which provides us with information about households tastes is the elasticity of substitution between home and foreign goods (parameter η). The posterior mean of this parameter is 0.56 indicating low possibility of substitution between home and foreign goods. Our estimate is slightly higher then estimate in [22].[13] The difference in estimates may be due to growth in possibility of substitution between home and foreign goods during the period under consideration[14].

Following eight parameters (ρ_a, σ_a, δ_H, θ_H, σ_H, δ_F, θ_F, σ_F) provide us with description of firms' behaviour and related exogenous processes in the Czech economy. First five of them are related to domestic producers and remaining three to domestic importers. The estimate of the parameter ρ_a which is the persistence of the technology shock is 0.9. It means that it takes approximately 22 quarters for the shock to be less then 10 per cent of the original magnitude. Hence, the technology shock is the long-lasting one. The estimate of a standard deviation of the technology shock σ_a is 0.76. The measure of the inflation indexation in case of domestic producers (parameter δ_H) is 0.8. It means that the producers who do not reoptimize prices adjust them according to 80 per cent of last period inflation. The probability of being unable to reoptimize price (parameter θ_H) is estimated to value 0.53. Expected average duration of price contract is therefore approximately 2.12 quarters. Estimated standard deviation of the domestic producers' cost-push shock is 0.85.

The estimate of the measure of the inflation indexation in case of importers (parameter δ_F) is 0.87. The estimate is close to the estimate of δ_H. Hence, there is the same inertia in both inflations. The estimate of the parameter θ_F is 0.62. This value means that the average duration of price contract is approximately 2.6 quarters. The estimated standard deviation of the cost-push shock σ_F is 11.28. This value is very high in comparison to the estimates of standard deviations of other shocks. This evidence is less striking when multiplying with the estimate of λ_F.[15] However the resulting value 2.67 is still high. Possible explanation is that the importers' cost-push shock and some other shock (probably UIP and/or terms of trade shock) are not sufficiently distinguishable within the model structure leading to the overestimate of cost-push shock and the underestimate of another shock(s).[16] However this is not the case in domestic inflation Phillips curve, where technology (long-lasting) and cost-push (short-lasting) shocks can be distinguished. The tendency of NK DSGE models to overemphasize the cost-push shocks is investigated by Peersman and Straub in [23].

The advantage of monetary policy formulation within this model compared to Taylor type rule is that it allows us to estimate the preferences of the cen-

[13] They estimated the value 0.38.

[14] Musil and Vašíček used data from 1Q1995 to 4Q2005 while we used data from 1Q1996 to 4Q2007.

[15] $\lambda_F = 0.237$

[16] Especially the estimate of the standard deviation of the UIP shock is quite low.

tral bank. With Taylor type rule, the inference on these preferences is mixed with sensitivity of concrete variables in policy function to interest rate. Another benefit of formulation adopted in this article is that the parameters are more "deep" than in the case of Taylor type rule. The estimated weight on output stabilization and interest rate smoothness is 0.08 and 0.53, respectively. This means that the Czech National Bank is not very interested in output stabilization. This could be a result of transformation development in the Czech Republic. The CNB focused on inflation stabilization and did not stabilize the output possibly affected by inevitable structural changes. The estimated value of parameter μ_r indicates that the CNB cares about smoothing development of interest rate, but not as much as about inflation targeting (μ_r is lesser than one). The most important objective of the CNB is inflation stabilization, because both estimated weights μ_y and μ_r are less than one. This result is in accordance with inflation targeting regime the CNB adopted in 1998.

5 Analysis of Behaviour

This section analyses behaviour of the estimated model. It uses posterior estimates of the parameters presented in Subsection 4.2 and simulates reaction of the model to each one of nine shocks. These are technology, uncovered interest parity, terms of trade, monetary, producers' cost-push, importers' cost-push, foreign output, foreign nominal interest rate, and foreign inflation rate shock. The magnitude of the simulated shocks is one per cent. This should be kept in mind all the time. The reason is that in the following text some of the shocks may be considered more important than the others in terms of impact on (macroeconomic) variables, but in the reality this shock is of smaller magnitude (measured by standard deviation of the shock) than the others. Once again, the impact of the shock as presented here is proportional to the standard deviation of the shock in reality.

5.1 Technology Shock

The responses of the variables to an one per cent technology shock are depicted in Fig. 2. The positive technology shock represents a decrease in marginal costs of domestic producers. The inflation of domestic production falls below the steady-state level and domestic production becomes more competitive (terms of trade increase). Households take advantage of the technology shock and consume more (consumption increases) and substitute domestic production for imports. The output increases as a result of growing demand.

The central bank lowers nominal interest rate to stabilize inflation at a targeted value. Real exchange rate increases because of expansive monetary policy. The raise in real exchange rate has two consequences. The first one is raise of the marginal costs of importers (imports become more expensive) and the second one is raise in competitiveness of exports resulting in further growth of output. Both of these consequences push overall inflation up towards the target.

The technology shock is very persistent, which follows from high estimated value of parameter ρ_a. Especially consumption and output are influenced by technology shock for long time. On the contrary, influence of the shock on inflation of domestic goods and overall inflation is quite short-lasting.

5.2 Uncovered Interest Parity Shock

The effect of an one per cent uncovered interest parity shock is shown in Fig. 3. The positive uncovered interest parity shock means that agents expect depreciation of real exchange rate ($E_t(q_{t+1} - q_t) > 0$, see Eq. (60d) in Appendix A). Real exchange rate initially falls in order to depreciate in following period. This decrease of real exchange rate represents decrease of importers' marginal costs (imports become cheaper) and worsening of domestic producers' export positions (leading to decrease in output). Import inflation decreases because of reduction in importers' marginal costs. Domestic inflation decreases too, because the lower output leads to lower marginal costs of domestic production. CPI inflation decreases because it is weighted mean of domestic and import inflation. The subsequent rise in real exchange rate affects importers and producers (hence output, domestic, import and CPI inflation) in opposite direction.

The central bank loosens monetary policy to raise CPI inflation back to the target. Lowering of nominal interest rate opposes the effect of the shock and causes appreciation of the real exchange rate in the following periods. Households increase their consumption in the first period in response to lower real interest rate.

The impact of the uncovered interest parity shock is apparently the most short-lasting of all nine shocks. Almost all variables reach the steady-state in ten periods (two and half years).

5.3 Terms of Trade Shock

Fig. 4 shows responses of key variables to an one per cent terms of trade shock. The positive shock improves competitiveness of domestic production over foreign production. Domestic and foreign households begin to prefer small economy's production to foreign economy's one as the result of the shock. Hence output and domestic inflation rise. Import inflation decreases because households substitute domestic goods for imports, which leads to a decrease in marginal costs of importers. The effect of rise in domestic inflation and fall in import inflation on CPI inflation partially cancels out and CPI inflation therefore increases only a little.

The central bank reacts to development of CPI inflation and tightens monetary policy. There are two channels how a rise in nominal interest rate leads to a fall in CPI inflation. The first one is based on uncovered interest parity condition. Monetary restriction causes a decrease of real exchange rate. Lower exchange rate means lower marginal costs of imports (imports become cheaper) and therefore a decrease of import inflation. Second effect of lower exchange rate is worsening of domestic producers' export position. The result is a decline of output and domestic inflation. The CPI inflation decreases because of a decrease in domestic and foreign inflation. The second channel is based on households' intertemporal substitution in consumption. Households postpone their consumption because of positive real interest rate, which leads to a decline of consumption. The CPI inflation also decreases in this case.

5.4 Monetary Shock

The responses of variables to an one per cent monetary shock are depicted in Fig. 5. The positive monetary shock within considered model structure is in fact unwanted monetary expansion as Fig. 5 shows. This is clear from Eq. (60j). The central bank lowers nominal interest rate to balance losses from consequences of monetary expansion and gain from smoothing nominal interest rate (lowering Δr_t).

Real exchange rate rises in the first period in order to appreciate as agents expect (because real interest rate falls). Growth of real exchange rate increases price of imports (importers' marginal costs rise) and competitiveness of domestic goods at international market. Domestic producers begin to produce more goods for export and their marginal costs increase. Domestic and import inflation rise because of higher marginal costs of both producers and importers. CPI inflation have to rise, too.

Development of households' consumption in subsequent periods is determined by real interest rate. If the real interest rate (gap) is negative, households prefer present consumption to future one. In other words, if real interest rate is negative, marginal utility from consumption in present period is lower than marginal utility in the next period. In case of utility function adopted in this text and estimated value of parameter h (close to one), the marginal utility is roughly speaking lower in period t than in period $t+1$ if growth of consumption is higher in period t than in the next period. It is exact if the parameter h equals 1. Because real interest rate is negative in the first three periods, growth of consumption declines. Real interest rate becomes positive and growth of consumption rises at period $t = 3$.

5.5 Importers' Cost-Push Shock

Fig. 7 shows effect of an one per cent importers' cost push shock. The shock represents temporary increase of importers' marginal costs. Hence, import inflation rises. The competitiveness of domestic production over imports increases because of higher prices of imports. Terms of trade rise and push output up. Because of higher import inflation and high degree of openness, overall inflation increases.

The central bank tightens monetary policy to push inflation back to the target. Direct consequence of monetary restriction is increase of real interest rate. Higher real interest rate leads to stabilization of the economy in two different ways. The agents expect depreciation (growth) of real exchange rate because of higher real interest rate. The real exchange rate therefore appreciates in initial period to depreciate later. Lower exchange rate decreases marginal costs of importers and import inflation falls. Deterioration of domestic producers' export position is another effect of a lower exchange rate. The decrease of demand for exports partially cancels out the increase in terms of trade.

Households postpone their consumption because price of present consumption rises. Consumption decreases. Producers have to lower their production. The decrease of households' consumption counteracts the increase in terms of trade. It means that households switch from imports to domestic production but also decrease the overall consumption. This is the second way how a rise in the real interest rate stabilizes inflation.

5.6 Foreign Output Shock

Fig. 8 shows how a rise of foreign output affects Czech economy. The rise in foreign output directly increases domestic output, because demand for domestic production in foreign economy rises. Domestic producers are able to produce more only with higher marginal costs. Hence domestic inflation rises too.

The central bank rises nominal interest rate in order to face inflationary pressures. Real exchange rate reacts to the change in real interest rate, decreases in first two periods and approaches the steady-state thereafter. Marginal costs of importers and competitiveness of exports decreases because of low real exchange rate. This facts create disinflationary pressure to both domestic and foreign inflation. Consumption develops according to real interest rate after a decrease in the first period. The central bank's intervention also helps to decrease inflation by reducing households' consumption.

5.7 Foreign Nominal Interest Rate Shock

The development of the Czech economy after a foreign nominal interest rate shock is depicted in Fig. 9. Foreign real interest rate increases due to the shock. A rise in foreign real interest rate causes the real exchange rate to go up because agents expect its appreciation. Domestic production becomes more competitive abroad and prices of imports increase because of higher real exchange rate. Overall inflation rises because domestic and import inflation do as well.

The central bank raises nominal interest rate to lower existing real interest rate differential. Lowering interest rate differential pushes real exchange rate back to its steady-state.

Households face unfavourable conditions on the market of domestic goods and imports and lower their consumption. After this decline, their consumption rises because positive real interest rate makes present consumption more expensive in comparison with future one. In other words, households substitute future consumption for present one.

5.8 Foreign Inflation Shock

Impulse responses to a foreign inflation rate shock are quite similar to a foreign nominal interest rate shock, but variables move in the opposite direction. The reason for this is that rise in foreign inflation causes a decrease in foreign real interest rate. This is clear from Eq. (18). The responses differ a little in magnitude, because initial value of foreign real interest rate is $\varepsilon_0^{r^*} = 1$ in the case of interest rate shock and $-E_0\pi_1^* = -a_1\varepsilon_0^{\pi^*} = -a_1$ in the case of inflation shock. The effects of the shocks also differ slightly in duration because of difference in persistence of the shocks measured by parameters a_1 and c_3. Impulse responses to an one per cent foreign inflation shock are depicted in Fig. 10. Their interpretation is analogous to that in previous subsection.

5.9 General Findings about the Behaviour

At the beginning of this section, it was noted that impact of an one per cent shocks should be considered together with estimates of their standard deviation. This subsection is going to do this. Some general aspects of behaviour of the Czech economy are discussed also in this subsection.

The technology shock has the most persistent impact on the Czech economy. This is the result of the high estimated value of the parameter ρ_a. It is clear from the impulse responses that technology shock affects the economy more than other shocks do. This fact remains true if the standard deviations of the shocks are taken into account. Only importer's cost-push shock has stronger impact in this case, but this will be discussed later. The technology shock is very important shock for the development of the Czech economy.

The uncovered interest parity shock is also very influencing due to high degree of openness of the Czech economy. If the standard deviation of this shock is considered, the impact is very small. Nevertheless, in this case we think the standard deviation of this shock is underestimated (see subsection 4.2 for more details) and this shock is more important in reality. This shock is very temporary.

The terms of trade shock influences the Czech economy a lot, no matter if its standard deviation is considered or not. This means that competitiveness of domestic production to imports is very important. This importance is raised by the degree of openness but on the other hand is reduced by the lower elasticity of substitution between home and foreign goods (parameter η).

If the standard deviation of the monetary shock is taken into account, the impact of this shock will diminish a lot. Hence the CNB controls nominal interest rate quite well in reality and does not destabilize the economy.

The producer's and importer's cost-push shocks are found less significant. The reason for this is that the effect of both shocks is affected by the degrees of price rigidity θ_H and θ_F, respectively. The higher is the degree of price rigidity (parameter θ_H or θ_F) the weaker is the effect of the shock, because the firms are less able to change their prices according to marginal costs. If standard deviations are considered, the importer's cost-push shock turns out to be very important. But we think the estimate of the standard deviation of this shock is overestimated because it includes volatility of real exchange rate shock (see subsection 4.2).

The effect of the foreign output shock is significant even if the standard deviation is considered. This is a direct consequence of the high degree of openness of the Czech economy. The reason for this is that foreign output gap represents the foreign households' consumption gap. Hence if the foreign output gap rises, the domestic production (export production) will rise proportionally to foreign output gap and the degree of openness α (see Eq. (60g)).

It is no surprise that the change in the foreign real interest rate affects the Czech economy significantly. It is a consequence of high degree of openness of the Czech economy. If standard deviation of foreign real interest rate shock is taken into account, the significance of this shock lessens noticeably.[17]

Based on the impulse responses it is evident that interaction of Czech and foreign economy is of great importance to stability of Czech economy. Czech output and inflation are noticeably influenced by development abroad. The factors which measure competitiveness of domestic goods to imports and for-

[17] Because foreign interest rate is $r_t^* - E_t\pi_{t+1}^*$, variance of foreign real interest rate shock is therefore $\sigma_{r^*}^2 + (a_1\sigma_{\pi^*})^2$.

eign production to exports are very important. These are real exchange rate and terms of trade.[18] The inflation of imports constitutes approximately 78 per cent of Czech CPI inflation.

The model includes two transmission channels of monetary policy. These are real exchange rate and real interest rate channels. They differ in duration and impact strength of the transmission.

The real exchange rate channel has relatively strong impact, which is supported by high degree of openness. The change in real interest rate has direct influence on export competitiveness leading to proportional (with coefficient $\alpha\eta$) change in domestic production. The change influences competitiveness of imports to domestic production, which results in further change in domestic output and overall inflation. This transmission channel seems to be very important in the case of Czech economy, but its transmission speed is high and real exchange rate stabilizes at the steady-state quickly.

The real interest rate channel consists in affecting relative price of present and future consumption, which results in a change in households' consumption. This change is weakened by households' persistence in consumption. The high openness of the Czech economy lowers the transmission effect too. Hence, the impact of the monetary policy through this transmission channel is weaker and the speed of the transmission is slower than in case of real exchange rate channel.

6 Conclusion

This article estimated the dynamic behavior of the Czech economy and CNB's preferences in conducting monetary policy. The estimate was done within the New Keynesian model of small open economy developed by Gali and Monacelli in [12]. This model structure includes nominal rigidities (monopolistic competition and Calvo style price setting behavior in sector of domestic producers and importers) and real rigidity (household's habit formation in consumption).

The central bank was treated as an optimizing agent minimizing its expected loss. Three objectives were incorporated in the central bank's loss function. They were inflation stabilization, output stabilization, and interest rate smoothing. The article used the solution algorithm for optimal commitment proposed by Dennis in [7].

The random walk Metropolis-Hastings algorithm was used to estimate the model's parameters. Two independent Markov chains (each containing 1 000 000 draws) were generated by the algorithm. Convergence diagnostics proposed by Brooks and Gelman in [4] and Geweke in [13] were carried out and both diagnostics indicated convergence of the Markov chain to the stationary

[18] See Eq. (60g), Eq. (60b) and Eq. (60c).

distribution.

We found important the estimate of parameter α (portion of foreign goods in domestic consumption at steady-state), which is 0.78. This value support the fact, that the Czech economy is very open economy. Such high degree of openness has direct effect on behavior of the Czech economy. The estimated values of other parameters are in most cases acceptable with regard to structural characteristics of the Czech economy. Nevertheless, particular problems with the estimation of same parameters occurred. The estimate of the parameter ϕ (the inverse elasticity of labor supply) may suffer from insufficient information about the parameter in the data. The high estimated value of the parameter σ_F (variance of the cost-push shock to importers) may be caused by poor discriminability of some shocks in the model structure resulting in the overestimating of the parameter σ_F and possibly the underestimating of the parameters σ_q and σ_s.

Treating the central bank as another optimizing agent enabled us to estimate the weights (representing the CNB's preferences) the CNB attaches to the three objectives mentioned above in providing monetary policy. We found that the CNB attaches the heaviest weight to the inflation stabilization, which is in accordance with the inflation targeting regime the CNB provides. The weight the CNB attaches to the output stabilization was found very low compared to the weight on the inflation stabilization (approximately 9 per cent of this weight). This value might indicate that the CNB cares only a little about output stabilization. The posterior estimate of the weight on the interest rate smoothing was 0.526 in terms of the weight on the inflation stabilization.

The development of the Czech economy is significantly influenced by development of technology and the foreign economy. The sensitivity about development abroad is direct consequence of high degree of openness of the Czech economy. The most important among foreign macroeconomic variables is foreign output gap, because the standard deviation of its shock is highest.

The low estimated value of the standard deviation of the monetary shock indicates, that the CNB is unimportant cause of fluctuations of the Czech economy. The monetary policy actions propagate in adopted model through two transmission channels. Based on the behavior analysis we found, that these channels differ in duration and strength of impact. The real exchange rate channel is of high influence and short duration. The importance of this channel is consequence of high degree of openness of the Czech economy. The real interest rate channel affects the economy more gradually because of household's habit formation in consumption.

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Figures and Tables

Table 1: Prior Densities

Parameter	Function	mean	std. deviation
β	point mass	0.99	–
α	Beta	0.7	0.1
h	Beta	0.8	0.1
σ	Gamma	0.5	0.2
ϕ	Gamma	2	0.35
η	Gamma	0.6	0.25
δ_H	Beta	0.8	0.1
δ_F	Beta	0.8	0.1
θ_H	Beta	0.5	0.1
θ_F	Beta	0.6	0.1
a_1	Gamma	0.7	0.1
b_2	Gamma	0.9	0.1
c_3	Gamma	0.8	0.1
ρ_a	Beta	0.85	0.1
μ_y	Gamma	0.3	0.15
μ_r	Gamma	0.6	0.15
σ_H	Gamma	1	0.5
σ_F	Gamma	11	1
σ_a	Gamma	0.8	0.1
σ_q	Gamma	0.1	0.05
σ_s	Gamma	0.7	0.1
$\sigma_{\pi*}$	Gamma	0.06	0.03
σ_{y*}	Gamma	0.25	0.05
σ_{r*}	Gamma	0.1	0.05
σ_r	Gamma	0.2	0.05

Table 2: Estimation Results

Parameter	Interpretation	Prior mean	5%	95%	Post. mean	5%	95%
α	degree of openness	0.70	0.52	0.85	0.78	0.72	0.84
h	habit in consumption	0.80	0.61	0.94	0.89	0.85	0.92
σ	inverse elasticity of intertemporal substitution	0.50	0.22	0.87	0.53	0.37	0.69
ϕ	inverse elasticity of labour supply	2.00	1.46	2.61	2.18	1.60	2.82
η	elasticity of substitution between home and foreign goods	0.60	0.26	1.06	0.56	0.39	0.78
δ_H	degree of inflation indexation in prices of products	0.80	0.61	0.94	0.80	0.63	0.94
δ_F	degree of inflation indexation in prices of imports	0.80	0.61	0.94	0.87	0.73	0.96
θ_H	fraction of non-optimizing producers	0.50	0.34	0.66	0.53	0.43	0.62
θ_F	fraction of non-optimizing importers	0.60	0.43	0.76	0.62	0.55	0.69
a_1	foreign inflation AR(1) parameter	0.70	0.54	0.87	0.68	0.53	0.83

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Table 2: Estimation Results

	Interpretation	Prior mean	5%	95%	Post. mean	5%	95%
b_2	foreign output AR(1) parameter	0.90	0.74	1.07	0.91	0.85	0.97
c_3	foreign interest rate AR(1) parameter	0.80	0.64	0.97	0.75	0.62	0.89
ρ_a	inertia of technology	0.85	0.66	0.97	0.90	0.84	0.95
μ_y	weight on output stabilization	0.30	0.10	0.58	0.08	0.04	0.13
μ_r	weight on interest rate smoothing	0.60	0.38	0.87	0.53	0.34	0.75
σ_H	std. deviation of producers' cost-push shock	1.00	0.34	1.94	0.85	0.63	1.16
σ_F	std. deviation of importers' cost-push shock	11.00	9.41	12.70	11 .28	9.82	12.79
σ_a	std. deviation of technology shock	0.80	0.64	0.97	0.76	0.61	0.93
σ_q	std. deviation of UIP shock	0.10	0.03	0.19	0.02	0.01	0.04
σ_s	std. deviation of terms of trade shock	0.70	0.54	0.87	0.62	0.50	0.76

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Table 2: Estimation Results

	Interpretation	Prior mean	5%	95%	Post. mean	5%	95%
$\sigma_{\pi*}$	std. deviation of foreign inflation shock	0.06	0.02	0.12	0.02	0.01	0.04
σ_{y*}	std. deviation of foreign output shock	0.25	0.17	0.34	0.26	0.21	0.30
σ_{r*}	std. deviation of foreign interest rate shock	0.10	0.03	0.19	0.03	0.02	0.04
σ_r	std. deviation of monetary shock	0.20	0.13	0.29	0.08	0.06	0.11

Table 3: Convergence Diagnostics

	Post Mean	**Post Std**	**2.5%**	**97.5%**	**NSE**	**p-value**	**PSRF**
β	0.990	0.000	0.990	0.990	0.000	1.000	1.000
α	0.776	0.037	0.699	0.844	0.002	0.120	1.009
h	0.887	0.019	0.848	0.920	0.001	0.877	1.000
σ	0.526	0.104	0.333	0.733	0.007	0.939	1.000
ϕ	2.181	0.370	1.518	2.968	0.010	0.707	1.000
η	0.556	0.121	0.374	0.840	0.010	0.210	1.015
δ_H	0.804	0.095	0.597	0.954	0.005	0.437	1.003
δ_F	0.869	0.071	0.707	0.972	0.004	0.778	1.000
θ_H	0.529	0.058	0.410	0.633	0.002	0.539	1.001
θ_F	0.624	0.041	0.541	0.703	0.001	0.917	1.000
a_1	0.675	0.091	0.504	0.860	0.001	0.959	1.000
b_2	0.913	0.038	0.832	0.982	0.001	0.091	1.001
c_3	0.753	0.081	0.597	0.913	0.001	0.314	1.000
ρ_a	0.897	0.033	0.828	0.955	0.002	0.065	1.014
μ_y	0.077	0.029	0.034	0.146	0.001	0.244	1.003
μ_r	0.528	0.133	0.304	0.813	0.010	0.300	1.011
σ_H	0.852	0.150	0.602	1.167	0.012	0.234	1.015
σ_F	11.284	0.904	9.574	13.105	0.051	0.866	1.000
σ_a	0.763	0.097	0.585	0.966	0.001	0.103	1.001
σ_q	0.021	0.012	0.005	0.051	0.000	0.099	1.002
σ_s	0.622	0.081	0.475	0.791	0.001	0.083	1.001
$\sigma_{\pi*}$	0.024	0.009	0.009	0.045	0.000	0.451	1.000

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Table 3: Convergence Diagnostics

	Post Mean	Post Std	2.5%	97.5%	NSE	p-value	PSRF
σ_{y*}	0.257	0.028	0.206	0.314	0.000	0.528	1.000
σ_{r*}	0.029	0.008	0.015	0.047	0.000	0.866	1.000
σ_{r}	0.082	0.015	0.054	0.111	0.000	0.429	1.001

Figure 1: Data used for the estimation

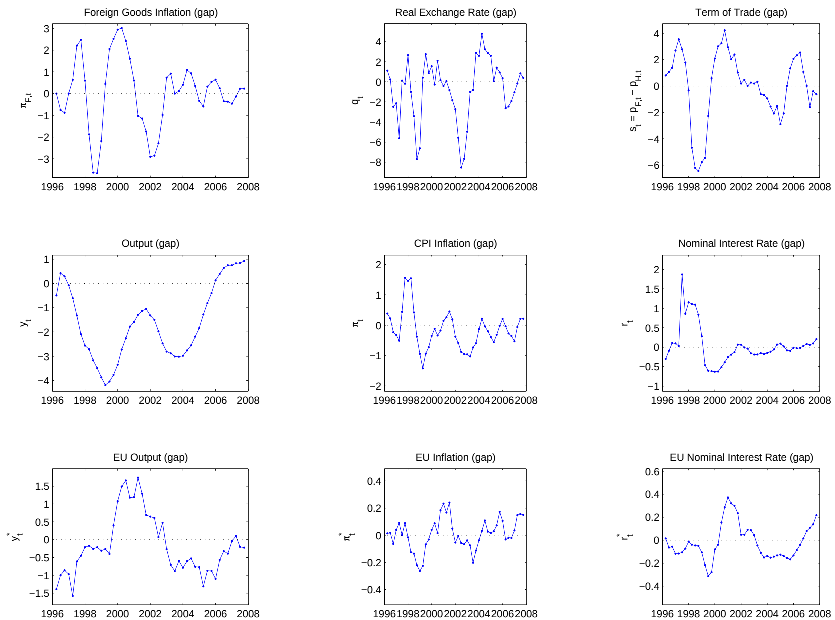


Figure 2: Technology shock

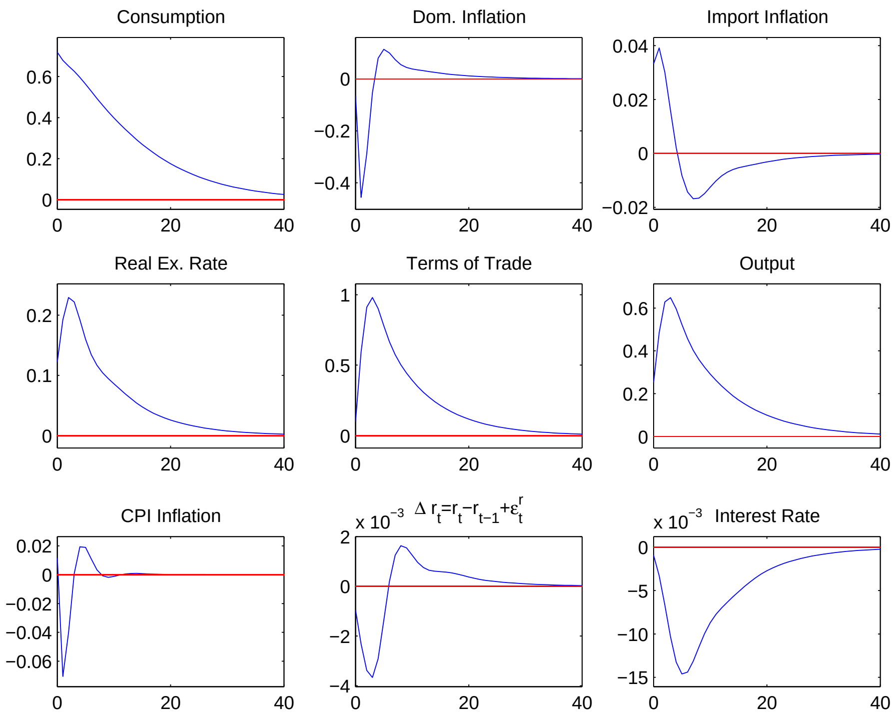


Figure 3: Uncovered interest parity shock

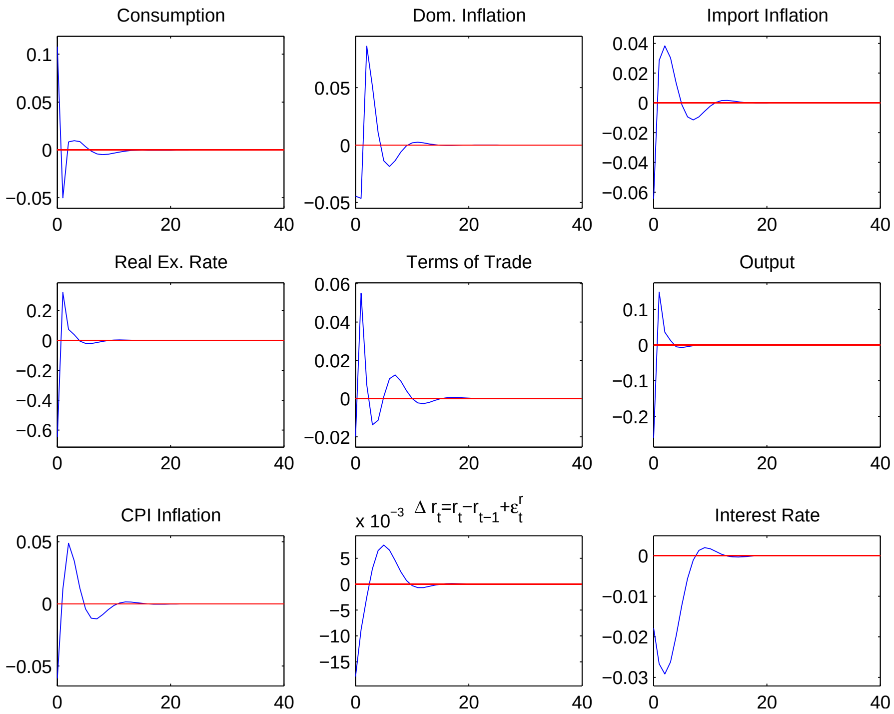


Figure 4: Terms of trade shock

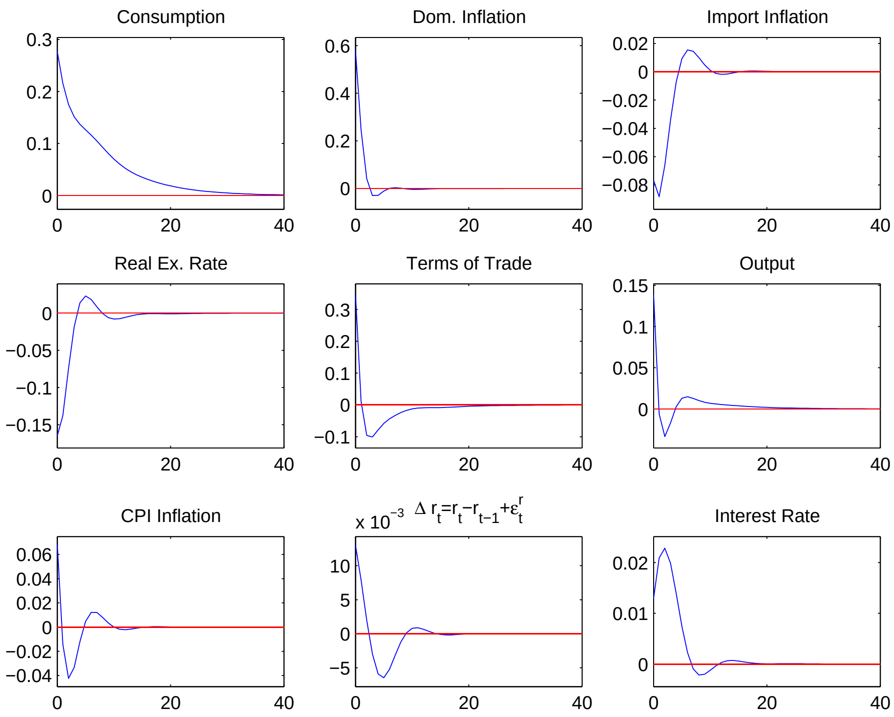


Figure 5: Monetary shock

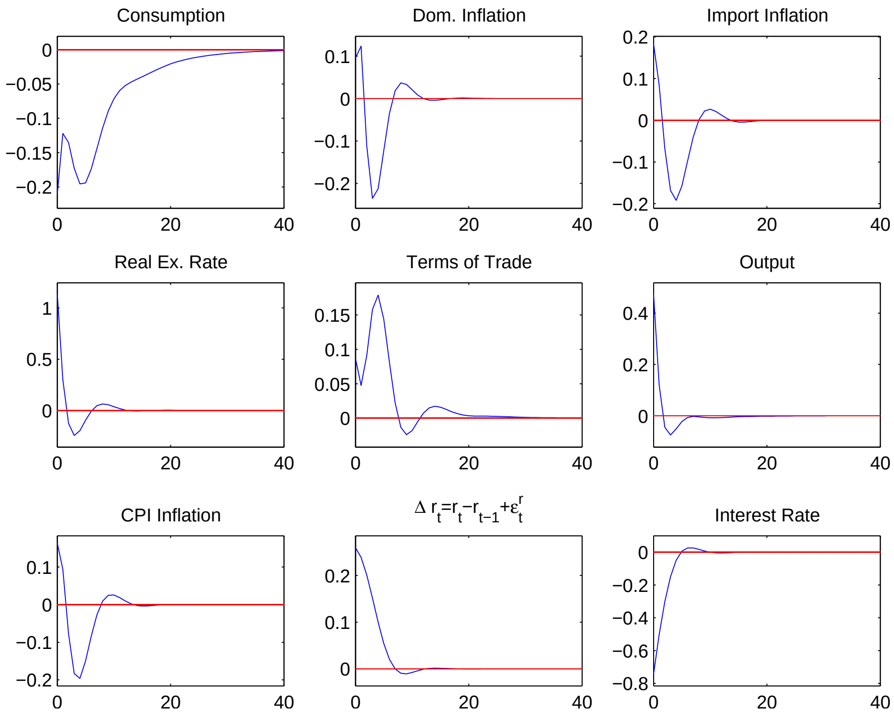


Figure 6: Producers' cost-push shock

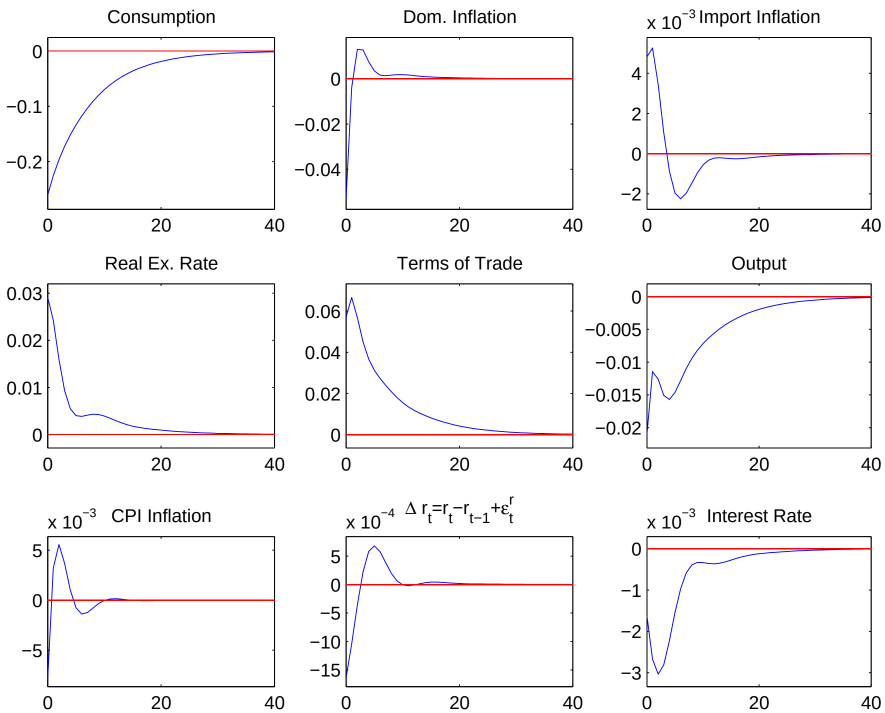


Figure 7: Importers' cost-push shock

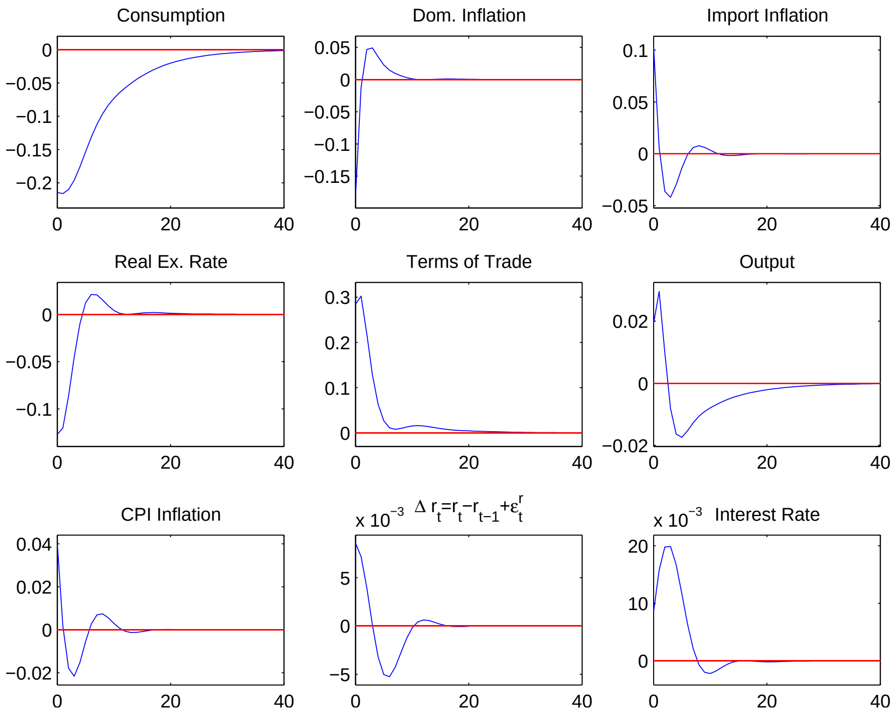


Figure 8: Foreign output shock

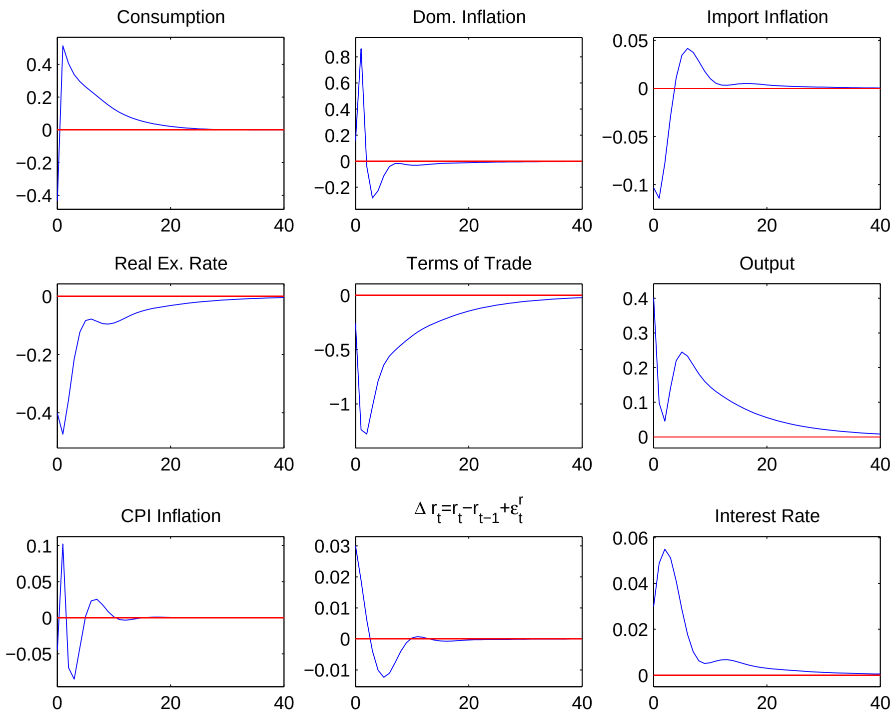


Figure 9: Foreign nominal interest rate shock

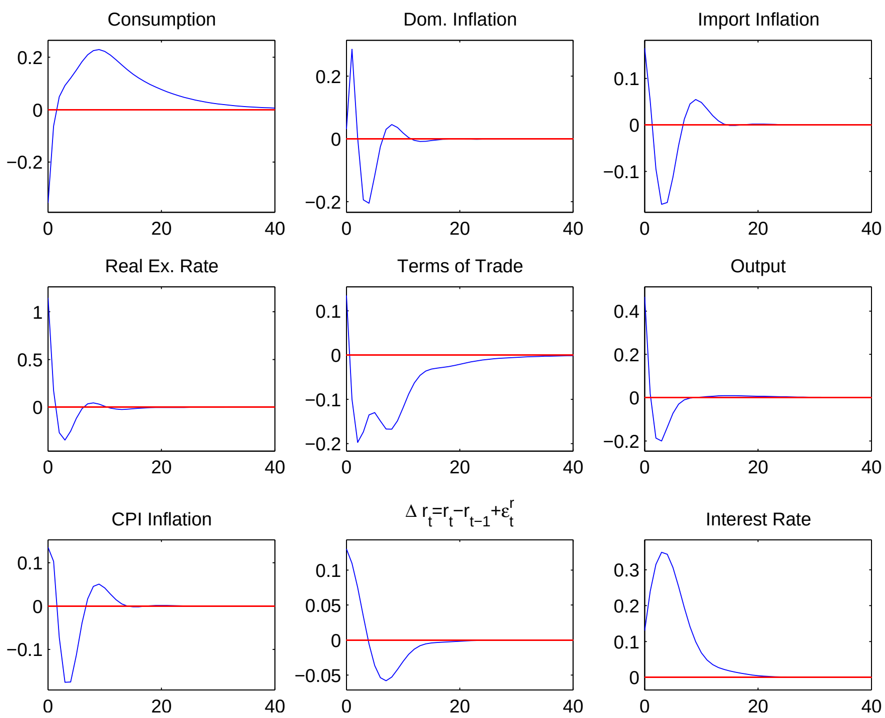
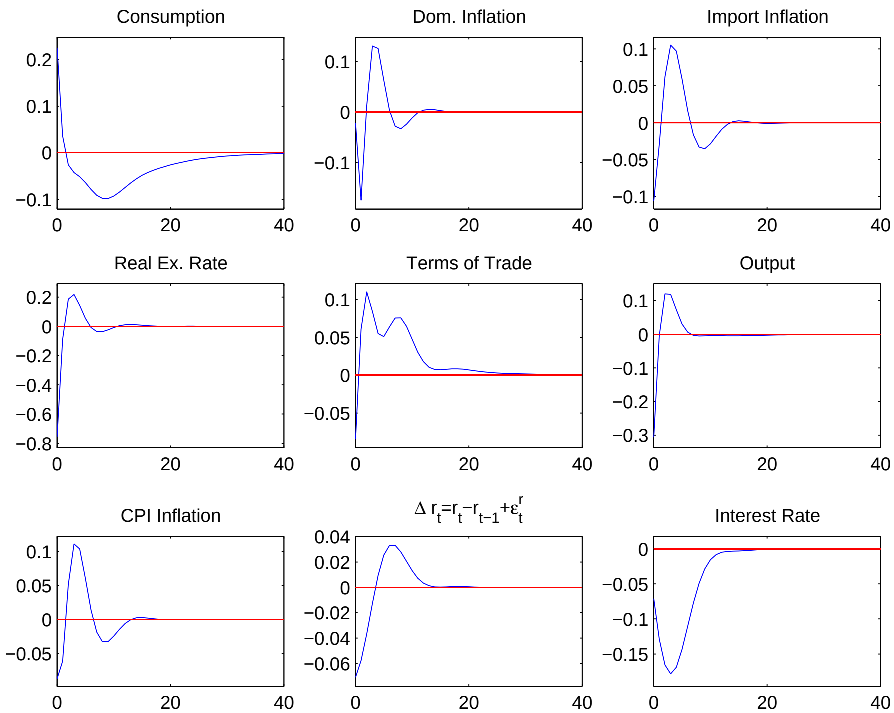


Figure 10: Foreign inflation rate shock



A Model without Monetary Policy

The model without monetary policy consists of the following equations:

$$c_t - hc_{t-1} = E_t(c_{t+1} - hc_t) - \frac{1-h}{\sigma}(r_t - E_t\pi_{t+1}), \tag{60a}$$

$$\begin{aligned} \pi_{H,t} &= \beta E_t(\pi_{H,t+1} - \delta_H \pi_{H,t}) + \delta_H \pi_{H,t-1} + \\ +\lambda_H &\left[\phi y_t - (1+\phi)a_t + \alpha s_t + \frac{\sigma}{1-h}(c_t - hc_{t-1})\right] + \lambda_H \varepsilon_t^H, \end{aligned} \tag{60b}$$

$$\pi_{F,t} = \beta E_t(\pi_{F,t+1} - \delta_F \pi_{F,t}) + \delta_F \pi_{F,t-1} + \lambda_F[q_t - (1-\alpha)s_t] + \lambda_F \varepsilon_t^F, \tag{60c}$$

$$E_t(q_{t+1} - q_t) = (r_t - E_t\pi_{t+1}) - (r_t^* - E_t\pi_{t+1}^*) + \varepsilon_t^q, \tag{60d}$$

$$c_t - hc_{t-1} = y_t^* - hy_{t-1}^* + \frac{1-h}{\sigma}q_t \tag{60e}$$

$$s_t - s_{t-1} = \pi_{F,t} - \pi_{H,t} + \varepsilon_t^s, \tag{60f}$$

$$y_t = (1-\alpha)c_t + \alpha\eta q_t + \alpha\eta s_t + \alpha y_t^*, \tag{60g}$$

$$\pi_t = (1-\alpha)\pi_{H,t} + \alpha\pi_{F,t}, \tag{60h}$$

$$\tilde{\pi}_t = \sum_{i=0}^{3} \pi_{t-i}/4, \tag{60i}$$

$$\Delta r_t = r_t - r_{t-1} + \varepsilon_t^r, \tag{60j}$$

$$a_t = \rho_a a_{t-1} + \varepsilon_t^a, \tag{60k}$$

$$\pi_t^* = a_1 \pi_{t-1}^* + \varepsilon_t^{\pi^*}, \tag{60l}$$

$$y_t^* = b_2 y_{t-1}^* + \varepsilon_t^{y^*}, \tag{60m}$$

$$r_t^* = c_3 r_{t-1}^* + \varepsilon_t^{r^*}, \tag{60n}$$

where $\rho_a \in (0,1)$ and $\varepsilon_t^j \sim \mathrm{N}(0, \sigma_j^2)$ for $j = a, s, q, H, F, r, \pi^*, y^*$, and r^*.

B Matrices in Central Bank's Loss Function

The vectors and matrices in central bank's loss function are

$$\begin{aligned} z_t = (&c_t,\ \pi_{H,t},\ \pi_{F,t},\ q_t,\ s_t,\ y_t,\ \pi_t,\ \Delta r_t,\ r_t,\ a_t,\ \pi_t^*,\ y_t^*,\ r_t^*, \\ &\pi_{t-1},\ \pi_{t-2},\ \tilde{\pi}_t)', \end{aligned} \tag{61}$$

$$x_t = (r_t), \tag{62}$$

$$
W = \begin{pmatrix}
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \cdots & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \cdots & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \cdots & 0 & 0 \\
0 & 0 & 0 & \mu_q & 0 & 0 & 0 & 0 & 0 & \cdots & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \cdots & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & \mu_y & 0 & 0 & 0 & \cdots & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \cdots & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & \mu_r & 0 & \cdots & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \cdots & 0 & 0 \\
\vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \ddots & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \cdots & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \cdots & 0 & 1
\end{pmatrix}_{16,16} \tag{63}
$$

$$
Q = (0). \tag{64}
$$

C Producer's Price Setting Behaviour

Optimization problem of the i-th producer is

$$
\begin{aligned}
\max_{\bar{P}_{H,t}} E_t \sum_{s=0}^{\infty} Q_{t,t+s} \theta_H^s Y_{H,t+s}(i) \Bigg[& \bar{P}_{H,t} \left(\frac{P_{H,t+s-1}}{P_{H,t-1}} \right)^{\delta_H} - \\
& - P_{H,t+s} MC_{H,t+s} \exp(\varepsilon_{t+s}^H) \Bigg], \text{ s.t.}
\end{aligned} \tag{65a}
$$

$$
Y_{H,t+s}(i) = \left[\frac{\bar{P}_{H,t}}{P_{H,t+s}} \left(\frac{P_{H,t+s-1}}{P_{H,t-1}} \right)^{\delta_H} \right]^{-\varepsilon} \underbrace{(C_{H,t+s} + C_{H,t+s}^*)}_{=Y_{t+s}}. \tag{65b}
$$

Hence, unconstrained optimization problem is

$$
\begin{aligned}
& \max_{\bar{P}_{H,t}} E_t \sum_{s=0}^{\infty} Q_{t,t+s} \theta_H^s \left[\frac{\bar{P}_{H,t}}{P_{H,t+s}} \left(\frac{P_{H,t+s-1}}{P_{H,t-1}} \right)^{\delta_H} \right]^{-\varepsilon} Y_{t+s} \\
& \left[\bar{P}_{H,t} \left(\frac{P_{H,t+s-1}}{P_{H,t-1}} \right)^{\delta_H} - P_{H,t+s} MC_{H,t+s} \exp(\varepsilon_{t+s}^H) \right].
\end{aligned} \tag{66}
$$

It is possible to write $\bar{P}_{H,t}$ instead of $\bar{P}_{H,t}(i)$, because producers share the same production technology and are price-takers at the labour market. Note that this is not possible in case of Y_t and $Y_t(i)$. The first one states for aggregate output and the second one for production of the i-th producer.

The FOC of the problem (66) is

$$0 = E_t \sum_{s=0}^{\infty} Q_{t,t+s}\theta_H^s \left[\frac{1}{P_{H,t+s}}\left(\frac{P_{H,t+s-1}}{P_{H,t-1}}\right)^{\delta_H}\right]^{-\varepsilon} Y_{t+s}\Bigg[(1-\varepsilon)\bar{P}_{H,t}^{-\varepsilon} \left(\frac{P_{H,t+s-1}}{P_{H,t-1}}\right)^{\delta_H} + \varepsilon \bar{P}_{H,t}^{-(\varepsilon+1)} P_{H,t+s} MC_{H,t+s}\exp(\varepsilon_{t+s}^H)\Bigg]. \tag{67}$$

Now, let's rearrange the FOC to get more favourable form:

$$0 = E_t \sum_{s=0}^{\infty} Q_{t,t+s}\theta_H^s \left[\frac{\bar{P}_{H,t}}{P_{H,t+s}}\left(\frac{P_{H,t+s-1}}{P_{H,t-1}}\right)^{\delta_H}\right]^{-\varepsilon} Y_{t+s} \left[(1-\varepsilon)\left(\frac{P_{H,t+s-1}}{P_{H,t-1}}\right)^{\delta_H} + \varepsilon \bar{P}_{H,t}^{-1} P_{H,t+s} MC_{H,t+s}\exp(\varepsilon_{t+s}^H)\right] \tag{68}$$

$$0 = E_t \sum_{s=0}^{\infty} Q_{t,t+s}\theta_H^s Y_{t+s}(i)\Bigg[\bar{P}_{H,t}\left(\frac{P_{H,t+s-1}}{P_{H,t-1}}\right)^{\delta_H} + + \frac{\varepsilon}{1-\varepsilon} P_{H,t+s} MC_{H,t+s}\exp(\varepsilon_{t+s}^H)\Bigg] \tag{69}$$

$$0 = E_t \sum_{s=0}^{\infty} (\beta\theta_H)^s \frac{P_t}{P_{t+s}}\left(\frac{C_t - hC_{t-1}}{C_{t+s} - hC_{t+s-1}}\right)^{\sigma} Y_{t+s}(i) \left[\bar{P}_{H,t}\left(\frac{P_{H,t+s-1}}{P_{H,t-1}}\right)^{\delta_H} + \frac{\varepsilon}{1-\varepsilon} P_{H,t+s} MC_{H,t+s}\exp(\varepsilon_{t+s}^H)\right]. \tag{70}$$

In the last equation, relations $E_t Q_{t,t+1} = \beta E_t \left\{ \frac{P_t}{P_{t+1}} \left(\frac{C_t - hC_{t-1}}{C_{t+1} - hC_t}\right)^{\sigma} \right\}$ which follows from equations (4) and (5), $Q_{t,t+s} = Q_{t,t+1}Q_{t+1,t+2}\cdots Q_{t+s-1,t+s}$, and $E_t Q_{t+s-1,t+s} = E_t E_{t+s-1} Q_{t+s-1,t+s}$ are employed.

The equation (70) have to hold at steady-state:

$$\begin{aligned} 0 = E_t \sum_{s=0}^{\infty} (\beta\theta_H)^s \frac{P}{P}\left(\frac{C(1-h)}{C(1-h)}\right)^{\sigma} Y(i)\Bigg[\bar{P}_H\left(\frac{P_H}{P_H}\right)^{\delta_H} + \\ + \frac{\varepsilon}{1-\varepsilon} P_H MC_H\Bigg] = \sum_{s=0}^{\infty} (\beta\theta_H)^s Y(i)\left[\bar{P}_H + \frac{\varepsilon}{1-\varepsilon} P_H MC_H\right] = \\ = \frac{Y(i)}{1-\beta\theta_H}\left[\bar{P}_H + \frac{\varepsilon}{1-\varepsilon} P_H MC_H\right]. \end{aligned} \tag{71}$$

The variables without time subscript stand for steady-state values. The term in the square brackets in the equation (71) have to be zero, because $Y(i)/(1-\beta\theta_H)$ is positive. In other words, nominal marginal revenues equal nominal marginal costs multiplied by $\varepsilon/(\varepsilon-1)$ at the steady-state:

$$\bar{P}_H = \frac{\varepsilon}{\varepsilon - 1} P_H MC_H, \tag{72}$$

where $\varepsilon/(\varepsilon-1)$ is the optimal mark-up in flexible price economy.

The equation (70) can be rewritten into the form

$$\begin{aligned}
&E_t \sum_{s=0}^{\infty} (\beta\theta_H)^s \frac{P_t}{P_{t+s}} \left(\frac{C_t - hC_{t-1}}{C_{t+s} - hC_{t+s-1}} \right)^{\sigma} Y_{t+s}(i) \bar{P}_{H,t} \\
&\left(\frac{P_{H,t+s-1}}{P_{H,t-1}} \right)^{\delta_H} = E_t \sum_{s=0}^{\infty} (\beta\theta_H)^s \frac{P_t}{P_{t+s}} \left(\frac{C_t - hC_{t-1}}{C_{t+s} - hC_{t+s-1}} \right)^{\sigma} Y_{t+s}(i) \\
&\frac{\varepsilon}{\varepsilon - 1} P_{H,t+s} MC_{H,t+s} \exp(\varepsilon^H_{t+s}) \; .
\end{aligned} \tag{73}$$

In the text below, the equation (73) is log-linearized around the steady-state. Following two approximations are employed through the log-linearization:

1. $X_t^{\nu} = X^{\nu}(1 + x_t)^{\nu} \approx X^{\nu}(1 + \nu x_t)$,
2. $X_t Y_t = XY(1 + x_t + y_t + x_t y_t) \approx XY(1 + x_t + y_t)$,

where capital letters without time subscript t stand for steady-state values, and lower-case letters are deviations of the variables from their steady-states.

Applying above approximations the log-linearized form of the equation (73) is

$$\begin{aligned}
&E_t \sum_{s=0}^{\infty} (\beta\theta_H)^s \frac{P}{P} \left(\frac{C - hC}{C - hC} \right)^{\sigma} Y(i) \bar{P}_H \left(\frac{P_H}{P_H} \right)^{\delta_H} (1 + y_{t+s}(i) + \\
&\quad + p_t - p_{t+s} + \frac{\sigma}{1-h}(c_t - hc_{t-1} - c_{t+s} + hc_{t+s-1}) + \bar{p}_{H,t} + \\
&\quad + \delta_H(p_{H,t+s-1} - p_{H,t-1})) = \\
&= E_t \sum_{s=0}^{\infty} (\beta\theta_H)^s \frac{P}{P} \left(\frac{C - hC}{C - hC} \right)^{\sigma} Y(i) \frac{\varepsilon}{\varepsilon - 1} P_H MC_H (1 + y_{t+s}(i) + \\
&\quad + p_t - p_{t+s} + \frac{\sigma}{1-h}(c_t - hc_{t-1} - c_{t+s} + hc_{t+s-1}) + p_{H,t+s} + \\
&\quad + mc_{H,t+s} + \varepsilon^H_{t+s}) \; .
\end{aligned} \tag{74}$$

With usage of (72) the steady-state values in equation (74) cancels out and we get

$$
\begin{aligned}
&E_t \sum_{s=0}^{\infty} (\beta\theta_H)^s (1 + y_{t+s}(i) + p_t - p_{t+s} + \frac{\sigma}{1-h}(c_t - hc_{t-1} - c_{t+s} + \\
&\quad + hc_{t+s-1}) + \bar{p}_{H,t} + \delta_H(p_{H,t+s-1} - p_{H,t-1})) = \\
&= E_t \sum_{s=0}^{\infty} (\beta\theta_H)^s (1 + y_{t+s}(i) + p_t - p_{t+s} + \frac{\sigma}{1-h}(c_t - hc_{t-1} - c_{t+s} + \\
&\quad + hc_{t+s-1}) + p_{H,t+s} + mc_{H,t+s} + \varepsilon^H_{t+s}) E_t \sum_{s=0}^{\infty} (\beta\theta_H)^s (\bar{p}_{H,t} + \\
&\quad + \delta_H(p_{H,t+s-1} - p_{H,t-1})) = \\
&= E_t \sum_{s=0}^{\infty} (\beta\theta_H)^s (p_{H,t+s} + mc_{H,t+s} + \varepsilon^H_{t+s})(\bar{p}_{H,t} - \delta_H p_{H,t-1}) \\
&\quad E_t \sum_{s=0}^{\infty} (\beta\theta_H)^s = \\
&= E_t \sum_{s=0}^{\infty} (\beta\theta_H)^s (p_{H,t+s} - \delta_H p_{H,t+s-1} + mc_{H,t+s} + \varepsilon^H_{t+s})(\bar{p}_{H,t} - \\
&\quad - \delta_H p_{H,t-1}) = \\
&= (1 - \beta\theta_H) E_t \sum_{s=0}^{\infty} (\beta\theta_H)^s (p_{H,t+s} - \delta_H p_{H,t+s-1} + mc_{H,t+s} + \varepsilon^H_{t+s}) \ .
\end{aligned}
$$

The last equation can be cast in the recursive form:

$$
\begin{aligned}
(\bar{p}_{H,t} - \delta_H p_{H,t-1}) =& (1 - \beta\theta_H)(p_{H,t} - \delta_H p_{H,t-1} + mc_{H,t} + \varepsilon^H_t) + \\
&+ \beta\theta_H (1 - \beta\theta_H) E_t \sum_{s=0}^{\infty} (\beta\theta_H)^s (p_{H,t+s+1} - \\
&- \delta_H p_{H,t+s} + mc_{H,t+s+1} + \varepsilon^H_{t+s+1}) = \\
=& (1 - \beta\theta_H)(p_{H,t} - \delta_H p_{H,t-1} + mc_{H,t} + \varepsilon^H_t) + \\
&+ \beta\theta_H (E_t \bar{p}_{H,t+1} - \delta_H p_{H,t}) \ . \qquad (75)
\end{aligned}
$$

Now, small digression has to be done in order to eliminate terms $\bar{p}_{H,t}$ and $\bar{p}_{H,t+1}$ in equation (75). Let's look at the equation for the aggregate domestic price level

$$
P_{H,t} = \left[(1 - \theta_H) \bar{P}^{1-\varepsilon}_{H,t} + \theta_H \left(P_{H,t-1} \left(\frac{P_{H,t-1}}{P_{H,t-2}} \right)^{\delta_H} \right)^{1-\varepsilon} \right]^{\frac{1}{1-\varepsilon}} . \qquad (76)
$$

The equation have to hold at steady-state, therefore we derive

$$P_H = \left[(1-\theta_H)\bar{P}_H^{1-\varepsilon} + \theta_H P_H^{1-\varepsilon}\right]^{\frac{1}{1-\varepsilon}} \tag{77}$$
$$(1-\theta_H)P_H^{1-\varepsilon} = (1-\theta_H)\bar{P}_H^{1-\varepsilon} \tag{78}$$
$$P_H = \bar{P}_H \ . \tag{79}$$

Notice that $MC_H = (\varepsilon-1)/\varepsilon$ as result of equation (79) and (72). The log-linearized form of the equation (76) is derived as follows

$$\begin{aligned}
P_H(1+p_{H,t}) =& \big[(1-\theta_H)P_H^{1-\varepsilon}(1+(1-\varepsilon)\bar{p}_{H,t}) + \theta_H P_H^{1-\varepsilon}(1+(1-\varepsilon) \\
& (p_{H,t-1}+\delta_H\pi_{H,t-1}))\big]^{\frac{1}{1-\varepsilon}} \\
P_H(1+p_{H,t}) =& \big[P_H^{1-\varepsilon}(1-\theta_H+(1-\theta_H)(1-\varepsilon)\bar{p}_{H,t}) + \theta_H + \theta_H(1-\varepsilon) \\
& (p_{H,t-1}+\delta_H\pi_{H,t-1}))\big]^{\frac{1}{1-\varepsilon}} \\
P_H(1+p_{H,t}) =& P_H(1+(1-\theta_H)\bar{p}_{H,t}) + \theta_H(p_{H,t-1}+\delta_H\pi_{H,t-1})) \\
p_{H,t} =& (1-\theta_H)\bar{p}_{H,t} + \theta_H p_{H,t-1} + \theta_H\delta_H\pi_{H,t-1} \ . \qquad (80)
\end{aligned}$$

The expression for $\bar{p}_{H,t}$ is obtained from equation (80) in the form

$$\bar{p}_{H,t} = \frac{1}{1-\theta_H}p_{H,t} - \frac{\theta_H}{1-\theta_H}p_{H,t-1} - \frac{\theta_H\delta_H}{1-\theta_H}\pi_{H,t-1} \ . \tag{81}$$

Substituting the right-hand side of the equation (81) for $\bar{p}_{H,t}$ and $\bar{p}_{H,t+1}$ in equation (75) we derive

$$\begin{aligned}
& -p_{H,t-1} - \beta(1-\theta_H)\delta_H p_{H,t-1} - \delta_H\pi_{H,t-1} = (-1-\beta-\beta(1- \\
& \quad -\theta_H)\delta_H)p_{H,t} + \lambda_H(mc_{H,t}+\varepsilon_t^H) + \beta E_t p_{H,t+1} - \beta\theta_H\delta_H\pi_{H,t} \\
& \pi_{H,t} + \beta(1-\theta_H)\delta_H\pi_{H,t} - \delta_H\pi_{H,t-1} = \beta E_t\pi_{H,t+1} - \beta\theta_H\delta_H\pi_{H,t} + \\
& \quad +\lambda_H(mc_{H,t}+\varepsilon_t^H) \\
& \pi_{H,t} = \beta E_t(\pi_{H,t+1}-\delta_H\pi_{H,t}) + \delta_H\pi_{H,t-1} + \lambda_H(mc_{H,t}+\varepsilon_t^H) \ . \qquad (82)
\end{aligned}$$

Equation (82) is the New Keynesian Phillips curve of the domestic goods inflation.

D Household's Optimization

This appendix derives first order conditions (5) and (6) of the household's optimization problem introduced in subsection 2.1. It also derives demand functions for domestic and foreign goods and overall price index (8). At the end of this appendix demand functions for the i-th domestic and foreign product and price indices of domestic goods and imports are stated, which are results of subsequent households' optimizations. These optimizations and their solutions are not given because they are analogous to that written below.

The household's optimization problem from subsection 2.1 is

$$\max_{\{C_{t+s},N_{t+s}\}_{s=0}^{\infty}} E_t \sum_{s=0}^{\infty} \beta^s \left(\frac{(C_{t+s} - hC_{t+s-1})^{1-\sigma}}{1-\sigma} - \frac{N_{t+s}^{1+\phi}}{1+\phi} \right) \tag{83}$$

subject to budget constraints

$$P_{t+s}C_{t+s} + E_t Q_{t+s,t+s+1} B_{t+s+1} = B_{t+s} + W_{t+s}N_{t+s} \qquad s = 0, 1, \ldots, \tag{84}$$

The budget constraint (84) is equality because of nonsatiation in consumption. The Bellman equation of this problem is

$$V(B_t) = \max_{C_t,N_t} \{U(C_t, N_t) + \beta E_t V(B_{t+1})\}, \qquad \text{s.t} \tag{85}$$

$$E_t B_{t+1} = R_t[B_t + W_t N_t - P_t C_t]. \tag{86}$$

The FOCs of this problem are

$$U_C(C_t, N_t) - \beta E_t V_B(B_{t+1}) R_t P_t = 0 \tag{87}$$

$$U_N(C_t, N_t) + \beta E_t V_B(B_{t+1}) R_t W_t = 0. \tag{88}$$

Combining FOCs together the equation

$$-\frac{U_N(C_t, N_t)}{U_C(C_t, N_t)} = \frac{W_t}{P_t} \tag{89}$$

is derived. Recall from household's utility function (1) that marginal utility of consumption is

$$U_C(C_t, N_t) = (C_t - hC_{t-1})^{-\sigma}, \tag{90}$$

and marginal disutility of labour is

$$U_N(C_t, N_t) = -N_t^{\phi}. \tag{91}$$

Substituting for U_C and U_N in equation (89) the equation

$$(C_t - hC_{t-1})^{-\sigma} \frac{W_t}{P_t} = N_t^{\phi} \tag{92}$$

is derived. This equation is the same as equation (6). The log-linearized form of this equation is computed as follows:

$$\begin{aligned}
[C(1+c_t) - hC(1+c_{t-1})]^{-\sigma} \frac{W(1+w_t)}{P(1+p_t)} &= [N(1+n_t)]^{\phi} \\
\left[(1-h)C(1+\frac{1}{1-h}(c_t - hc_{t-1}))\right]^{-\sigma} \frac{W(1+w_t)}{P(1+p_t)} &= N^{\phi}(1+\phi n_t) \\
[(1-h)C]^{-\sigma} \frac{W}{P}\left(1 - \frac{\sigma}{1-h}(c_t - hc_{t-1})\right)\frac{1+w_t}{1+p_t} &= N^{\phi}(1+\phi n_t) \\
[(1-h)C]^{-\sigma} \frac{W}{P}\left(1 - \frac{\sigma}{1-h}(c_t - hc_{t-1}) + w_t - p_t\right) &= N^{\phi}(1+\phi n_t) \\
-\frac{\sigma}{1-h}(c_t - hc_{t-1}) + w_t - p_t &= \phi n_t. \qquad (93)
\end{aligned}$$

The last equation follows from the fact that the equation Eq. (92) have to hold at the steady-state.

Now let's compute derivative of the value function $V(\cdot)$ at the point B_{t+1}. Because derivative of $V(\cdot)$ at the point B_t is[19]

$$V_B(B_t) = \beta E_t V_D(D_{t+1}) R_t = \frac{U_C(C_t, N_t)}{P_t}\ , \qquad (94)$$

the derivative of value function at the point B_{t+1} is

$$V_B(B_{t+1}) = \frac{U_C(C_{t+1}, N_{t+1})}{P_{t+1}}\ . \qquad (95)$$

Substituting for $V_B(B_{t+1})$ from the last equation to the Eq. (87) the intertemporal Euler equation:

$$\beta R_t E_t \left\{ \frac{P_t}{P_{t+1}} \left(\frac{C_{t+1} - hC_t}{C_t - hC_{t-1}} \right)^{-\sigma} \right\} = 1 \qquad (96)$$

[19]The envelope theorem is used. The last equality follows from equation (87)

is derived. The log-linearized form of the Euler equation is

$$\beta R(1+r_t)E_t\left\{\frac{P(1+p_t)}{P(1+p_{t+1})}\left(\frac{(1-h)C(1+\frac{1}{1-h}(c_{t+1}-hc_t))}{(1-h)C(1+\frac{1}{1-h}(c_t-hc_{t-1}))}\right)^{-\sigma}\right\}=1$$

$$\beta R(1+r_t)E_t\left\{\frac{1+p_t}{1+p_{t+1}}\left(\frac{1+\frac{1}{1-h}(c_{t+1}-hc_t)}{1+\frac{1}{1-h}(c_t-hc_{t-1})}\right)^{-\sigma}\right\}=1$$

$$\beta R(1+r_t)E_t\left\{1+p_t-p_{t+1}-\frac{\sigma}{1-h}\big[(c_{t+1}-hc_t)-(c_t-hc_{t-1})\big]\right\}=1$$

$$\beta R\left(1+r_t-E_t\pi_{t+1}-\frac{\sigma}{1-h}\big[E_t(c_{t+1}-hc_t)-(c_t-hc_{t-1})\big]\right)=1$$

$$r_t-E_t\pi_{t+1}-\frac{\sigma}{1-h}\big[E_t(c_{t+1}-hc_t)-(c_t-hc_{t-1})\big]=0$$

$$c_t-hc_{t-1}=E_t(c_{t+1}-hc_t)-\frac{1-h}{\sigma}(r_t-E_t\pi_{t+1}). \tag{97}$$

In the derivation above the equality $\beta R=1$ is used. This equality is the equation (96) at the steady-state.

The appendix continues with derivation of the household's demand functions $C_{H,t}$ and $C_{F,t}$. The overall price index P_t is going to be derived too.

If the aggregate concumption index is defined by the CES function

$$C_t=\left[(1-\alpha)^{\frac{1}{\eta}}C_{H,t}^{\frac{\eta-1}{\eta}}+\alpha^{\frac{1}{\eta}}C_{F,t}^{\frac{\eta-1}{\eta}}\right]^{\frac{\eta}{\eta-1}}, \tag{98}$$

the task is to solve the optimization problem

$$\max_{C_{H,t},C_{F,t}}\left[(1-\alpha)^{\frac{1}{\eta}}C_{H,t}^{\frac{\eta-1}{\eta}}+\alpha^{\frac{1}{\eta}}C_{F,t}^{\frac{\eta-1}{\eta}}\right]^{\frac{\eta}{\eta-1}},\qquad \text{s.t} \tag{99}$$

$$P_tC_t=P_{H,t}C_{H,t}+P_{F,t}C_{F,t}, \tag{100}$$

with given total expenditure P_tC_t. The FOCs of this maximization problem are

$$\frac{\eta}{\eta-1}\left[(1-\alpha)^{\frac{1}{\eta}}C_{H,t}^{\frac{\eta-1}{\eta}}+\alpha^{\frac{1}{\eta}}C_{F,t}^{\frac{\eta-1}{\eta}}\right]^{\frac{1}{\eta-1}}(1-\alpha)^{\frac{1}{\eta}}\frac{\eta-1}{\eta}C_{H,t}^{-\frac{1}{\eta}}-$$
$$-\lambda_tP_{H,t}=0, \tag{101}$$

$$\frac{\eta}{\eta-1}\left[(1-\alpha)^{\frac{1}{\eta}}C_{H,t}^{\frac{\eta-1}{\eta}}+\alpha^{\frac{1}{\eta}}C_{F,t}^{\frac{\eta-1}{\eta}}\right]^{\frac{1}{\eta-1}}\alpha^{\frac{1}{\eta}}\frac{\eta-1}{\eta}C_{F,t}^{-\frac{1}{\eta}}-$$
$$-\lambda_tP_{F,t}=0. \tag{102}$$

Combining the above FOCs together yields

$$\begin{aligned}
\frac{1}{P_{H,t}}(1-\alpha)^{\frac{1}{\eta}}C_{H,t}^{-\frac{1}{\eta}} &= \frac{1}{P_{F,t}}\alpha^{\frac{1}{\eta}}C_{F,t}^{-\frac{1}{\eta}} \\
C_{H,t} &= \left(\frac{P_{H,t}}{P_{F,t}}\right)^{-\eta}\frac{1-\alpha}{\alpha}C_{F,t} \ . \qquad (103)
\end{aligned}$$

The equation (103) is now substituted to equation (100):

$$\begin{aligned}
P_tC_t &= \left[P_{H,t}\left(\frac{P_{H,t}}{P_{F,t}}\right)^{-\eta}\frac{1-\alpha}{\alpha}+P_{F,t}\right]C_{F,t} \\
P_tC_t &= \left[\frac{1-\alpha}{\alpha}P_{H,t}^{1-\eta}+P_{F,t}^{1-\eta}\right]\frac{C_{F,t}}{P_{F,t}^{-\eta}} \ , \qquad (104)
\end{aligned}$$

and to Eq. (98):

$$\begin{aligned}
C_t &= \left[(1-\alpha)^{\frac{1}{\eta}}\left[\left(\frac{P_{H,t}}{P_{F,t}}\right)^{-\eta}\frac{1-\alpha}{\alpha}C_{F,t}\right]^{\frac{\eta-1}{\eta}}+\alpha^{\frac{1}{\eta}}C_{F,t}^{\frac{\eta-1}{\eta}}\right]^{\frac{\eta}{\eta-1}} \\
C_t &= \left[\frac{1-\alpha}{\alpha^{\frac{\eta-1}{\eta}}}\left(\frac{P_{H,t}}{P_{F,t}}\right)^{1-\eta}+\alpha^{\frac{1}{\eta}}\right]^{\frac{\eta}{\eta-1}}C_{F,t} \ . \qquad (105)
\end{aligned}$$

The overall price index P_t is derived, if equations (104) and (105) are put together:

$$\begin{aligned}
P_t\left[\frac{1-\alpha}{\alpha^{\frac{\eta-1}{\eta}}}\left(\frac{P_{H,t}}{P_{F,t}}\right)^{1-\eta}+\alpha^{\frac{1}{\eta}}\right]^{\frac{\eta}{\eta-1}}C_{F,t} &= \left[\frac{1-\alpha}{\alpha}P_{H,t}^{1-\eta}+P_{F,t}^{1-\eta}\right]\frac{C_{F,t}}{P_{F,t}^{-\eta}} \\
P_t\left[\frac{1-\alpha}{\alpha^{\frac{\eta-1}{\eta}}}P_{H,t}^{1-\eta}+\alpha^{\frac{1}{\eta}}P_{F,t}^{1-\eta}\right]^{\frac{\eta}{\eta-1}} &= \frac{1-\alpha}{\alpha}P_{H,t}^{1-\eta}+P_{F,t}^{1-\eta} \\
P_t\left[(1-\alpha)P_{H,t}^{1-\eta}+\alpha P_{F,t}^{1-\eta}\right]^{\frac{\eta}{\eta-1}} &= (1-\alpha)P_{H,t}^{1-\eta}+\alpha P_{F,t}^{1-\eta} \\
\left[(1-\alpha)P_{H,t}^{1-\eta}+\alpha P_{F,t}^{1-\eta}\right]^{\frac{1}{1-\eta}} &= P_t. \qquad (106)
\end{aligned}$$

The equation (106) is the same as the equation (8). If we assume that price of domestic goods and imports equals at the steady-state[20], the log-linearized form of the above equation is

$$\begin{aligned}
P(1+p_t) &= \left[(1-\alpha)(P(1+p_{H,t}))^{1-\eta}+\alpha(P(1+p_{F,t}))^{1-\eta}\right]^{\frac{1}{1-\eta}} \\
P(1+p_t) &= \left\{P^{1-\eta}\left[(1-\alpha)(1+(1-\eta)p_{H,t})+\alpha(1+(1-\eta)p_{F,t})\right]\right\}^{\frac{1}{1-\eta}} \\
P(1+p_t) &= \left\{P^{1-\eta}\left[1+(1-\eta)((1-\alpha)p_{H,t}+\alpha p_{F,t})\right]\right\}^{\frac{1}{1-\eta}} \\
1+p_t &= 1+\frac{1}{1-\eta}(1-\eta)((1-\alpha)p_{H,t}+\alpha p_{F,t}) \\
p_t &= (1-\alpha)p_{H,t}+\alpha p_{F,t} \ . \qquad (107)
\end{aligned}$$

[20] If this condition holds then $P = P_H = P_F$.

The household's demand for imports is obtained, if Eq. (104) and Eq. (106) are combined:

$$
\begin{aligned}
P_t C_t &= \frac{1}{\alpha} P_t^{1-\eta} \frac{C_{F,t}}{P_{F,t}^{-\eta}} \\
C_{F,t} &= \alpha \left(\frac{P_{F,t}}{P_t} \right)^{-\eta} C_t \ . \qquad (108)
\end{aligned}
$$

Finally, the household's demand for domestic goods is derived from equations (103) and (108):

$$
\begin{aligned}
C_{H,t} &= \left(\frac{P_{H,t}}{P_{F,t}} \right)^{-\eta} \frac{1-\alpha}{\alpha} \alpha \left(\frac{P_{F,t}}{P_t} \right)^{-\eta} C_t \\
C_{H,t} &= (1-\alpha) \left(\frac{P_{H,t}}{P_t} \right)^{-\eta} C_t \ . \qquad (109)
\end{aligned}
$$

Given CES aggregate functions for domestic goods and imports:

$$
\begin{aligned}
C_{H,t} &= \left(\int_0^1 C_{H,t}(i)^{\frac{\varepsilon-1}{\varepsilon}} \mathrm{d}i \right)^{\frac{\varepsilon}{\varepsilon-1}} , \qquad (110) \\
C_{F,t} &= \left(\int_0^1 C_{F,t}(i)^{\frac{\varepsilon-1}{\varepsilon}} \mathrm{d}i \right)^{\frac{\varepsilon}{\varepsilon-1}} , \qquad (111)
\end{aligned}
$$

the demand functions

$$
\begin{aligned}
C_{H,t}(i) &= \left(\frac{P_{H,t}(i)}{P_{H,t}} \right)^{-\varepsilon} C_{H,t} \ , \qquad (112) \\
C_{F,t}(i) &= \left(\frac{P_{F,t}(i)}{P_{F,t}} \right)^{-\varepsilon} C_{F,t} \ , \qquad (113)
\end{aligned}
$$

and price indices

$$
\begin{aligned}
P_{H,t} &= \left(\int_0^1 P_{H,t}(i)^{1-\varepsilon} \mathrm{d}i \right)^{\frac{1}{1-\varepsilon}} , \qquad (114) \\
P_{F,t} &= \left(\int_0^1 P_{F,t}(i)^{1-\varepsilon} \mathrm{d}i \right)^{\frac{1}{1-\varepsilon}} \qquad (115)
\end{aligned}
$$

are derived by analogy.

E Goods-Market Clearing Condition

This appendix derives log-linearized goods-market clearing condition (31). Before it proceeds recall the demand functions derived in Appendix D:

- Domestic household's demand for domestic goods is

$$C_{H,t} = (1-\alpha)\left(\frac{P_{H,t}}{P_t}\right)^{-\eta} C_t \,. \tag{116}$$

- Domestic household's demand for the i-th domestic product is

$$C_{H,t}(i) = \left(\frac{P_{H,t}(i)}{P_{H,t}}\right)^{-\varepsilon} C_{H,t} \,. \tag{117}$$

If C_t^*, $C_{H,t}^*$ and $C_{H,t}^*(i)$ stand for foreign household's overall consumption, consumption of goods produced in small open economy, and consumption of the i-th commodity produced in small open economy, respectively, then following equalities hold by analogy[21]:

- Foreign consumption of the goods produced in small economy (demand for export) is

$$C_{H,t}^* = \alpha\left(\frac{P_{H,t}}{Z_t P_t^*}\right)^{-\eta} C_t^* \,. \tag{118}$$

- Foreign household's demand for the i-th domestic product (demand for export of the i-th domestic product)is

$$C_{H,t}^*(i) = \left(\frac{P_{H,t}(i)}{P_{H,t}}\right)^{-\varepsilon} C_{H,t}^* \,. \tag{119}$$

The aggregate output of the home economy is defined by the CES function

$$Y_t = \left(\int_0^1 Y_t(i)^{\frac{\varepsilon-1}{\varepsilon}}\mathrm{d}i\right)^{\frac{\varepsilon}{\varepsilon-1}} \,. \tag{120}$$

Equilibrium on the market of the i-th product will arise, if production of the i-th commodity equals consumption of this commodity. Consumption of the i-th product divides into domestic and foreign consumption. Hence goods-market eqiuilibrium conditions are

$$Y_t(i) = C_{H,t}(i) + C_{H,t}^*(i)\,, \qquad \text{for } i \in [0,1]. \tag{121}$$

[21] The same elasticities of substitution between different types of goods are supposed within both economies.

Substituting Eq. (116)–(119) yields

$$
\begin{aligned}
Y_t(i) &= \left(\frac{P_{H,t}(i)}{P_{H,t}}\right)^{-\varepsilon} C_{H,t} + \left(\frac{P_{H,t}(i)}{P_{H,t}}\right)^{-\varepsilon} C^*_{H,t} \\
Y_t(i) &= \left(\frac{P_{H,t}(i)}{P_{H,t}}\right)^{-\varepsilon} \left[(1-\alpha)\left(\frac{P_{H,t}}{P_t}\right)^{-\eta} C_t + \alpha\left(\frac{P_{H,t}}{Z_t P^*_t}\right)^{-\eta} C^*_t\right] \\
Y_t(i)^{\frac{\varepsilon-1}{\varepsilon}} &= \left(\frac{P_{H,t}(i)}{P_{H,t}}\right)^{1-\varepsilon} \left[(1-\alpha)\left(\frac{P_{H,t}}{P_t}\right)^{-\eta} C_t + \alpha\left(\frac{P_{H,t}}{Z_t P^*_t}\right)^{-\eta} C^*_t\right]^{\frac{\varepsilon-1}{\varepsilon}}
\end{aligned}
\tag{122}
$$

Now lets integrate with respect to i both sides of the previous equation:

$$
\begin{aligned}
\int_0^1 Y_t(i)^{\frac{\varepsilon-1}{\varepsilon}} \mathrm{d}i &= \left(\frac{1}{P_{H,t}}\right)^{1-\varepsilon} \left[(1-\alpha)\left(\frac{P_{H,t}}{P_t}\right)^{-\eta} C_t + \right. \\
&\quad \left. +\alpha\left(\frac{P_{H,t}}{Z_t P^*_t}\right)^{-\eta} C^*_t\right]^{\frac{\varepsilon-1}{\varepsilon}} \int_0^1 P_{H,t}(i)^{1-\varepsilon} \mathrm{d}i \\
Y_t^{\frac{\varepsilon-1}{\varepsilon}} &= \left(\frac{1}{P_{H,t}}\right)^{1-\varepsilon} \left[(1-\alpha)\left(\frac{P_{H,t}}{P_t}\right)^{-\eta} C_t + \right. \\
&\quad \left. +\alpha\left(\frac{P_{H,t}}{Z_t P^*_t}\right)^{-\eta} C^*_t\right]^{\frac{\varepsilon-1}{\varepsilon}} P_{H,t}^{1-\varepsilon} \\
Y_t &= (1-\alpha)\left(\frac{P_{H,t}}{P_t}\right)^{-\eta} C_t + \alpha\left(\frac{P_{H,t}}{Z_t P^*_t}\right)^{-\eta} C^*_t \\
Y_t &= C_{H,t} + C^*_{H,t}\ .
\end{aligned}
$$

This equation states that aggregate output equals sum of aggregate domestic and foreign consumption of the goods produced in the small economy.

The log-linear approximation of Eq. (123) is

$$
\begin{aligned}
Y(1+y_t) &= C_H(1+c_{H,t}) + C^*_H(1+c^*_{H,t}) \\
Y y_t &= C_H c_{H,t} + C^*_H c^*_{H,t} \\
y_t &= \frac{C_H}{Y} c_{H,t} + \frac{C^*_H}{Y} c^*_{H,t} \\
y_t &= (1-\alpha) c_{H,t} + \alpha c^*_{H,t}\ .
\end{aligned}
\tag{123}
$$

The gap $c_{H,t}$ can be computed from Eq. (116), Eq. (107) and log-linearized form of Eq. (12) as follows

$$\begin{aligned}
C_H(1+c_{H,t}) &= (1-\alpha)\left(\frac{P_H(1+p_{H,t})}{P(1+p_t)}\right)^{-\eta} C(1+c_t) \\
1+c_{H,t} &= (1+p_{H,t}-p_t)^{-\eta}(1+c_t) \\
1+c_{H,t} &= 1-\eta(p_{H,t}-p_t)+c_t \\
c_{H,t} &= -\eta(p_{H,t}-p_t)+c_t \\
c_{H,t} &= -\eta(p_{H,t}-(1-\alpha)p_{H,t}-\alpha p_{F,t})+c_t \\
c_{H,t} &= -\eta(\alpha p_{H,t}-\alpha p_{F,t})+c_t \\
c_{H,t} &= -\eta(\alpha(p_{F,t}-s_t)-\alpha p_{F,t})+c_t \\
c_{H,t} &= \alpha\eta s_t+c_t\ . \qquad (124)
\end{aligned}$$

Equation for gap $c^*_{H,t}$ is derived with use of Eq. (118), Eq (16), and log-linearized form of Eq (14) and Eq (12) in the following way:

$$\begin{aligned}
C^*_H(1+c^*_{H,t}) &= \alpha\left(\frac{P_H(1+p_{H,t})}{Z(1+z_t)P^*(1+p^*_t)}\right)^{-\eta} C^*(1+c^*_t) \\
C^*_H(1+c^*_{H,t}) &= \alpha\left(\frac{P_H}{ZP^*}(1+p_{H,t}-z_t-p^*_t)\right)^{-\eta} C^*(1+c^*_t) \\
1+c^*_{H,t} &= (1-\eta(p_{H,t}-z_t-p^*_t))(1+c^*_t) \\
c^*_{H,t} &= -\eta(p_{H,t}-z_t-p^*_t)+c^*_t \\
c^*_{H,t} &= -\eta(p_{H,t}-\psi_t-p_{F,t})+c^*_t \\
c^*_{H,t} &= -\eta(-\psi_t-s_t)+c^*_t \\
c^*_{H,t} &= -\eta(-q_t+(1-\alpha)s_t-s_t)+c^*_t \\
c^*_{H,t} &= \eta q_t+\alpha\eta s_t+c^*_t\ . \qquad (125)
\end{aligned}$$

Substituting for $c_{H,t}$ and $c^*_{H,t}$ back to Eq. (123) yields[22]

$$y_t = (1-\alpha)c_t+\alpha y^*_t+\alpha\eta s_t+\alpha\eta q_t \qquad (126)$$

which is the goods-market clearing condition presented in subsection 2.5.

F Producer's Real Marginal Costs

This appendix derives log-linearized form of the producer's real marginal costs. This form is ready to be substituted to New Keynesian Phillips curve (82).

Recall the i-th producer's production function

$$Y_t(i) = A_t N_t(i). \qquad (127)$$

[22] Note that in foreign economy the equality $y^*_t = c^*_t$ holds.

It is clear from the production function that level of technology (productivity of labour) is the same for all producers. The producer is unable to influence the wage because we assume perfect competition on the labour market. The total costs of the i-th producers are therefore

$$TC_{H,t}(i) = W_t N_t(i) = \frac{W_t Y_t(i)}{A_t}, \tag{128}$$

and his real marginal costs are

$$MC_{H,t}(i) = \frac{W_t}{A_t P_{H,t}} \ . \tag{129}$$

The index i in the above equation of the real marginal cost can be omitted. It means that all producers produce with the same marginal costs. Now it is straightforward to derive log-linear approximation of real marginal costs MC_t:

$$\begin{aligned} MC_H(1+mc_{H,t}) &= \frac{W(1+w_t)}{A(1+a_t)P_H(1+p_{H,t})} \\ 1+mc_{H,t} &= \frac{1+w_t}{1+a_t+p_{H,t}} \\ 1+mc_{H,t} &= 1+w_t-a_t-p_{H,t} \\ mc_{H,t} &= w_t-a_t-p_{H,t} \ . \end{aligned} \tag{130}$$

The last equation can be with use of Eq. (93), Eq. (107), and log-linearized form of Eq. (12) rewritten as follows

$$\begin{aligned} mc_{H,t} &= w_t - a_t - p_{H,t} - p_t + p_t = \\ &= \phi n_t + \frac{\sigma}{1-h}(c_t - hc_{t-1}) - a_t - p_{H,t} + p_t = \\ &= \phi n_t + \frac{\sigma}{1-h}(c_t - hc_{t-1}) - a_t + s_t - p_{F,t} + (1-\alpha)p_{H,t} + \\ &\quad + \alpha p_{F,t} = \\ &= \phi n_t + \frac{\sigma}{1-h}(c_t - hc_{t-1}) - a_t + \alpha s_t \ . \end{aligned} \tag{131}$$

The finall task to do is to rule the term n_t out of the equation (131).To deal with this task remember that production of the i-th producer is

$$\begin{aligned} Y_t(i) = C_{H,t}(i) + C^*_{H,t}(i) &= \left(\frac{P_{H,t}(i)}{P_{H,t}}\right)^{-\varepsilon} C_{H,t} + \left(\frac{P_{H,t}(i)}{P_{H,t}}\right)^{-\varepsilon} C^*_{H,t} = \\ &= \left(\frac{P_{H,t}(i)}{P_{H,t}}\right)^{-\varepsilon} (C_{H,t} + C^*_{H,t}) = \left(\frac{P_{H,t}(i)}{P_{H,t}}\right)^{-\varepsilon} Y_t, \end{aligned} \tag{132}$$

assuming the market-clearing condition holds. See Appendix E for more details. From Eq.(114) it is clear that the equality

$$1 = \int_0^1 \left(\frac{P_{H,t}(i)}{P_{H,t}}\right)^{1-\varepsilon} \mathrm{d}i \tag{133}$$

holds. The first order log-linearization of this equation yields

$$
\begin{aligned}
1 &= \int_0^1 \left(\frac{P_H(1+p_{H,t}(i))}{P_H(1+p_{H,t})} \right)^{1-\varepsilon} \mathrm{d}i \\
1 &\approx \int_0^1 (1+p_{H,t}(i)-p_{H,t})^{1-\varepsilon} \mathrm{d}i \\
1 &\approx \int_0^1 \big[1+(1-\varepsilon)(p_{H,t}(i)-p_{H,t})\big] \mathrm{d}i \\
1 &\approx 1+(1-\varepsilon)\int_0^1 \big(p_{H,t}(i)-p_{H,t}\big)\mathrm{d}i \\
0 &\approx \int_0^1 \big(p_{H,t}(i)-p_{H,t}\big)\mathrm{d}i \ . \qquad (134)
\end{aligned}
$$

Now return to the task of deriving log-linearized form of the overall labour N_t. The overall labour is

$$
N_t = \int_0^1 N_t(i)\mathrm{d}i \ . \qquad (135)
$$

Hence, with use of Eq. (132) it is possible to write overall labour as

$$
N_t = \int_0^1 N_t(i)\mathrm{d}i = \int_0^1 \frac{Y_t(i)}{A_t}\mathrm{d}i = \frac{Y_t}{A_t}\int_0^1 \left(\frac{P_{H,t}(i)}{P_{H,t}} \right)^{-\varepsilon} \mathrm{d}i \ , \qquad (136)
$$

and its log-linearized form as

$$
\begin{aligned}
N(1+n_t) &= \frac{Y(1+y_t)}{A(1+a_t)} \int_0^1 \left(\frac{P_H(1+pH,t(i))}{P_H(1+p_{H,t})} \right)^{-\varepsilon} \mathrm{d}i \\
1+n_t &= (1+y_t-a_t)\int_0^1 \big[1-\varepsilon(pH,t(i)-p_{H,t})\big]\mathrm{d}i \\
1+n_t &= 1+y_t-a_t-\varepsilon \underbrace{\int_0^1 (pH,t(i)-p_{H,t})\mathrm{d}i}_{\approx 0} \\
n_t &= y_t-a_t \ . \qquad (137)
\end{aligned}
$$

Substituting for n_t to Eq.(131) results in the final form of the log-linearized producer's real marginal costs:

$$mc_{H,t} = \phi y_t - (1+\phi)a_t + \frac{\sigma}{1-h}(c_t - hc_{t-1}) + \alpha s_t \; . \qquad (138)$$