Nevlastní integrál

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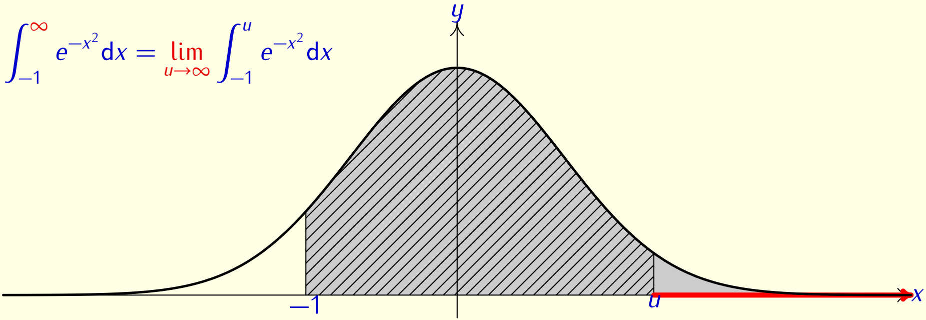
Obsah

1 Nevlastní integrál

Nevlastní integrál je rozšířením pojmu Riemannova integrálu. Riemannův integrál je definovaný pouze pro *ohraničené* funkce a *konečné* obory integrace.
Body, ve kterých funkce není ohraničená a nevlastní body $\pm\infty$ budeme souhrnně nazývat *singulárními body*.

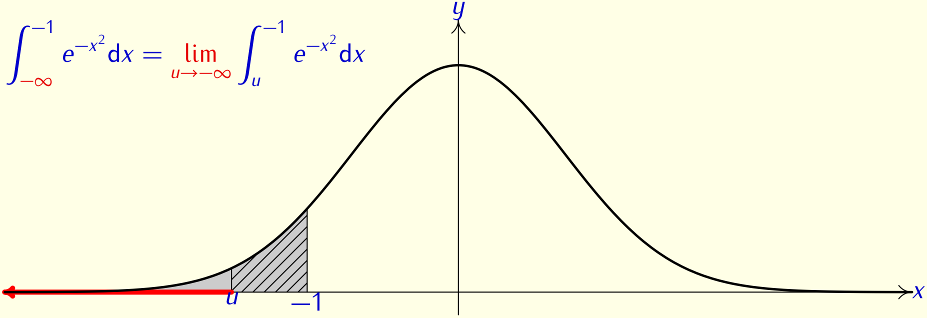
Integrál $\int_a^b f(x)\,\mathrm{d}x$ nazýváme nevlastní, pokud alespoň jedno z čísel a, b je rovno $\pm\infty$, nebo funkce $f(x)$ *není ohraničená* na *uzavřeném* intervalu $[a,b]$ (tj. alespoň v jednom bodě intervalu funkce má singulární bod - nemusí jít vždy o body a nebo b, ale singulární bod může být i uvnitř intervalu).



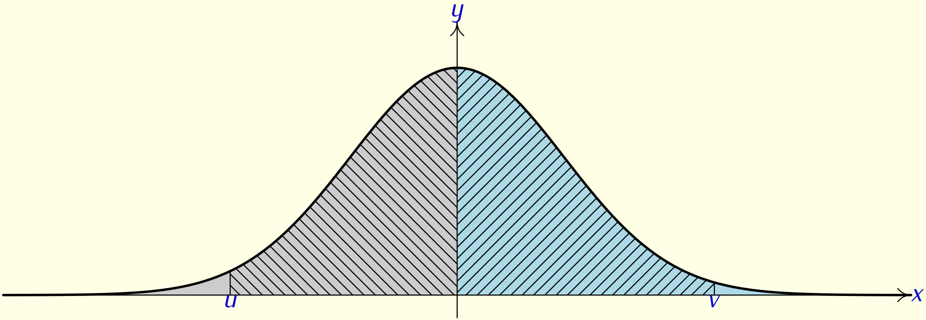
$$\int_{-1}^{\infty} e^{-x^2} dx = \lim_{u\to\infty} \int_{-1}^{u} e^{-x^2} dx$$

Definice. Nechť $b \in \mathbb{R} \cup \{+\infty\}$ a nechť funkce $f(x)$ je integrovatelná na každém intervalu $[a, u]$, kde $a < u < b$. Dále nechť buď platí $b = \infty$ nebo nechť $f(x)$ není ohraničená v okolí bodu b. Existuje-li vlastní limita $\lim\limits_{u\to b^-} \int_a^u f(x)\, dx = B$, říkáme že *nevlastní integrál konverguje* a píšeme $\int_a^b f(x)\, dx = B$. Pokud limita neexistuje, nebo je nevlastní, říkáme, že integrál $\int_a^b f(x)\, dx$ *diverguje*.

$$\int_{-\infty}^{-1} e^{-x^2}\,\mathrm{d}x = \lim_{u\to-\infty} \int_u^{-1} e^{-x^2}\,\mathrm{d}x$$



Definice. Nechť $a \in \mathbb{R} \cup \{-\infty\}$ a nechť funkce $f(x)$ je integrovatelná na každém intervalu $[u, b]$, kde $a < u < b$. Dále nechť buď platí $a = -\infty$ nebo nechť $f(x)$ není ohraničená v okolí bodu a. Existuje-li vlastní limita $\lim\limits_{u\to a^+} \int_u^b f(x)\,\mathrm{d}x = A$, říkáme že *nevlastní integrál konverguje* a píšeme $\int_a^b f(x)\,\mathrm{d}x = A$. Pokud limita neexistuje, nebo je nevlastní, říkáme, že integrál $\int_a^b f(x)\,\mathrm{d}x$ *diverguje*.



$$\int_{-\infty}^{\infty} e^{-x^2}\,\mathrm{d}x = \int_{-\infty}^{0} e^{-x^2}\,\mathrm{d}x + \int_{0}^{\infty} e^{-x^2}\,\mathrm{d}x$$

$$= \lim_{u\to-\infty} \int_{u}^{0} e^{-x^2}\,\mathrm{d}x + \lim_{v\to\infty} \int_{0}^{v} e^{-x^2}\,\mathrm{d}x$$

Pokud singulární bod leží uvnitř intervalu (a, b), $a, b \in \mathbb{R} \cup \{\pm\infty\}$, nebo pokud jsou singulárními body obě meze, rozdělíme interval přes který integrujeme na několik podintervalů opakovaným využitím aditivity Riemannova integrálu vzheldem k mezím a integrujeme na každém intervalu samostatně.

Integrujte $\displaystyle\int_1^{\infty} \frac{1}{x(x^2+1)}\,\mathrm{d}x.$

Integrujte $\int_1^\infty \frac{1}{x(x^2+1)}\,\mathrm{d}x$.

$$I = \lim_{u\to\infty} \int_1^u \frac{1}{x(x^2+1)}\,\mathrm{d}x$$

Použijeme definici nevlastního integrálu a rozepíšeme jej jako limitu Riemannova integrálu.

Integrujte $\int_1^{\infty} \frac{1}{x(x^2+1)}\,\mathrm{d}x$.

$$I = \lim_{u\to\infty} \int_1^u \frac{1}{x(x^2+1)}\,\mathrm{d}x$$

$$\int \frac{1}{x(x^2+1)}\,\mathrm{d}x = \int \frac{1}{x} - \frac{x}{x^2+1}\,\mathrm{d}x =$$

Pro výpočet neurčitého integrálu rozložíme na parciální zlomky.

Integrujte $\int_1^{\infty} \frac{1}{x(x^2+1)}\,\mathrm{d}x$.

$$I = \lim_{u\to\infty} \int_1^{u} \frac{1}{x(x^2+1)}\,\mathrm{d}x$$

$$\int \frac{1}{x(x^2+1)}\,\mathrm{d}x = \int \frac{1}{x} - \frac{x}{x^2+1}\,\mathrm{d}x = \ln x - \frac{1}{2}\ln(x^2+1)$$

Užijeme základní vzorce a pravidla pro integraci.

Integrujte $\int_1^{\infty} \frac{1}{x(x^2+1)}\,\mathrm{d}x$.

$$I = \lim_{u\to\infty} \int_1^u \frac{1}{x(x^2+1)}\,\mathrm{d}x$$

$$\int \frac{1}{x(x^2+1)}\,\mathrm{d}x = \int \frac{1}{x} - \frac{x}{x^2+1}\,\mathrm{d}x = \ln x - \frac{1}{2}\ln(x^2+1)$$

$$\int_1^u \frac{1}{x(x^2+1)}\,\mathrm{d}x = \ln u - \frac{1}{2}\ln(u^2+1) + \frac{1}{2}\ln(2)$$

Vypočteme Riemannův integral pomocí Newtonovy–Leibnizovy věty.

Integrujte $\int_1^\infty \frac{1}{x(x^2+1)}\,dx$.

$$I = \lim_{u\to\infty} \int_1^u \frac{1}{x(x^2+1)}\,dx$$

$$\int \frac{1}{x(x^2+1)}\,dx = \int \frac{1}{x} - \frac{x}{x^2+1}\,dx = \ln x - \frac{1}{2}\ln(x^2+1)$$

$$\int_1^u \frac{1}{x(x^2+1)}\,dx = \ln u - \frac{1}{2}\ln(u^2+1) + \frac{1}{2}\ln(2)$$

$$I = \lim_{u\to\infty} \left[\ln u - \frac{1}{2}\ln(u^2+1) + \frac{1}{2}\ln(2)\right]$$

Nyní užijeme limitní přechod $u \to \infty$.

Integrujte $\int_1^\infty \frac{1}{x(x^2+1)}\,dx$.

$$I = \lim_{u\to\infty} \int_1^u \frac{1}{x(x^2+1)}\,dx$$

$$\int \frac{1}{x(x^2+1)}\,dx = \int \frac{1}{x} - \frac{x}{x^2+1}\,dx = \ln x - \frac{1}{2}\ln(x^2+1)$$

$$\int_1^u \frac{1}{x(x^2+1)}\,dx = \ln u - \frac{1}{2}\ln(u^2+1) + \frac{1}{2}\ln(2)$$

$$I = \lim_{u\to\infty}\left[\ln u - \frac{1}{2}\ln(u^2+1) + \frac{1}{2}\ln(2)\right] =$$

$$= \frac{1}{2}\ln 2 + \frac{1}{2}\ln\left(\lim_{u\to\infty}\frac{u^2}{u^2+1}\right) = \frac{1}{2}\ln 2 + \frac{1}{2}\ln 1 = \frac{1}{2}\ln 2.$$

- Výraz je typu $\infty - \infty$.
- Sečtením logaritmů převedeme na logaritmus podílu, se kterým se lépe zachází.

Integrujte $\int_1^\infty \frac{1}{x(x^2+1)}\,\mathrm{d}x$.

$$I = \lim_{u\to\infty} \int_1^u \frac{1}{x(x^2+1)}\,\mathrm{d}x$$

$$\int \frac{1}{x(x^2+1)}\,\mathrm{d}x = \int \frac{1}{x} - \frac{x}{x^2+1}\,\mathrm{d}x = \ln x - \frac{1}{2}\ln(x^2+1)$$

$$\int_1^u \frac{1}{x(x^2+1)}\,\mathrm{d}x = \ln u - \frac{1}{2}\ln(u^2+1) + \frac{1}{2}\ln(2)$$

$$I = \lim_{u\to\infty}\left[\ln u - \frac{1}{2}\ln(u^2+1) + \frac{1}{2}\ln(2)\right] =$$

$$= \frac{1}{2}\ln 2 + \frac{1}{2}\ln\left(\lim_{u\to\infty}\frac{u^2}{u^2+1}\right) = \frac{1}{2}\ln 2 + \frac{1}{2}\ln 1 = \frac{1}{2}\ln 2.$$

Integrál konverguje, jeho hodnota je $I = \frac{1}{2}\ln 2$.

Integrujte $\displaystyle\int_2^{\infty} \frac{1}{x \ln x}\, dx$.

Rozepíšeme

$$I = \lim_{u\to\infty} \int_2^{u} \frac{1}{x \ln x}\, dx.$$

Neurčitý integrál splňuje

$$\int \frac{1}{x \ln x}\, dx = \int \frac{\frac{1}{x}}{\ln x}\, dx = \ln|\ln x|$$

a proto

$$I = \int_2^{\infty} \frac{1}{x \ln x}\, dx = \lim_{u\to\infty} \int_2^{u} \frac{1}{x \ln x}\, dx = \lim_{u\to\infty} \Big[\ln|\ln u| - \ln|\ln 2|\Big] = \infty$$

a nevlastní integrál tedy diverguje.

Integrujte $\displaystyle\int_1^{\infty} \frac{1}{x\sqrt{x+1}}\,\mathrm{d}x.$

Integrujte $\displaystyle\int_1^\infty \frac{1}{x\sqrt{x+1}}\,\mathrm{d}x.$

$$I = \lim_{u\to\infty}\int_1^u \frac{1}{x\sqrt{x+1}}\,\mathrm{d}x$$

použijeme definici nevlastního integrálu.

Integrujte $\displaystyle\int_1^{\infty} \frac{1}{x\sqrt{x+1}}\,\mathrm{d}x.$

$$I = \lim_{u\to\infty} \int_1^u \frac{1}{x\sqrt{x+1}}\,\mathrm{d}x$$

$$\int \frac{1}{x\sqrt{x+1}}\,\mathrm{d}x$$

Vypočteme neurčitý integrál.

Integrujte $\displaystyle\int_1^{\infty} \frac{1}{x\sqrt{x+1}}\,\mathrm{d}x.$

$$I = \lim_{u\to\infty} \int_1^{u} \frac{1}{x\sqrt{x+1}}\,\mathrm{d}x$$

$$\int \frac{1}{x\sqrt{x+1}}\,\mathrm{d}x$$

$x+1=t^2$

Substitucí odstraníme odmocninu.

Integrujte $\displaystyle\int_1^{\infty} \frac{1}{x\sqrt{x+1}}\,dx.$

$$I = \lim_{u\to\infty} \int_1^{u} \frac{1}{x\sqrt{x+1}}\,dx$$

$$\int \frac{1}{x\sqrt{x+1}}\,dx$$

$x+1=t^2$
$x=t^2-1$

Vypočteme x...

Integrujte $\int_1^{\infty} \frac{1}{x\sqrt{x+1}}\,\mathrm{d}x.$

$$I = \lim_{u\to\infty} \int_1^{u} \frac{1}{x\sqrt{x+1}}\,\mathrm{d}x$$

$$\int \frac{1}{x\sqrt{x+1}}\,\mathrm{d}x \qquad \boxed{\begin{aligned} x+1 &= t^2 \\ x &= t^2-1 \\ \mathrm{d}x &= 2t\,\mathrm{d}t \end{aligned}}$$

. . . a odsud nalezneme vztah mezi diferenciály.

Integrujte $\displaystyle\int_1^\infty \frac{1}{x\sqrt{x+1}}\,\mathrm{d}x.$

$$I = \lim_{u\to\infty} \int_1^u \frac{1}{x\sqrt{x+1}}\,\mathrm{d}x$$

$$\int \frac{1}{x\sqrt{x+1}}\,\mathrm{d}x \quad \boxed{\begin{array}{c} x+1=t^2 \\ x=t^2-1 \\ \mathrm{d}x = 2t\,\mathrm{d}t \end{array}} = \int \frac{1}{(t^2-1)t} 2t\,\mathrm{d}t$$

Dosadíme substituci. . .

Integrujte $\displaystyle\int_1^\infty \frac{1}{x\sqrt{x+1}}\,\mathrm{d}x.$

$$I = \lim_{u\to\infty}\int_1^u \frac{1}{x\sqrt{x+1}}\,\mathrm{d}x$$

$$\int \frac{1}{x\sqrt{x+1}}\,\mathrm{d}x \quad \boxed{\begin{array}{c} x+1=t^2 \\ x=t^2-1 \\ \mathrm{d}x = 2t\,\mathrm{d}t \end{array}} = \int \frac{1}{(t^2-1)t}2t\,\mathrm{d}t = \int \frac{2}{t^2-1}\,\mathrm{d}t$$

. . . a upravíme.

Integrujte $\displaystyle\int_1^\infty \frac{1}{x\sqrt{x+1}}\,\mathrm{d}x.$

$$I = \lim_{u\to\infty}\int_1^u \frac{1}{x\sqrt{x+1}}\,\mathrm{d}x$$

$$\int \frac{1}{x\sqrt{x+1}}\,\mathrm{d}x \quad \boxed{\begin{array}{c} x+1=t^2 \\ x=t^2-1 \\ \mathrm{d}x = 2t\,\mathrm{d}t \end{array}} = \int \frac{1}{(t^2-1)t}2t\,\mathrm{d}t = \int \frac{2}{t^2-1}\,\mathrm{d}t = \ln\frac{t-1}{t+1}$$

Rozložíme na parciální zlomky (zde přeskočeno) a zintegrujeme.

$$\int \frac{2}{t^2-1}\,\mathrm{d}t = \int \frac{1}{t-1} - \frac{1}{t+1}\,\mathrm{d}t = \ln|t-1| - \ln|t+1|$$

$$= \ln\frac{|t-1|}{|t+1|} = \ln\frac{t-1}{t+1}$$

Integrujte $\displaystyle\int_1^{\infty} \frac{1}{x\sqrt{x+1}}\,\mathrm{d}x.$

$$I = \lim_{u\to\infty} \int_1^{u} \frac{1}{x\sqrt{x+1}}\,\mathrm{d}x$$

$$\int \frac{1}{x\sqrt{x+1}}\,\mathrm{d}x \quad \boxed{\begin{array}{c} x+1=t^2 \\ x=t^2-1 \\ \mathrm{d}x=2t\,\mathrm{d}t \end{array}} = \int \frac{1}{(t^2-1)t} 2t\,\mathrm{d}t = \int \frac{2}{t^2-1}\,\mathrm{d}t = \ln \frac{t-1}{t+1}$$

$$= \ln \frac{\sqrt{x+1}-1}{\sqrt{x+1}+1}$$

Dosadíme substituci a vrátíme se tak zpět k proměnné x.

Integrujte $\displaystyle\int_1^{\infty} \frac{1}{x\sqrt{x+1}}\,\mathrm{d}x.$

$$I = \lim_{u\to\infty} \int_1^u \frac{1}{x\sqrt{x+1}}\,\mathrm{d}x \qquad\qquad \int \frac{1}{x\sqrt{x+1}}\,\mathrm{d}x = \ln\frac{\sqrt{x+1}-1}{\sqrt{x+1}+1}$$

$$\int_1^u \frac{1}{x\sqrt{x+1}}\,\mathrm{d}x$$

Užijeme primitivní funkci k výpočtu určitého integrálu.

Integrujte $\displaystyle\int_1^{\infty} \frac{1}{x\sqrt{x+1}}\,\mathrm{d}x.$

$$I = \lim_{u\to\infty} \int_1^u \frac{1}{x\sqrt{x+1}}\,\mathrm{d}x \qquad \int \frac{1}{x\sqrt{x+1}}\,\mathrm{d}x = \ln \frac{\sqrt{x+1}-1}{\sqrt{x+1}+1}$$

$$\int_1^u \frac{1}{x\sqrt{x+1}}\,\mathrm{d}x = \left[\ln \frac{\sqrt{x+1}-1}{\sqrt{x+1}+1}\right]_1^u$$

Newtonova–Leibnizova formule.

Integrujte $\displaystyle\int_1^\infty \frac{1}{x\sqrt{x+1}}\,\mathrm{d}x.$

$$I = \lim_{u\to\infty} \int_1^u \frac{1}{x\sqrt{x+1}}\,\mathrm{d}x$$

$$\int \frac{1}{x\sqrt{x+1}}\,\mathrm{d}x = \ln \frac{\sqrt{x+1}-1}{\sqrt{x+1}+1}$$

$$\int_1^u \frac{1}{x\sqrt{x+1}}\,\mathrm{d}x = \left[\ln \frac{\sqrt{x+1}-1}{\sqrt{x+1}+1}\right]_1^u = \ln \frac{\sqrt{u+1}-1}{\sqrt{u+1}+1} - \ln \frac{\sqrt{2}-1}{\sqrt{2}+1}$$

Určitý integrál.

Integrujte $\displaystyle\int_1^{\infty} \frac{1}{x\sqrt{x+1}}\,\mathrm{d}x.$

$$I = \lim_{u\to\infty} \int_1^{u} \frac{1}{x\sqrt{x+1}}\,\mathrm{d}x \qquad \int \frac{1}{x\sqrt{x+1}}\,\mathrm{d}x = \ln\frac{\sqrt{x+1}-1}{\sqrt{x+1}+1}$$

$$\int_1^{u} \frac{1}{x\sqrt{x+1}}\,\mathrm{d}x = \left[\ln\frac{\sqrt{x+1}-1}{\sqrt{x+1}+1}\right]_1^{u} = \ln\frac{\sqrt{u+1}-1}{\sqrt{u+1}+1} - \ln\frac{\sqrt{2}-1}{\sqrt{2}+1}$$

$$I = -\ln\frac{\sqrt{2}-1}{\sqrt{2}+1} + \lim_{u\to\infty} \ln\frac{\sqrt{u+1}-1}{\sqrt{u+1}+1}$$

Nevlastní integrál je limitou určitého integrálu.

Integrujte $\displaystyle\int_1^\infty \frac{1}{x\sqrt{x+1}}\,\mathrm{d}x.$

$$I=\lim_{u\to\infty}\int_1^u \frac{1}{x\sqrt{x+1}}\,\mathrm{d}x \qquad \int \frac{1}{x\sqrt{x+1}}\,\mathrm{d}x=\ln\frac{\sqrt{x+1}-1}{\sqrt{x+1}+1}$$

$$\int_1^u \frac{1}{x\sqrt{x+1}}\,\mathrm{d}x=\left[\ln\frac{\sqrt{x+1}-1}{\sqrt{x+1}+1}\right]_1^u=\ln\frac{\sqrt{u+1}-1}{\sqrt{u+1}+1}-\ln\frac{\sqrt{2}-1}{\sqrt{2}+1}$$

$$I=-\ln\frac{\sqrt{2}-1}{\sqrt{2}+1}+\lim_{u\to\infty}\ln\frac{\sqrt{u+1}-1}{\sqrt{u+1}+1}$$

$$=\ln\frac{\sqrt{2}+1}{\sqrt{2}-1}+\ln\left(\lim_{u\to\infty}\frac{\sqrt{u+1}-1}{\sqrt{u+1}+1}\right)$$

Užijeme větu o limitě funkce se spojitou vnější složkou.

Integrujte $\displaystyle\int_1^{\infty} \frac{1}{x\sqrt{x+1}}\,\mathrm{d}x.$

$$I = \lim_{u\to\infty} \int_1^{u} \frac{1}{x\sqrt{x+1}}\,\mathrm{d}x \qquad \int \frac{1}{x\sqrt{x+1}}\,\mathrm{d}x = \ln \frac{\sqrt{x+1}-1}{\sqrt{x+1}+1}$$

$$\int_1^{u} \frac{1}{x\sqrt{x+1}}\,\mathrm{d}x = \left[\ln \frac{\sqrt{x+1}-1}{\sqrt{x+1}+1}\right]_1^{u} = \ln \frac{\sqrt{u+1}-1}{\sqrt{u+1}+1} - \ln \frac{\sqrt{2}-1}{\sqrt{2}+1}$$

$$I = -\ln \frac{\sqrt{2}-1}{\sqrt{2}+1} + \lim_{u\to\infty} \ln \frac{\sqrt{u+1}-1}{\sqrt{u+1}+1}$$

$$= \ln \frac{\sqrt{2}+1}{\sqrt{2}-1} + \ln\left(\lim_{u\to\infty} \frac{\sqrt{u+1}-1}{\sqrt{u+1}+1}\right) = \ln \frac{\sqrt{2}+1}{\sqrt{2}-1} + \ln\left(\lim_{u\to\infty} \frac{(\sqrt{u+1}-1)'}{(\sqrt{u+1}+1)'}\right)$$

Vnitřní složka je neurčitý výraz typu $\frac{\infty}{\infty}$ a lze použít l'Hospitalovo pravidlo.

Integrujte $\int_1^{\infty} \frac{1}{x\sqrt{x+1}}\,dx.$

$$I = \lim_{u\to\infty} \int_1^u \frac{1}{x\sqrt{x+1}}\,dx \qquad \int \frac{1}{x\sqrt{x+1}}\,dx = \ln\frac{\sqrt{x+1}-1}{\sqrt{x+1}+1}$$

$$\int_1^u \frac{1}{x\sqrt{x+1}}\,dx = \left[\ln\frac{\sqrt{x+1}-1}{\sqrt{x+1}+1}\right]_1^u = \ln\frac{\sqrt{u+1}-1}{\sqrt{u+1}+1} - \ln\frac{\sqrt{2}-1}{\sqrt{2}+1}$$

$$I = -\ln\frac{\sqrt{2}-1}{\sqrt{2}+1} + \lim_{u\to\infty}\ln\frac{\sqrt{u+1}-1}{\sqrt{u+1}+1}$$

$$= \ln\frac{\sqrt{2}+1}{\sqrt{2}-1} + \ln\left(\lim_{u\to\infty}\frac{\sqrt{u+1}-1}{\sqrt{u+1}+1}\right) = \ln\frac{\sqrt{2}+1}{\sqrt{2}-1} + \ln\left(\lim_{u\to\infty}\frac{(\sqrt{u+1})'}{(\sqrt{u+1})'}\right)$$

$$= \ln\frac{\sqrt{2}+1}{\sqrt{2}-1} + \ln 1$$

Čitatel a jmenovatel se zkrátí.

Integrujte $\displaystyle\int_1^\infty \frac{1}{x\sqrt{x+1}}\,dx.$

$$I = \lim_{u\to\infty} \int_1^u \frac{1}{x\sqrt{x+1}}\,dx \qquad \int \frac{1}{x\sqrt{x+1}}\,dx = \ln \frac{\sqrt{x+1}-1}{\sqrt{x+1}+1}$$

$$\int_1^u \frac{1}{x\sqrt{x+1}}\,dx = \left[\ln \frac{\sqrt{x+1}-1}{\sqrt{x+1}+1}\right]_1^u = \ln \frac{\sqrt{u+1}-1}{\sqrt{u+1}+1} - \ln \frac{\sqrt{2}-1}{\sqrt{2}+1}$$

$$I = -\ln \frac{\sqrt{2}-1}{\sqrt{2}+1} + \lim_{u\to\infty} \ln \frac{\sqrt{u+1}-1}{\sqrt{u+1}+1}$$

$$= \ln \frac{\sqrt{2}+1}{\sqrt{2}-1} + \ln\left(\lim_{u\to\infty} \frac{\sqrt{u+1}-1}{\sqrt{u+1}+1}\right) = \ln \frac{\sqrt{2}+1}{\sqrt{2}-1} + \ln\left(\lim_{u\to\infty} \frac{(\sqrt{u+1})'}{(\sqrt{u+1})'}\right)$$

$$= \ln \frac{\sqrt{2}+1}{\sqrt{2}-1} + \ln 1 = \ln \frac{\sqrt{2}+1}{\sqrt{2}-1}$$

$\ln 1 = 0$

Integrujte $\displaystyle\int_1^\infty \frac{1}{x\sqrt{x+1}}\,dx.$

$$I = \lim_{u\to\infty} \int_1^u \frac{1}{x\sqrt{x+1}}\,dx \qquad \int \frac{1}{x\sqrt{x+1}}\,dx = \ln\frac{\sqrt{x+1}-1}{\sqrt{x+1}+1}$$

$$\int_1^u \frac{1}{x\sqrt{x+1}}\,dx = \left[\ln\frac{\sqrt{x+1}-1}{\sqrt{x+1}+1}\right]_1^u = \ln\frac{\sqrt{u+1}-1}{\sqrt{u+1}+1} - \ln\frac{\sqrt{2}-1}{\sqrt{2}+1}$$

$$I = -\ln\frac{\sqrt{2}-1}{\sqrt{2}+1} + \lim_{u\to\infty}\ln\frac{\sqrt{u+1}-1}{\sqrt{u+1}+1}$$

$$= \ln\frac{\sqrt{2}+1}{\sqrt{2}-1} + \ln\left(\lim_{u\to\infty}\frac{\sqrt{u+1}-1}{\sqrt{u+1}+1}\right) = \ln\frac{\sqrt{2}+1}{\sqrt{2}-1} + \ln\left(\lim_{u\to\infty}\frac{(\sqrt{u+1})'}{(\sqrt{u+1})'}\right)$$

$$= \ln\frac{\sqrt{2}+1}{\sqrt{2}-1} + \ln 1 = \ln\frac{\sqrt{2}+1}{\sqrt{2}-1}$$

Problém je vyřešen, integrál konverguje.

Integrujte $I = \int_0^{\infty} xe^{-x^2}\,\mathrm{d}x$

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$$I = \lim_{u\to\infty} \int_0^{u} xe^{-x^2}\,\mathrm{d}x$$

Použijeme definici nevlastního integrálu. Singulárním bodem je $+\infty$.

Integrujte $I = \int_0^{\infty} x e^{-x^2} \, dx$

$$I = \lim_{u\to\infty} \int_0^{u} x e^{-x^2} \, dx$$

$$\int x e^{-x^2} \, dx$$

Nejdříve vypočteme neurčitý integrál.

Integrujte $I = \int_0^{\infty} x e^{-x^2}\,\mathrm{d}x$

$$I = \lim_{u\to\infty} \int_0^{u} x e^{-x^2}\,\mathrm{d}x$$

$$\int x e^{-x^2}\,\mathrm{d}x$$

$$-x^2 = t$$

Složená funkce "volá" po substituci $(-x^2)$.

Integrujte $I = \int_0^{\infty} xe^{-x^2}\,dx$

$$I = \lim_{u\to\infty} \int_0^{u} xe^{-x^2}\,dx$$

$$\int xe^{-x^2}\,dx$$

$$\begin{aligned} -x^2 &= t \\ -2x\,dx &= dt \\ x\,dx &= -\frac{1}{2}\,dt \end{aligned}$$

Nalezneme vztah mezi diferenciály. Všimněme si, že diferenciál nalevo vychází $x\,dx$, což v integrálu přesně potřebujeme.

Integrujte $I = \int_0^{\infty} x e^{-x^2}\, \mathrm{d}x$

$$I = \lim_{u\to\infty} \int_0^{u} x e^{-x^2}\, \mathrm{d}x$$

$$\int x e^{-x^2}\, \mathrm{d}x \quad \boxed{\begin{aligned} -x^2 &= t \\ -2x\,\mathrm{d}x &= \mathrm{d}t \\ x\,\mathrm{d}x &= -\frac{1}{2}\,\mathrm{d}t \end{aligned}} = -\frac{1}{2}\int e^t\, \mathrm{d}t$$

Dosadíme substituci.

Integrujte $I = \int_0^{\infty} xe^{-x^2}\,\mathrm{d}x$

$$I = \lim_{u\to\infty} \int_0^{u} xe^{-x^2}\,\mathrm{d}x$$

$$\int xe^{-x^2}\,\mathrm{d}x \quad \boxed{\begin{aligned} -x^2 &= t \\ -2x\,\mathrm{d}x &= \mathrm{d}t \\ x\,\mathrm{d}x &= -\frac{1}{2}\,\mathrm{d}t \end{aligned}} = -\frac{1}{2}\int e^t\,\mathrm{d}t = -\frac{1}{2}e^t$$

Vypočteme integrál.

Integrujte $I = \int_0^\infty x e^{-x^2}\,dx$

$$I = \lim_{u\to\infty} \int_0^u x e^{-x^2}\,dx$$

$$\int x e^{-x^2}\,dx \quad \boxed{\begin{aligned} -x^2 &= t \\ -2x\,dx &= dt \\ x\,dx &= -\frac{1}{2}\,dt \end{aligned}} \quad = -\frac{1}{2}\int e^t\,dt = -\frac{1}{2}e^t = \frac{1}{2}e^{-x^2}$$

Zpětná substituce zařídí návrat k proměnné x.

Integrujte $I = \int_0^{\infty} x e^{-x^2}\, dx$

$$I = \lim_{u \to \infty} \int_0^u x e^{-x^2}\, dx$$

$$\int x e^{-x^2}\, dx \quad \boxed{\begin{aligned} -x^2 &= t \\ -2x\, dx &= dt \\ x\, dx &= -\frac{1}{2}\, dt \end{aligned}} = -\frac{1}{2} \int e^t\, dt = -\frac{1}{2} e^t = \frac{1}{2} e^{-x^2}$$

$$\int_0^u x e^{-x^2}\, dx$$

Vypočítáme určitý integrál.

Integrujte $I = \int_0^{\infty} x e^{-x^2}\,\mathrm{d}x$

$$I = \lim_{u\to\infty} \int_0^{u} x e^{-x^2}\,\mathrm{d}x$$

$$\int x e^{-x^2}\,\mathrm{d}x \quad \boxed{\begin{aligned} -x^2 &= t \\ -2x\,\mathrm{d}x &= \mathrm{d}t \\ x\,\mathrm{d}x &= -\frac{1}{2}\,\mathrm{d}t \end{aligned}} = -\frac{1}{2}\int e^t\,\mathrm{d}t = -\frac{1}{2}e^t = \frac{1}{2}e^{-x^2}$$

$$\int_0^{u} x e^{-x^2}\,\mathrm{d}x = \left[-\frac{1}{2}e^{-x^2}\right]_0^{u}$$

Neučitý integrál známe a můžeme použít Newtonovu-Leibnizovu větu.

Integrujte $I = \int_0^{\infty} x e^{-x^2}\,\mathrm{d}x$

$$I = \lim_{u\to\infty} \int_0^{u} x e^{-x^2}\,\mathrm{d}x$$

$$\int x e^{-x^2}\,\mathrm{d}x \quad \boxed{\begin{aligned} -x^2 &= t \\ -2x\,\mathrm{d}x &= \mathrm{d}t \\ x\,\mathrm{d}x &= -\frac{1}{2}\,\mathrm{d}t \end{aligned}} = -\frac{1}{2}\int e^t\,\mathrm{d}t = -\frac{1}{2}e^t = \frac{1}{2}e^{-x^2}$$

$$\int_0^{u} x e^{-x^2}\,\mathrm{d}x = \left[-\frac{1}{2}e^{-x^2}\right]_0^{u} = -\frac{1}{2}e^{-u^2} - \left(-\frac{1}{2}e^{0}\right)$$

Dosadíme meze.

Integrujte $I = \int_0^{\infty} x e^{-x^2}\,\mathrm{d}x$

$$I = \lim_{u\to\infty} \int_0^u x e^{-x^2}\,\mathrm{d}x$$

$$\int x e^{-x^2}\,\mathrm{d}x \quad \boxed{\begin{aligned} -x^2 &= t \\ -2x\,\mathrm{d}x &= \mathrm{d}t \\ x\,\mathrm{d}x &= -\frac{1}{2}\,\mathrm{d}t \end{aligned}} = -\frac{1}{2}\int e^t\,\mathrm{d}t = -\frac{1}{2}e^t = \frac{1}{2}e^{-x^2}$$

$$\int_0^u x e^{-x^2}\,\mathrm{d}x = \left[-\frac{1}{2}e^{-x^2}\right]_0^u = -\frac{1}{2}e^{-u^2} - \left(-\frac{1}{2}e^0\right) = -\frac{1}{2}e^{-u^2} + \frac{1}{2}$$

Upravíme.

Integrujte $I = \int_0^{\infty} x e^{-x^2}\,\mathrm{d}x$

$$I = \lim_{u\to\infty} \int_0^{u} x e^{-x^2}\,\mathrm{d}x$$

$$\int x e^{-x^2}\,\mathrm{d}x \quad \boxed{\begin{aligned} -x^2 &= t \\ -2x\,\mathrm{d}x &= \mathrm{d}t \\ x\,\mathrm{d}x &= -\frac{1}{2}\,\mathrm{d}t \end{aligned}} = -\frac{1}{2}\int e^t\,\mathrm{d}t = -\frac{1}{2}e^t = \frac{1}{2}e^{-x^2}$$

$$\int_0^{u} x e^{-x^2}\,\mathrm{d}x = \left[-\frac{1}{2}e^{-x^2}\right]_0^u = -\frac{1}{2}e^{-u^2} - \left(-\frac{1}{2}e^{0}\right) = -\frac{1}{2}e^{-u^2} + \frac{1}{2}$$

$$I = \frac{1}{2} - \lim_{u\to\infty} \frac{1}{2}e^{-u^2}$$

Nevlastní integrál je (podle definice) limitou určitého integrálu.

Integrujte $I = \int_0^{\infty} x e^{-x^2}\,\mathrm{d}x$

$$I = \lim_{u\to\infty} \int_0^{u} x e^{-x^2}\,\mathrm{d}x$$

$$\int x e^{-x^2}\,\mathrm{d}x \quad \boxed{\begin{aligned} -x^2 &= t \\ -2x\,\mathrm{d}x &= \mathrm{d}t \\ x\,\mathrm{d}x &= -\frac{1}{2}\,\mathrm{d}t \end{aligned}} = -\frac{1}{2}\int e^t\,\mathrm{d}t = -\frac{1}{2}e^t = \frac{1}{2}e^{-x^2}$$

$$\int_0^u x e^{-x^2}\,\mathrm{d}x = \left[-\frac{1}{2}e^{-x^2}\right]_0^u = -\frac{1}{2}e^{-u^2} - \left(-\frac{1}{2}e^0\right) = -\frac{1}{2}e^{-u^2} + \frac{1}{2}$$

$$I = \frac{1}{2} - \lim_{u\to\infty}\frac{1}{2}e^{-u^2} = \frac{1}{2} - \frac{1}{2}e^{-\infty}$$

$\infty^2 = \infty$ (při počítání s limitami)

Integrujte $I = \int_0^{\infty} x e^{-x^2}\,\mathrm{d}x$

$$I = \lim_{u\to\infty} \int_0^{u} x e^{-x^2}\,\mathrm{d}x$$

$$\int x e^{-x^2}\,\mathrm{d}x \quad \boxed{\begin{aligned} -x^2 &= t \\ -2x\,\mathrm{d}x &= \mathrm{d}t \\ x\,\mathrm{d}x &= -\frac{1}{2}\,\mathrm{d}t \end{aligned}} = -\frac{1}{2}\int e^t\,\mathrm{d}t = -\frac{1}{2}e^t = \frac{1}{2}e^{-x^2}$$

$$\int_0^{u} x e^{-x^2}\,\mathrm{d}x = \left[-\frac{1}{2}e^{-x^2}\right]_0^{u} = -\frac{1}{2}e^{-u^2} - \left(-\frac{1}{2}e^{0}\right) = -\frac{1}{2}e^{-u^2} + \frac{1}{2}$$

$$I = \frac{1}{2} - \lim_{u\to\infty} \frac{1}{2}e^{-u^2} = \frac{1}{2} - \frac{1}{2}e^{-\infty} = \frac{1}{2}$$

$e^{-\infty} = 0$ (při počítání s limitami)

Integrujte $I = \int_0^{\infty} x e^{-x^2}\,dx$

$$I = \lim_{u\to\infty} \int_0^{u} x e^{-x^2}\,dx$$

$$\int x e^{-x^2}\,dx \quad \boxed{\begin{aligned} -x^2 &= t \\ -2x\,dx &= dt \\ x\,dx &= -\frac{1}{2}\,dt \end{aligned}} = -\frac{1}{2}\int e^t\,dt = -\frac{1}{2}e^t = \frac{1}{2}e^{-x^2}$$

$$\int_0^u x e^{-x^2}\,dx = \left[-\frac{1}{2}e^{-x^2}\right]_0^u = -\frac{1}{2}e^{-u^2} - \left(-\frac{1}{2}e^{0}\right) = -\frac{1}{2}e^{-u^2} + \frac{1}{2}$$

$$I = \frac{1}{2} - \lim_{u\to\infty}\frac{1}{2}e^{-u^2} = \frac{1}{2} - \frac{1}{2}e^{-\infty} = \frac{1}{2}$$

Vyřešeno.

Integrujte $\quad I = \int_0^{\infty} x^2 e^{-x} \, \mathrm{d}x.$

Integrujte $I = \int_0^{\infty} x^2 e^{-x} \, \mathrm{d}x.$

$$I = \lim_{u \to \infty} \int_0^u x^2 e^{-x} \, \mathrm{d}x$$

Použijeme definici nevlastního integrálu.

Integrujte $I = \int_0^\infty x^2 e^{-x}\,\mathrm{d}x.$

$$I = \lim_{u\to\infty} \int_0^u x^2 e^{-x}\,\mathrm{d}x$$

$$\int x^2 e^{-x}\,\mathrm{d}x$$

Nejprve budeme hledat primitivní funkci.

Integrujte $I = \int_0^\infty x^2 e^{-x}\,\mathrm{d}x.$

$$I = \lim_{u\to\infty} \int_0^u x^2 e^{-x}\,\mathrm{d}x$$

$$\int x^2 e^{-x}\,\mathrm{d}x = -x^2 e^{-x} + 2\int x e^{-x}\,\mathrm{d}x$$

Použijeme integraci per partés při volbě $\boxed{\begin{array}{ll} u = x^2 & u' = 2x \\ v' = e^{-x} & v = -e^{-x} \end{array}}$.

Integrujte $\quad I = \int_0^{\infty} x^2 e^{-x}\,\mathrm{d}x.$

$$I = \lim_{u\to\infty} \int_0^{u} x^2 e^{-x}\,\mathrm{d}x$$

$$\int x^2 e^{-x}\,\mathrm{d}x = -x^2 e^{-x} + 2\int x e^{-x}\,\mathrm{d}x = -x^2 e^{-x} + 2\left(-x e^{-x} + \int e^{-x}\,\mathrm{d}x\right)$$

Použijeme opět integraci per partés, nyní při volbě $\begin{array}{ll} u = x & u' = 1 \\ v' = e^{-x} & v = -e^{-x} \end{array}$.

Integrujte $I = \int_0^{\infty} x^2 e^{-x}\,\mathrm{d}x.$

$$I = \lim_{u\to\infty} \int_0^u x^2 e^{-x}\,\mathrm{d}x$$

$$\int x^2 e^{-x}\,\mathrm{d}x = -x^2 e^{-x} + 2\int x e^{-x}\,\mathrm{d}x = -x^2 e^{-x} + 2\left(-x e^{-x} + \int e^{-x}\,\mathrm{d}x\right)$$

$$= -x^2 e^{-x} + 2\left(-x e^{-x} - e^{-x}\right)$$

Zintegrujeme.

Integrujte $I = \int_0^\infty x^2 e^{-x}\,\mathrm{d}x.$

$$I = \lim_{u\to\infty} \int_0^u x^2 e^{-x}\,\mathrm{d}x$$

$$\int x^2 e^{-x}\,\mathrm{d}x = -x^2 e^{-x} + 2\int x e^{-x}\,\mathrm{d}x = -x^2 e^{-x} + 2\left(-x e^{-x} + \int e^{-x}\,\mathrm{d}x\right)$$

$$= -x^2 e^{-x} + 2\left(-x e^{-x} - e^{-x}\right) = -e^{-x}(x^2 + 2x + 2)$$

Vytkneme opakující se člen $-e^{-x}$ před závorku.

Integrujte $I = \int_0^\infty x^2 e^{-x}\,\mathrm{d}x.$

$$I = \lim_{u\to\infty} \int_0^u x^2 e^{-x}\,\mathrm{d}x$$

$$\int x^2 e^{-x}\,\mathrm{d}x = -x^2 e^{-x} + 2\int x e^{-x}\,\mathrm{d}x = -x^2 e^{-x} + 2\left(-x e^{-x} + \int e^{-x}\,\mathrm{d}x\right)$$

$$= -x^2 e^{-x} + 2\left(-x e^{-x} - e^{-x}\right) = -e^{-x}(x^2 + 2x + 2)$$

$$\int_0^u x^2 e^{-x}\,\mathrm{d}x = \left[-e^{-x}(x^2 + 2x + 2)\right]_0^u$$

Nyní budeme počítat určitý integrál. Protože známe primitivní funkci, můžeme využít Newtonovu-Leibnizovu větu.

Integrujte $I = \int_0^{\infty} x^2 e^{-x}\,\mathrm{d}x.$

$$I = \lim_{u\to\infty} \int_0^u x^2 e^{-x}\,\mathrm{d}x$$

$$\int x^2 e^{-x}\,\mathrm{d}x = -x^2 e^{-x} + 2\int x e^{-x}\,\mathrm{d}x = -x^2 e^{-x} + 2\left(-xe^{-x} + \int e^{-x}\,\mathrm{d}x\right)$$

$$= -x^2 e^{-x} + 2\left(-xe^{-x} - e^{-x}\right) = -e^{-x}(x^2 + 2x + 2)$$

$$\int_0^u x^2 e^{-x}\,\mathrm{d}x = \left[-e^{-x}(x^2 + 2x + 2)\right]_0^u$$

$$= -e^{-u}(u^2 + 2u + 2) - \left[-e^{0}(0 + 0 + 2)\right]$$

Dosadíme meze.

Integrujte $\quad I = \int_0^{\infty} x^2 e^{-x}\,\mathrm{d}x.$

$$I = \lim_{u\to\infty} \int_0^u x^2 e^{-x}\,\mathrm{d}x$$

$$\int x^2 e^{-x}\,\mathrm{d}x = -x^2e^{-x} + 2\int xe^{-x}\,\mathrm{d}x = -x^2e^{-x} + 2\left(-xe^{-x} + \int e^{-x}\,\mathrm{d}x\right)$$

$$= -x^2e^{-x} + 2\,(-xe^{-x} - e^{-x}) = -e^{-x}(x^2 + 2x + 2)$$

$$\int_0^u x^2 e^{-x}\,\mathrm{d}x = \left[-e^{-x}(x^2+2x+2)\right]_0^u$$

$$= -e^{-u}(u^2+2u+2) - \left[-e^{0}(0+0+2)\right] = -e^{-u}(u^2+2u+2) + 2$$

Upravíme. Nyní je nutno vypočítat limitu.

Integrujte $\quad I = \int_0^{\infty} x^2 e^{-x}\,\mathrm{d}x.$

$$I = \lim_{u\to\infty} \int_0^u x^2 e^{-x}\,\mathrm{d}x$$

$$\int x^2 e^{-x}\,\mathrm{d}x = -x^2 e^{-x} + 2\int x e^{-x}\,\mathrm{d}x = -x^2 e^{-x} + 2\left(-x e^{-x} + \int e^{-x}\,\mathrm{d}x\right)$$

$$= -x^2 e^{-x} + 2\left(-x e^{-x} - e^{-x}\right) = -e^{-x}(x^2 + 2x + 2)$$

$$\int_0^u x^2 e^{-x}\,\mathrm{d}x = \left[-e^{-x}(x^2 + 2x + 2)\right]_0^u$$

$$= -e^{-u}(u^2 + 2u + 2) - \left[-e^0(0 + 0 + 2)\right] = -e^{-u}(u^2 + 2u + 2) + 2$$

$$I = 2 - \lim_{u\to\infty} e^{-u}(u^2 + 2u + 2)$$

$\lim_{u\to\infty} e^{-u} = 0$ a vychází neurčitý výraz typu $\boxed{0 \times \infty}$.

Integrujte $I = \int_0^{\infty} x^2 e^{-x}\,\mathrm{d}x.$

$$I = \lim_{u\to\infty} \int_0^u x^2 e^{-x}\,\mathrm{d}x$$

$$\int x^2 e^{-x}\,\mathrm{d}x = -x^2 e^{-x} + 2\int x e^{-x}\,\mathrm{d}x = -x^2 e^{-x} + 2\left(-x e^{-x} + \int e^{-x}\,\mathrm{d}x\right)$$

$$= -x^2 e^{-x} + 2\left(-x e^{-x} - e^{-x}\right) = -e^{-x}(x^2 + 2x + 2)$$

$$\int_0^u x^2 e^{-x}\,\mathrm{d}x = \left[-e^{-x}(x^2 + 2x + 2)\right]_0^u$$

$$= -e^{-u}(u^2 + 2u + 2) - \left[-e^0(0 + 0 + 2)\right] = -e^{-u}(u^2 + 2u + 2) + 2$$

$$I = 2 - \lim_{u\to\infty} e^{-u}(u^2 + 2u + 2) = 2 - \lim_{u\to\infty} \frac{u^2 + 2u + 2}{e^u}$$

Převedeme součin na podíl, abychom mohli použít l'Hospitalovo pravidlo.

Integrujte $I = \int_0^{\infty} x^2 e^{-x}\,\mathrm{d}x.$

$$I = \lim_{u\to\infty} \int_0^u x^2 e^{-x}\,\mathrm{d}x$$

$$\int x^2 e^{-x}\,\mathrm{d}x = -x^2e^{-x} + 2\int xe^{-x}\,\mathrm{d}x = -x^2e^{-x} + 2\left(-xe^{-x} + \int e^{-x}\,\mathrm{d}x\right)$$

$$= -x^2e^{-x} + 2\left(-xe^{-x} - e^{-x}\right) = -e^{-x}(x^2+2x+2)$$

$$\int_0^u x^2 e^{-x}\,\mathrm{d}x = \left[-e^{-x}(x^2+2x+2)\right]_0^u$$

$$= -e^{-u}(u^2+2u+2) - \left[-e^0(0+0+2)\right] = -e^{-u}(u^2+2u+2) + 2$$

$$I = 2 - \lim_{u\to\infty} e^{-u}(u^2+2u+2) = 2 - \lim_{u\to\infty}\frac{u^2+2u+2}{e^u}$$

$$= 2 - \lim_{u\to\infty}\frac{2u+2}{e^u}$$

Po aplikaci l'Hospitalova pravidla máme stále neurčitý výraz $\frac{\infty}{\infty}$. Použijeme tedy l'Hospitalovo pravdlo ještě jednou.

Integrujte $\quad I = \int_0^{\infty} x^2 e^{-x}\,\mathrm{d}x.$

$$I = \lim_{u\to\infty} \int_0^u x^2 e^{-x}\,\mathrm{d}x$$

$$\int x^2 e^{-x}\,\mathrm{d}x = -x^2 e^{-x} + 2\int x e^{-x}\,\mathrm{d}x = -x^2 e^{-x} + 2\left(-x e^{-x} + \int e^{-x}\,\mathrm{d}x\right)$$

$$= -x^2 e^{-x} + 2\left(-x e^{-x} - e^{-x}\right) = -e^{-x}(x^2 + 2x + 2)$$

$$\int_0^u x^2 e^{-x}\,\mathrm{d}x = \left[-e^{-x}(x^2 + 2x + 2)\right]_0^u$$

$$= -e^{-u}(u^2 + 2u + 2) - \left[-e^0(0 + 0 + 2)\right] = -e^{-u}(u^2 + 2u + 2) + 2$$

$$I = 2 - \lim_{u\to\infty} e^{-u}(u^2 + 2u + 2) = 2 - \lim_{u\to\infty} \frac{u^2 + 2u + 2}{e^u}$$

$$= 2 - \lim_{u\to\infty} \frac{2u + 2}{e^u} = 2 - \lim_{u\to\infty} \frac{2}{e^u}$$

Nyní dostáváme $\lim_{u\to\infty} \frac{2}{e^u} = \frac{2}{e^{\infty}} = \frac{2}{\infty} = 0.$

Integrujte $\quad I = \int_0^{\infty} x^2 e^{-x}\,\mathrm{d}x.$

$$I = \lim_{u\to\infty} \int_0^u x^2 e^{-x}\,\mathrm{d}x$$

$$\int x^2 e^{-x}\,\mathrm{d}x = -x^2 e^{-x} + 2\int x e^{-x}\,\mathrm{d}x = -x^2 e^{-x} + 2\left(-x e^{-x} + \int e^{-x}\,\mathrm{d}x\right)$$

$$= -x^2 e^{-x} + 2\left(-x e^{-x} - e^{-x}\right) = -e^{-x}(x^2 + 2x + 2)$$

$$\int_0^u x^2 e^{-x}\,\mathrm{d}x = \left[-e^{-x}(x^2 + 2x + 2)\right]_0^u$$

$$= -e^{-u}(u^2 + 2u + 2) - \left[-e^0(0 + 0 + 2)\right] = -e^{-u}(u^2 + 2u + 2) + 2$$

$$I = 2 - \lim_{u\to\infty} e^{-u}(u^2 + 2u + 2) = 2 - \lim_{u\to\infty} \frac{u^2 + 2u + 2}{e^u}$$

$$= 2 - \lim_{u\to\infty} \frac{2u + 2}{e^u} = 2 - \lim_{u\to\infty} \frac{2}{e^u} = 2 - 0$$

Nyní dostáváme $\lim_{u\to\infty} \frac{2}{e^u} = \frac{2}{e^{\infty}} = \frac{2}{\infty} = 0.$

Integrujte $I = \int_0^{\infty} x^2 e^{-x}\, dx.$

$$I = \lim_{u\to\infty} \int_0^u x^2 e^{-x}\, dx$$

$$\int x^2 e^{-x}\, dx = -x^2 e^{-x} + 2\int x e^{-x}\, dx = -x^2 e^{-x} + 2\left(-xe^{-x} + \int e^{-x}\, dx\right)$$

$$= -x^2 e^{-x} + 2\left(-xe^{-x} - e^{-x}\right) = -e^{-x}(x^2 + 2x + 2)$$

$$\int_0^u x^2 e^{-x}\, dx = \left[-e^{-x}(x^2 + 2x + 2)\right]_0^u$$

$$= -e^{-u}(u^2 + 2u + 2) - \left[-e^0(0 + 0 + 2)\right] = -e^{-u}(u^2 + 2u + 2) + 2$$

$$I = 2 - \lim_{u\to\infty} e^{-u}(u^2 + 2u + 2) = 2 - \lim_{u\to\infty} \frac{u^2 + 2u + 2}{e^u}$$

$$= 2 - \lim_{u\to\infty} \frac{2u + 2}{e^u} = 2 - \lim_{u\to\infty} \frac{2}{e^u} = 2 - 0 = 2$$

Hotovo, integrál konveruje a jeho hodnota je 2.

Find $I = \int_1^{\infty} \frac{\operatorname{arctg} x}{x^2 + 1} \, \mathrm{d}x$

Find $I = \int_1^{\infty} \frac{\operatorname{arctg} x}{x^2+1}\,\mathrm{d}x$

$$I = \lim_{u\to\infty} \int_1^{u} \frac{\operatorname{arctg} x}{x^2+1}\,\mathrm{d}x$$

We start with the definition of the improper integral.

Find $I = \int_1^{\infty} \frac{\operatorname{arctg} x}{x^2+1}\,\mathrm{d}x$

$$I = \lim_{u\to\infty} \int_1^{u} \frac{\operatorname{arctg} x}{x^2+1}\,\mathrm{d}x$$

$$\int \frac{\operatorname{arctg} x}{x^2+1}\,\mathrm{d}x$$

We evaluate the indefinite integral first.

Find $I = \int_1^{\infty} \frac{\operatorname{arctg} x}{x^2+1}\,\mathrm{d}x$

$$I = \lim_{u\to\infty} \int_1^{u} \frac{\operatorname{arctg} x}{x^2+1}\,\mathrm{d}x$$

$$\int \frac{\operatorname{arctg} x}{x^2+1}\,\mathrm{d}x$$

$\operatorname{arctg} x = t$

We use the substitution $\operatorname{arctg} x = t$.

Find $I = \int_1^{\infty} \frac{\operatorname{arctg} x}{x^2+1}\,\mathrm{d}x$

$$I = \lim_{u\to\infty} \int_1^{u} \frac{\operatorname{arctg} x}{x^2+1}\,\mathrm{d}x$$

$$\int \frac{\operatorname{arctg} x}{x^2+1}\,\mathrm{d}x$$

$$\operatorname{arctg} x = t$$
$$\frac{1}{x^2+1}\,\mathrm{d}x = \mathrm{d}t$$

With this substitution we have $\frac{1}{x^2+1}\,\mathrm{d}x = \mathrm{d}t$ and the term $\frac{1}{x^2+1}\,\mathrm{d}x$ is present in the integral.

Find $I = \int_1^{\infty} \frac{\operatorname{arctg} x}{x^2+1}\,dx$

$$I = \lim_{u\to\infty} \int_1^{u} \frac{\operatorname{arctg} x}{x^2+1}\,dx$$

$$\int \frac{\operatorname{arctg} x}{x^2+1}\,dx \quad \boxed{\begin{array}{c}\operatorname{arctg} x = t\\ \frac{1}{x^2+1}\,dx = dt\end{array}} = \int t\,dt$$

We substitute,...

Find $I = \int_1^{\infty} \frac{\operatorname{arctg} x}{x^2+1}\,\mathrm{d}x$

$$I = \lim_{u\to\infty} \int_1^{u} \frac{\operatorname{arctg} x}{x^2+1}\,\mathrm{d}x$$

$$\int \frac{\operatorname{arctg} x}{x^2+1}\,\mathrm{d}x \quad \boxed{\begin{array}{c}\operatorname{arctg} x = t\\ \frac{1}{x^2+1}\,\mathrm{d}x = \mathrm{d}t\end{array}} = \int t\,\mathrm{d}t = \frac{t^2}{2}$$

... evaluate the integral ...

Find $I = \int_1^{\infty} \frac{\operatorname{arctg} x}{x^2+1}\,dx$

$$I = \lim_{u\to\infty} \int_1^{u} \frac{\operatorname{arctg} x}{x^2+1}\,dx$$

$$\int \frac{\operatorname{arctg} x}{x^2+1}\,dx \quad \boxed{\begin{gathered}\operatorname{arctg} x = t\\ \frac{1}{x^2+1}\,dx = dt\end{gathered}} = \int t\,dt = \frac{t^2}{2} = \frac{\operatorname{arctg}^2 x}{2}$$

... and return to the variable x.

Find $I = \int_1^{\infty} \frac{\operatorname{arctg} x}{x^2+1}\,\mathrm{d}x$

$$I = \lim_{u\to\infty} \int_1^{u} \frac{\operatorname{arctg} x}{x^2+1}\,\mathrm{d}x$$

$$\int \frac{\operatorname{arctg} x}{x^2+1}\,\mathrm{d}x \quad \boxed{\begin{array}{c} \operatorname{arctg} x = t \\ \frac{1}{x^2+1}\,\mathrm{d}x = \mathrm{d}t \end{array}} = \int t\,\mathrm{d}t = \frac{t^2}{2} = \frac{\operatorname{arctg}^2 x}{2}$$

$$\int_1^{u} \frac{\operatorname{arctg} x}{x^2+1}\,\mathrm{d}x$$

We continue with the definite integral.

Find $I = \int_1^{\infty} \frac{\operatorname{arctg} x}{x^2+1}\,\mathrm{d}x$

$$I = \lim_{u\to\infty} \int_1^{u} \frac{\operatorname{arctg} x}{x^2+1}\,\mathrm{d}x$$

$$\int \frac{\operatorname{arctg} x}{x^2+1}\,\mathrm{d}x \quad \boxed{\begin{array}{c}\operatorname{arctg} x = t\\ \frac{1}{x^2+1}\,\mathrm{d}x = \mathrm{d}t\end{array}} = \int t\,\mathrm{d}t = \frac{t^2}{2} = \frac{\operatorname{arctg}^2 x}{2}$$

$$\int_1^{u} \frac{\operatorname{arctg} x}{x^2+1}\,\mathrm{d}x = \left[\frac{\operatorname{arctg}^2 x}{2}\right]_1^u$$

The antiderivative is known.

Find $I = \int_1^{\infty} \frac{\operatorname{arctg} x}{x^2+1}\,dx$

$$I = \lim_{u\to\infty} \int_1^{u} \frac{\operatorname{arctg} x}{x^2+1}\,dx$$

$$\int \frac{\operatorname{arctg} x}{x^2+1}\,dx \quad \boxed{\begin{array}{c}\operatorname{arctg} x = t\\ \frac{1}{x^2+1}\,dx = dt\end{array}} = \int t\,dt = \frac{t^2}{2} = \frac{\operatorname{arctg}^2 x}{2}$$

$$\int_1^{u} \frac{\operatorname{arctg} x}{x^2+1}\,dx = \left[\frac{\operatorname{arctg}^2 x}{2}\right]_1^u = \frac{\operatorname{arctg}^2 u}{2} - \frac{\operatorname{arctg}^2 1}{2}$$

Newton–Leibniz formula yields the value of the integral.

Find $I = \int_1^{\infty} \frac{\operatorname{arctg} x}{x^2+1}\,dx$

$$I = \lim_{u\to\infty} \int_1^{u} \frac{\operatorname{arctg} x}{x^2+1}\,dx$$

$$\int \frac{\operatorname{arctg} x}{x^2+1}\,dx \quad \boxed{\begin{aligned} \operatorname{arctg} x &= t \\ \frac{1}{x^2+1}\,dx &= dt \end{aligned}} = \int t\,dt = \frac{t^2}{2} = \frac{\operatorname{arctg}^2 x}{2}$$

$$\int_1^{u} \frac{\operatorname{arctg} x}{x^2+1}\,dx = \left[\frac{\operatorname{arctg}^2 x}{2}\right]_1^u = \frac{\operatorname{arctg}^2 u}{2} - \frac{\operatorname{arctg}^2 1}{2} = \frac{\operatorname{arctg}^2 u}{2} - \frac{(\pi/4)^2}{2}$$

Simplifications can be made.

Find $I = \int_1^\infty \frac{\operatorname{arctg} x}{x^2+1}\,dx$

$$I = \lim_{u\to\infty} \int_1^u \frac{\operatorname{arctg} x}{x^2+1}\,dx$$

$$\int \frac{\operatorname{arctg} x}{x^2+1}\,dx \quad \boxed{\begin{aligned}\operatorname{arctg} x &= t\\ \frac{1}{x^2+1}\,dx &= dt\end{aligned}} = \int t\,dt = \frac{t^2}{2} = \frac{\operatorname{arctg}^2 x}{2}$$

$$\int_1^u \frac{\operatorname{arctg} x}{x^2+1}\,dx = \left[\frac{\operatorname{arctg}^2 x}{2}\right]_1^u = \frac{\operatorname{arctg}^2 u}{2} - \frac{\operatorname{arctg}^2 1}{2} = \frac{\operatorname{arctg}^2 u}{2} - \frac{(\pi/4)^2}{2}$$

$$= \frac{\operatorname{arctg}^2 u}{2} - \frac{\pi^2}{32}$$

Find $I = \int_1^{\infty} \frac{\operatorname{arctg} x}{x^2+1}\,dx$

$$I = \lim_{u\to\infty} \int_1^u \frac{\operatorname{arctg} x}{x^2+1}\,dx$$

$$\int \frac{\operatorname{arctg} x}{x^2+1}\,dx \quad \boxed{\begin{array}{c}\operatorname{arctg} x = t\\ \frac{1}{x^2+1}\,dx = dt\end{array}} = \int t\,dt = \frac{t^2}{2} = \frac{\operatorname{arctg}^2 x}{2}$$

$$\int_1^u \frac{\operatorname{arctg} x}{x^2+1}\,dx = \left[\frac{\operatorname{arctg}^2 x}{2}\right]_1^u = \frac{\operatorname{arctg}^2 u}{2} - \frac{\operatorname{arctg}^2 1}{2} = \frac{\operatorname{arctg}^2 u}{2} - \frac{(\pi/4)^2}{2}$$

$$= \frac{\operatorname{arctg}^2 u}{2} - \frac{\pi^2}{32}$$

$$I = \lim_{u\to\infty} \frac{\operatorname{arctg}^2 u}{2} - \frac{\pi^2}{32}$$

We continue with the improper integral. It is a limit of the definite integral.

Find $I = \int_1^{\infty} \frac{\operatorname{arctg} x}{x^2+1}\,dx$

$$I = \lim_{u\to\infty} \int_1^{u} \frac{\operatorname{arctg} x}{x^2+1}\,dx$$

$$\int \frac{\operatorname{arctg} x}{x^2+1}\,dx \quad \boxed{\begin{array}{c}\operatorname{arctg} x = t\\ \frac{1}{x^2+1}\,dx = dt\end{array}} = \int t\,dt = \frac{t^2}{2} = \frac{\operatorname{arctg}^2 x}{2}$$

$$\int_1^{u} \frac{\operatorname{arctg} x}{x^2+1}\,dx = \left[\frac{\operatorname{arctg}^2 x}{2}\right]_1^u = \frac{\operatorname{arctg}^2 u}{2} - \frac{\operatorname{arctg}^2 1}{2} = \frac{\operatorname{arctg}^2 u}{2} - \frac{(\pi/4)^2}{2}$$
$$= \frac{\operatorname{arctg}^2 u}{2} - \frac{\pi^2}{32}$$

$$I = \lim_{u\to\infty} \frac{\operatorname{arctg}^2 u}{2} - \frac{\pi^2}{32} = \frac{(\pi/2)^2}{2} - \frac{\pi^2}{32}$$

The function $y = \operatorname{arctg} x$ has an horizontal asymptote $y = \frac{\pi}{2}$ in $+\infty$. This is the value of the limit $\lim_{u\to\infty} \operatorname{arctg} u$.

Find $I = \int_1^{\infty} \frac{\operatorname{arctg} x}{x^2+1}\,\mathrm{d}x$

$$I = \lim_{u\to\infty} \int_1^u \frac{\operatorname{arctg} x}{x^2+1}\,\mathrm{d}x$$

$$\int \frac{\operatorname{arctg} x}{x^2+1}\,\mathrm{d}x \quad \boxed{\begin{aligned}\operatorname{arctg} x &= t\\ \frac{1}{x^2+1}\,\mathrm{d}x &= \mathrm{d}t\end{aligned}} = \int t\,\mathrm{d}t = \frac{t^2}{2} = \frac{\operatorname{arctg}^2 x}{2}$$

$$\int_1^u \frac{\operatorname{arctg} x}{x^2+1}\,\mathrm{d}x = \left[\frac{\operatorname{arctg}^2 x}{2}\right]_1^u = \frac{\operatorname{arctg}^2 u}{2} - \frac{\operatorname{arctg}^2 1}{2} = \frac{\operatorname{arctg}^2 u}{2} - \frac{(\pi/4)^2}{2}$$

$$= \frac{\operatorname{arctg}^2 u}{2} - \frac{\pi^2}{32}$$

$$I = \lim_{u\to\infty} \frac{\operatorname{arctg}^2 u}{2} - \frac{\pi^2}{32} = \frac{(\pi/2)^2}{2} - \frac{\pi^2}{32} = \frac{\pi^2}{8} - \frac{\pi^2}{32}$$

We simplify.

Find $I = \int_1^{\infty} \frac{\operatorname{arctg} x}{x^2+1}\,dx$

$$I = \lim_{u\to\infty} \int_1^{u} \frac{\operatorname{arctg} x}{x^2+1}\,dx$$

$$\int \frac{\operatorname{arctg} x}{x^2+1}\,dx \quad \boxed{\begin{aligned}\operatorname{arctg} x &= t\\ \frac{1}{x^2+1}\,dx &= dt\end{aligned}} = \int t\,dt = \frac{t^2}{2} = \frac{\operatorname{arctg}^2 x}{2}$$

$$\int_1^{u} \frac{\operatorname{arctg} x}{x^2+1}\,dx = \left[\frac{\operatorname{arctg}^2 x}{2}\right]_1^u = \frac{\operatorname{arctg}^2 u}{2} - \frac{\operatorname{arctg}^2 1}{2} = \frac{\operatorname{arctg}^2 u}{2} - \frac{(\pi/4)^2}{2}$$

$$= \frac{\operatorname{arctg}^2 u}{2} - \frac{\pi^2}{32}$$

$$I = \lim_{u\to\infty} \frac{\operatorname{arctg}^2 u}{2} - \frac{\pi^2}{32} = \frac{(\pi/2)^2}{2} - \frac{\pi^2}{32} = \frac{\pi^2}{8} - \frac{\pi^2}{32} = \frac{3\pi^2}{32}$$

Find $I = \int_1^{\infty} \frac{\operatorname{arctg} x}{x^2+1}\,dx$

$$I = \lim_{u\to\infty} \int_1^{u} \frac{\operatorname{arctg} x}{x^2+1}\,dx$$

$$\int \frac{\operatorname{arctg} x}{x^2+1}\,dx \quad \boxed{\begin{aligned} \operatorname{arctg} x &= t \\ \frac{1}{x^2+1}\,dx &= dt \end{aligned}} = \int t\,dt = \frac{t^2}{2} = \frac{\operatorname{arctg}^2 x}{2}$$

$$\int_1^{u} \frac{\operatorname{arctg} x}{x^2+1}\,dx = \left[\frac{\operatorname{arctg}^2 x}{2}\right]_1^u = \frac{\operatorname{arctg}^2 u}{2} - \frac{\operatorname{arctg}^2 1}{2} = \frac{\operatorname{arctg}^2 u}{2} - \frac{(\pi/4)^2}{2}$$
$$= \frac{\operatorname{arctg}^2 u}{2} - \frac{\pi^2}{32}$$

$$I = \lim_{u\to\infty} \frac{\operatorname{arctg}^2 u}{2} - \frac{\pi^2}{32} = \frac{(\pi/2)^2}{2} - \frac{\pi^2}{32} = \frac{\pi^2}{8} - \frac{\pi^2}{32} = \frac{3\pi^2}{32}$$

The integral is evaluated.

Find $I = \int_{-\infty}^{\infty} \frac{1}{e^{-x} + e^{x}} \, \mathrm{d}x$

Find $I = \int_{-\infty}^{\infty} \frac{1}{e^{-x} + e^{x}}\,\mathrm{d}x$

$$\int_{-\infty}^{\infty} \frac{1}{e^{-x} + e^{x}}\,\mathrm{d}x$$

We start with the integral.

Find $I = \int_{-\infty}^{\infty} \frac{1}{e^{-x} + e^{x}}\, dx$

$$\int_{-\infty}^{\infty} \frac{1}{e^{-x} + e^{x}}\, dx = \int_{-\infty}^{0} \frac{1}{e^{-x} + e^{x}}\, dx + \int_{0}^{\infty} \frac{1}{e^{-x} + e^{x}}\, dx$$

There are two singularities: $\pm\infty$.

Find $I = \int_{-\infty}^{\infty} \frac{1}{e^{-x} + e^{x}}\, \mathrm{d}x$

$$\int_{-\infty}^{\infty} \frac{1}{e^{-x} + e^{x}}\, \mathrm{d}x = \int_{-\infty}^{0} \frac{1}{e^{-x} + e^{x}}\, \mathrm{d}x + \int_{0}^{\infty} \frac{1}{e^{-x} + e^{x}}\, \mathrm{d}x$$

$$= \lim_{u \to -\infty} \int_{u}^{0} \frac{1}{e^{-x} + e^{x}}\, \mathrm{d}x + \lim_{u \to \infty} \int_{0}^{u} \frac{1}{e^{-x} + e^{x}}\, \mathrm{d}x$$

We divide into two integrals on half-lines.

Find $I = \int_{-\infty}^{\infty} \frac{1}{e^{-x} + e^{x}}\, \mathrm{d}x$

$$\int_{-\infty}^{\infty} \frac{1}{e^{-x} + e^{x}}\, \mathrm{d}x = \int_{-\infty}^{0} \frac{1}{e^{-x} + e^{x}}\, \mathrm{d}x + \int_{0}^{\infty} \frac{1}{e^{-x} + e^{x}}\, \mathrm{d}x$$

$$= \lim_{u\to-\infty} \int_{u}^{0} \frac{1}{e^{-x} + e^{x}}\, \mathrm{d}x + \lim_{u\to\infty} \int_{0}^{u} \frac{1}{e^{-x} + e^{x}}\, \mathrm{d}x$$

$$\int \frac{1}{e^{-x} + e^{x}}\, \mathrm{d}x$$

We evaluate the indefinite integral.

Find $I = \int_{-\infty}^{\infty} \frac{1}{e^{-x} + e^{x}} \, dx$

$$\int_{-\infty}^{\infty} \frac{1}{e^{-x} + e^{x}} \, dx = \int_{-\infty}^{0} \frac{1}{e^{-x} + e^{x}} \, dx + \int_{0}^{\infty} \frac{1}{e^{-x} + e^{x}} \, dx$$

$$= \lim_{u \to -\infty} \int_{u}^{0} \frac{1}{e^{-x} + e^{x}} \, dx + \lim_{u \to \infty} \int_{0}^{u} \frac{1}{e^{-x} + e^{x}} \, dx$$

$$\int \frac{1}{e^{-x} + e^{x}} \, dx = \int \frac{e^{x}}{1 + (e^{x})^{2}} \, dx$$

We simplify the integrand...

Find $I = \int_{-\infty}^{\infty} \frac{1}{e^{-x} + e^{x}}\,\mathrm{d}x$

$$\int_{-\infty}^{\infty} \frac{1}{e^{-x} + e^{x}}\,\mathrm{d}x = \int_{-\infty}^{0} \frac{1}{e^{-x} + e^{x}}\,\mathrm{d}x + \int_{0}^{\infty} \frac{1}{e^{-x} + e^{x}}\,\mathrm{d}x$$

$$= \lim_{u\to-\infty} \int_{u}^{0} \frac{1}{e^{-x} + e^{x}}\,\mathrm{d}x + \lim_{u\to\infty} \int_{0}^{u} \frac{1}{e^{-x} + e^{x}}\,\mathrm{d}x$$

$$\int \frac{1}{e^{-x} + e^{x}}\,\mathrm{d}x = \int \frac{e^{x}}{1 + (e^{x})^{2}}\,\mathrm{d}x \quad \boxed{\begin{aligned} e^{x} &= t \\ e^{x}\,\mathrm{d}x &= \mathrm{d}t \end{aligned}}$$

. . . and substitute.

Find $I = \int_{-\infty}^{\infty} \frac{1}{e^{-x}+e^{x}}\,\mathrm{d}x$

$$\int_{-\infty}^{\infty} \frac{1}{e^{-x}+e^{x}}\,\mathrm{d}x = \int_{-\infty}^{0} \frac{1}{e^{-x}+e^{x}}\,\mathrm{d}x + \int_{0}^{\infty} \frac{1}{e^{-x}+e^{x}}\,\mathrm{d}x$$

$$= \lim_{u\to-\infty}\int_{u}^{0} \frac{1}{e^{-x}+e^{x}}\,\mathrm{d}x + \lim_{u\to\infty}\int_{0}^{u} \frac{1}{e^{-x}+e^{x}}\,\mathrm{d}x$$

$$\int \frac{1}{e^{-x}+e^{x}}\,\mathrm{d}x = \int \frac{e^{x}}{1+(e^{x})^2}\,\mathrm{d}x \quad \boxed{\begin{array}{c} e^x = t \\ e^x\,\mathrm{d}x = \mathrm{d}t \end{array}} = \int \frac{1}{1+t^2}\,\mathrm{d}t$$

The substitution gives this integral...

Find $I = \int_{-\infty}^{\infty} \frac{1}{e^{-x} + e^{x}} \,\mathrm{d}x$

$$\int_{-\infty}^{\infty} \frac{1}{e^{-x} + e^{x}} \,\mathrm{d}x = \int_{-\infty}^{0} \frac{1}{e^{-x} + e^{x}} \,\mathrm{d}x + \int_{0}^{\infty} \frac{1}{e^{-x} + e^{x}} \,\mathrm{d}x$$

$$= \lim_{u\to-\infty} \int_{u}^{0} \frac{1}{e^{-x} + e^{x}} \,\mathrm{d}x + \lim_{u\to\infty} \int_{0}^{u} \frac{1}{e^{-x} + e^{x}} \,\mathrm{d}x$$

$$\int \frac{1}{e^{-x} + e^{x}} \,\mathrm{d}x = \int \frac{e^{x}}{1 + (e^{x})^{2}} \,\mathrm{d}x \quad \boxed{\begin{array}{c} e^{x} = t \\ e^{x}\,\mathrm{d}x = \mathrm{d}t \end{array}} \quad = \int \frac{1}{1 + t^{2}} \,\mathrm{d}t = \operatorname{arctg} t$$

. . . which can be integrated by direct formula.

Find $I = \int_{-\infty}^{\infty} \frac{1}{e^{-x}+e^{x}}\,\mathrm{d}x$

$$\int_{-\infty}^{\infty} \frac{1}{e^{-x}+e^{x}}\,\mathrm{d}x = \int_{-\infty}^{0} \frac{1}{e^{-x}+e^{x}}\,\mathrm{d}x + \int_{0}^{\infty} \frac{1}{e^{-x}+e^{x}}\,\mathrm{d}x$$

$$= \lim_{u\to-\infty} \int_{u}^{0} \frac{1}{e^{-x}+e^{x}}\,\mathrm{d}x + \lim_{u\to\infty} \int_{0}^{u} \frac{1}{e^{-x}+e^{x}}\,\mathrm{d}x$$

$$\int \frac{1}{e^{-x}+e^{x}}\,\mathrm{d}x = \int \frac{e^{x}}{1+(e^{x})^2}\,\mathrm{d}x \quad \boxed{\begin{array}{c} e^{x} = t \\ e^{x}\,\mathrm{d}x = \mathrm{d}t \end{array}} \quad = \int \frac{1}{1+t^2}\,\mathrm{d}t = \operatorname{arctg} t$$

$$= \operatorname{arctg} e^{x}$$

Finally we return to the original variable.

Find $I = \int_{-\infty}^{\infty} \frac{1}{e^{-x} + e^{x}}\,\mathrm{d}x$

$$\int_{-\infty}^{\infty} \frac{1}{e^{-x} + e^{x}}\,\mathrm{d}x = \int_{-\infty}^{0} \frac{1}{e^{-x} + e^{x}}\,\mathrm{d}x + \int_{0}^{\infty} \frac{1}{e^{-x} + e^{x}}\,\mathrm{d}x$$

$$= \lim_{u\to-\infty} \int_{u}^{0} \frac{1}{e^{-x} + e^{x}}\,\mathrm{d}x + \lim_{u\to\infty} \int_{0}^{u} \frac{1}{e^{-x} + e^{x}}\,\mathrm{d}x$$

$$\int \frac{1}{e^{-x} + e^{x}}\,\mathrm{d}x = \int \frac{e^{x}}{1 + (e^{x})^2}\,\mathrm{d}x \quad \boxed{\begin{array}{c} e^{x} = t \\ e^{x}\,\mathrm{d}x = \mathrm{d}t \end{array}} \quad = \int \frac{1}{1 + t^2}\,\mathrm{d}t = \operatorname{arctg} t$$

$$= \operatorname{arctg} e^{x}$$

$$\int_{u}^{0} \frac{1}{e^{-x} + e^{x}}\,\mathrm{d}x$$

Find $I = \int_{-\infty}^{\infty} \frac{1}{e^{-x}+e^{x}}\,dx$

$$\int_{-\infty}^{\infty} \frac{1}{e^{-x}+e^{x}}\,dx = \int_{-\infty}^{0} \frac{1}{e^{-x}+e^{x}}\,dx + \int_{0}^{\infty} \frac{1}{e^{-x}+e^{x}}\,dx$$

$$= \lim_{u\to-\infty} \int_{u}^{0} \frac{1}{e^{-x}+e^{x}}\,dx + \lim_{u\to\infty} \int_{0}^{u} \frac{1}{e^{-x}+e^{x}}\,dx$$

$$\int \frac{1}{e^{-x}+e^{x}}\,dx = \int \frac{e^{x}}{1+(e^{x})^2}\,dx \quad \boxed{\begin{array}{c} e^{x} = t \\ e^{x}\,dx = dt \end{array}} \quad = \int \frac{1}{1+t^2}\,dt = \operatorname{arctg} t$$

$$= \operatorname{arctg} e^{x}$$

$$\int_{u}^{0} \frac{1}{e^{-x}+e^{x}}\,dx = [\operatorname{arctg} e^{x}]_{u}^{0}$$

Find $I = \int_{-\infty}^{\infty} \frac{1}{e^{-x}+e^{x}}\,\mathrm{d}x$

$$\int_{-\infty}^{\infty} \frac{1}{e^{-x}+e^{x}}\,\mathrm{d}x = \int_{-\infty}^{0} \frac{1}{e^{-x}+e^{x}}\,\mathrm{d}x + \int_{0}^{\infty} \frac{1}{e^{-x}+e^{x}}\,\mathrm{d}x$$

$$= \lim_{u\to-\infty} \int_{u}^{0} \frac{1}{e^{-x}+e^{x}}\,\mathrm{d}x + \lim_{u\to\infty} \int_{0}^{u} \frac{1}{e^{-x}+e^{x}}\,\mathrm{d}x$$

$$\int \frac{1}{e^{-x}+e^{x}}\,\mathrm{d}x = \int \frac{e^{x}}{1+(e^{x})^{2}}\,\mathrm{d}x \quad \boxed{\begin{array}{c} e^{x}=t \\ e^{x}\,\mathrm{d}x = \mathrm{d}t \end{array}} \quad = \int \frac{1}{1+t^{2}}\,\mathrm{d}t = \operatorname{arctg} t$$

$$= \operatorname{arctg} e^{x}$$

$$\int_{u}^{0} \frac{1}{e^{-x}+e^{x}}\,\mathrm{d}x = \left[\operatorname{arctg} e^{x}\right]_{u}^{0} = \operatorname{arctg} e^{0} - \operatorname{arctg} e^{u}$$

$$\text{Find } I=\int_{-\infty}^{\infty}\frac{1}{e^{-x}+e^{x}}\,dx$$

$$\int_{-\infty}^{\infty}\frac{1}{e^{-x}+e^{x}}\,dx=\int_{-\infty}^{0}\frac{1}{e^{-x}+e^{x}}\,dx+\int_{0}^{\infty}\frac{1}{e^{-x}+e^{x}}\,dx$$

$$=\lim_{u\to-\infty}\int_{u}^{0}\frac{1}{e^{-x}+e^{x}}\,dx+\lim_{u\to\infty}\int_{0}^{u}\frac{1}{e^{-x}+e^{x}}\,dx$$

$$\int\frac{1}{e^{-x}+e^{x}}\,dx=\int\frac{e^{x}}{1+(e^{x})^{2}}\,dx\ \boxed{\begin{aligned}e^{x}&=t\\ e^{x}\,dx&=dt\end{aligned}}=\int\frac{1}{1+t^{2}}\,dt=\operatorname{arctg}t$$

$$=\operatorname{arctg}e^{x}$$

$$\int_{u}^{0}\frac{1}{e^{-x}+e^{x}}\,dx=\left[\operatorname{arctg}e^{x}\right]_{u}^{0}=\operatorname{arctg}e^{0}-\operatorname{arctg}e^{u}=\operatorname{arctg}1-\operatorname{arctg}e^{u}$$

Find $I = \int_{-\infty}^{\infty} \frac{1}{e^{-x} + e^{x}}\,dx$

$$\int_{-\infty}^{\infty} \frac{1}{e^{-x} + e^{x}}\,dx = \int_{-\infty}^{0} \frac{1}{e^{-x} + e^{x}}\,dx + \int_{0}^{\infty} \frac{1}{e^{-x} + e^{x}}\,dx$$

$$= \lim_{u\to-\infty} \int_{u}^{0} \frac{1}{e^{-x} + e^{x}}\,dx + \lim_{u\to\infty} \int_{0}^{u} \frac{1}{e^{-x} + e^{x}}\,dx$$

$$\int \frac{1}{e^{-x} + e^{x}}\,dx = \int \frac{e^{x}}{1 + (e^{x})^2}\,dx \quad \boxed{\begin{array}{c} e^x = t \\ e^x\,dx = dt \end{array}} \quad = \int \frac{1}{1+t^2}\,dt = \operatorname{arctg} t$$

$$= \operatorname{arctg} e^x$$

$$\int_{u}^{0} \frac{1}{e^{-x} + e^{x}}\,dx = [\operatorname{arctg} e^x]_u^0 = \operatorname{arctg} e^0 - \operatorname{arctg} e^u = \operatorname{arctg} 1 - \operatorname{arctg} e^u$$

$$= \frac{\pi}{4} - \operatorname{arctg} e^u$$

Find $I = \int_{-\infty}^{\infty} \frac{1}{e^{-x} + e^{x}}\,\mathrm{d}x$

$$\int_{-\infty}^{\infty} \frac{1}{e^{-x} + e^{x}}\,\mathrm{d}x = \int_{-\infty}^{0} \frac{1}{e^{-x} + e^{x}}\,\mathrm{d}x + \int_{0}^{\infty} \frac{1}{e^{-x} + e^{x}}\,\mathrm{d}x$$

$$= \lim_{u\to-\infty} \int_{u}^{0} \frac{1}{e^{-x} + e^{x}}\,\mathrm{d}x + \lim_{u\to\infty} \int_{0}^{u} \frac{1}{e^{-x} + e^{x}}\,\mathrm{d}x$$

$$\int \frac{1}{e^{-x} + e^{x}}\,\mathrm{d}x = \int \frac{e^{x}}{1 + (e^{x})^2}\,\mathrm{d}x \quad \boxed{\begin{array}{c} e^{x} = t \\ e^{x}\,\mathrm{d}x = \mathrm{d}t \end{array}} \quad = \int \frac{1}{1+t^2}\,\mathrm{d}t = \operatorname{arctg} t$$

$$= \operatorname{arctg} e^{x}$$

$$\int_{u}^{0} \frac{1}{e^{-x} + e^{x}}\,\mathrm{d}x = \left[\operatorname{arctg} e^{x}\right]_{u}^{0} = \operatorname{arctg} e^{0} - \operatorname{arctg} e^{u} = \operatorname{arctg} 1 - \operatorname{arctg} e^{u}$$

$$= \frac{\pi}{4} - \operatorname{arctg} e^{u}$$

$$\int_{-\infty}^{0} \frac{1}{e^{-x} + e^{x}}\,\mathrm{d}x$$

Find $I = \int_{-\infty}^{\infty} \frac{1}{e^{-x} + e^{x}}\,\mathrm{d}x$

$$\int_{-\infty}^{\infty} \frac{1}{e^{-x}+e^{x}}\,\mathrm{d}x = \int_{-\infty}^{0} \frac{1}{e^{-x}+e^{x}}\,\mathrm{d}x + \int_{0}^{\infty} \frac{1}{e^{-x}+e^{x}}\,\mathrm{d}x$$

$$= \lim_{u\to-\infty} \int_{u}^{0} \frac{1}{e^{-x}+e^{x}}\,\mathrm{d}x + \lim_{u\to\infty} \int_{0}^{u} \frac{1}{e^{-x}+e^{x}}\,\mathrm{d}x$$

$$\int \frac{1}{e^{-x}+e^{x}}\,\mathrm{d}x = \int \frac{e^{x}}{1+(e^{x})^{2}}\,\mathrm{d}x \quad \boxed{\begin{array}{c} e^{x} = t \\ e^{x}\,\mathrm{d}x = \mathrm{d}t \end{array}} \quad = \int \frac{1}{1+t^{2}}\,\mathrm{d}t = \operatorname{arctg} t$$

$$= \operatorname{arctg} e^{x}$$

$$\int_{u}^{0} \frac{1}{e^{-x}+e^{x}}\,\mathrm{d}x = \left[\operatorname{arctg} e^{x}\right]_{u}^{0} = \operatorname{arctg} e^{0} - \operatorname{arctg} e^{u} = \operatorname{arctg} 1 - \operatorname{arctg} e^{u}$$

$$= \frac{\pi}{4} - \operatorname{arctg} e^{u}$$

$$\int_{-\infty}^{0} \frac{1}{e^{-x}+e^{x}}\,\mathrm{d}x = \lim_{u\to-\infty} \left(\frac{\pi}{4} - \operatorname{arctg} e^{u}\right)$$

Find $I=\int_{-\infty}^{\infty}\frac{1}{e^{-x}+e^{x}}\,\mathrm{d}x$

$$\int_{-\infty}^{\infty}\frac{1}{e^{-x}+e^{x}}\,\mathrm{d}x=\int_{-\infty}^{0}\frac{1}{e^{-x}+e^{x}}\,\mathrm{d}x+\int_{0}^{\infty}\frac{1}{e^{-x}+e^{x}}\,\mathrm{d}x$$

$$=\lim_{u\to-\infty}\int_{u}^{0}\frac{1}{e^{-x}+e^{x}}\,\mathrm{d}x+\lim_{u\to\infty}\int_{0}^{u}\frac{1}{e^{-x}+e^{x}}\,\mathrm{d}x$$

$$\int\frac{1}{e^{-x}+e^{x}}\,\mathrm{d}x=\int\frac{e^{x}}{1+(e^{x})^2}\,\mathrm{d}x\quad\boxed{\begin{array}{c}e^{x}=t\\ e^{x}\,\mathrm{d}x=\mathrm{d}t\end{array}}\quad=\int\frac{1}{1+t^2}\,\mathrm{d}t=\operatorname{arctg}t$$

$$=\operatorname{arctg}e^{x}$$

$$\int_{u}^{0}\frac{1}{e^{-x}+e^{x}}\,\mathrm{d}x=\left[\operatorname{arctg}e^{x}\right]_{u}^{0}=\operatorname{arctg}e^{0}-\operatorname{arctg}e^{u}=\operatorname{arctg}1-\operatorname{arctg}e^{u}$$

$$=\frac{\pi}{4}-\operatorname{arctg}e^{u}$$

$$\int_{-\infty}^{0}\frac{1}{e^{-x}+e^{x}}\,\mathrm{d}x=\lim_{u\to-\infty}\left(\frac{\pi}{4}-\operatorname{arctg}e^{u}\right)=\frac{\pi}{4}-\operatorname{arctg}e^{-\infty}$$

Find $I = \int_{-\infty}^{\infty} \frac{1}{e^{-x}+e^{x}}\,\mathrm{d}x$

$$\int_{-\infty}^{\infty} \frac{1}{e^{-x}+e^{x}}\,\mathrm{d}x = \int_{-\infty}^{0} \frac{1}{e^{-x}+e^{x}}\,\mathrm{d}x + \int_{0}^{\infty} \frac{1}{e^{-x}+e^{x}}\,\mathrm{d}x$$

$$= \lim_{u\to-\infty} \int_{u}^{0} \frac{1}{e^{-x}+e^{x}}\,\mathrm{d}x + \lim_{u\to\infty} \int_{0}^{u} \frac{1}{e^{-x}+e^{x}}\,\mathrm{d}x$$

$$\int \frac{1}{e^{-x}+e^{x}}\,\mathrm{d}x = \int \frac{e^{x}}{1+(e^{x})^2}\,\mathrm{d}x \quad \boxed{\begin{array}{c} e^{x} = t \\ e^{x}\,\mathrm{d}x = \mathrm{d}t \end{array}} \quad = \int \frac{1}{1+t^2}\,\mathrm{d}t = \operatorname{arctg} t$$

$$= \operatorname{arctg} e^{x}$$

$$\int_{u}^{0} \frac{1}{e^{-x}+e^{x}}\,\mathrm{d}x = [\operatorname{arctg} e^{x}]_{u}^{0} = \operatorname{arctg} e^{0} - \operatorname{arctg} e^{u} = \operatorname{arctg} 1 - \operatorname{arctg} e^{u}$$

$$= \frac{\pi}{4} - \operatorname{arctg} e^{u}$$

$$\int_{-\infty}^{0} \frac{1}{e^{-x}+e^{x}}\,\mathrm{d}x = \lim_{u\to-\infty} \left(\frac{\pi}{4} - \operatorname{arctg} e^{u}\right) = \frac{\pi}{4} - \operatorname{arctg} e^{-\infty} = \frac{\pi}{4} - \operatorname{arctg} 0$$

Find $I = \int_{-\infty}^{\infty} \frac{1}{e^{-x} + e^{x}}\,dx$

$$\int_{-\infty}^{\infty} \frac{1}{e^{-x} + e^{x}}\,dx = \int_{-\infty}^{0} \frac{1}{e^{-x} + e^{x}}\,dx + \int_{0}^{\infty} \frac{1}{e^{-x} + e^{x}}\,dx$$

$$= \lim_{u\to-\infty} \int_{u}^{0} \frac{1}{e^{-x} + e^{x}}\,dx + \lim_{u\to\infty} \int_{0}^{u} \frac{1}{e^{-x} + e^{x}}\,dx$$

$$\int \frac{1}{e^{-x} + e^{x}}\,dx = \int \frac{e^{x}}{1 + (e^{x})^2}\,dx \quad \boxed{\begin{array}{c} e^{x} = t \\ e^{x}\,dx = dt \end{array}} \quad = \int \frac{1}{1+t^2}\,dt = \operatorname{arctg} t$$

$$= \operatorname{arctg} e^{x}$$

$$\int_{u}^{0} \frac{1}{e^{-x} + e^{x}}\,dx = [\operatorname{arctg} e^{x}]_{u}^{0} = \operatorname{arctg} e^{0} - \operatorname{arctg} e^{u} = \operatorname{arctg} 1 - \operatorname{arctg} e^{u}$$

$$= \frac{\pi}{4} - \operatorname{arctg} e^{u}$$

$$\int_{-\infty}^{0} \frac{1}{e^{-x} + e^{x}}\,dx = \lim_{u\to-\infty} \left(\frac{\pi}{4} - \operatorname{arctg} e^{u}\right) = \frac{\pi}{4} - \operatorname{arctg} e^{-\infty} = \frac{\pi}{4} - \operatorname{arctg} 0$$

$$= \frac{\pi}{4}$$

Find $I = \int_{-\infty}^{\infty} \frac{1}{e^{-x} + e^{x}} \, \mathrm{d}x$

$$\int_{-\infty}^{\infty} \frac{1}{e^{-x} + e^{x}} \, \mathrm{d}x = \int_{-\infty}^{0} \frac{1}{e^{-x} + e^{x}} \, \mathrm{d}x + \int_{0}^{\infty} \frac{1}{e^{-x} + e^{x}} \, \mathrm{d}x$$

$$= \lim_{u \to -\infty} \int_{u}^{0} \frac{1}{e^{-x} + e^{x}} \, \mathrm{d}x + \lim_{u \to \infty} \int_{0}^{u} \frac{1}{e^{-x} + e^{x}} \, \mathrm{d}x$$

$$\int \frac{1}{e^{-x} + e^{x}} \, \mathrm{d}x = \operatorname{arctg} e^{x}$$

$$\int_{-\infty}^{0} \frac{1}{e^{-x} + e^{x}} \, \mathrm{d}x = \frac{\pi}{4}$$

$$\int_{0}^{u} \frac{1}{e^{-x} + e^{x}} \, \mathrm{d}x$$

Find $I = \int_{-\infty}^{\infty} \frac{1}{e^{-x} + e^{x}}\,\mathrm{d}x$

$$\int_{-\infty}^{\infty} \frac{1}{e^{-x} + e^{x}}\,\mathrm{d}x = \int_{-\infty}^{0} \frac{1}{e^{-x} + e^{x}}\,\mathrm{d}x + \int_{0}^{\infty} \frac{1}{e^{-x} + e^{x}}\,\mathrm{d}x$$

$$= \lim_{u\to-\infty} \int_{u}^{0} \frac{1}{e^{-x} + e^{x}}\,\mathrm{d}x + \lim_{u\to\infty} \int_{0}^{u} \frac{1}{e^{-x} + e^{x}}\,\mathrm{d}x$$

$$\int \frac{1}{e^{-x} + e^{x}}\,\mathrm{d}x = \operatorname{arctg} e^{x}$$

$$\int_{-\infty}^{0} \frac{1}{e^{-x} + e^{x}}\,\mathrm{d}x = \frac{\pi}{4}$$

$$\int_{0}^{u} \frac{1}{e^{-x} + e^{x}}\,\mathrm{d}x = \left[\operatorname{arctg} e^{x}\right]_{0}^{u}$$

Find $I = \int_{-\infty}^{\infty} \frac{1}{e^{-x} + e^{x}} \, dx$

$$\int_{-\infty}^{\infty} \frac{1}{e^{-x}+e^{x}}\,dx = \int_{-\infty}^{0} \frac{1}{e^{-x}+e^{x}}\,dx + \int_{0}^{\infty} \frac{1}{e^{-x}+e^{x}}\,dx$$

$$= \lim_{u\to-\infty} \int_{u}^{0} \frac{1}{e^{-x}+e^{x}}\,dx + \lim_{u\to\infty} \int_{0}^{u} \frac{1}{e^{-x}+e^{x}}\,dx$$

$$\int \frac{1}{e^{-x}+e^{x}}\,dx = \operatorname{arctg} e^{x}$$

$$\int_{-\infty}^{0} \frac{1}{e^{-x}+e^{x}}\,dx = \frac{\pi}{4}$$

$$\int_{0}^{u} \frac{1}{e^{-x}+e^{x}}\,dx = \left[\operatorname{arctg} e^{x}\right]_{0}^{u} = \operatorname{arctg} e^{u} - \operatorname{arctg} e^{0}$$

Find $I = \int_{-\infty}^{\infty} \frac{1}{e^{-x} + e^{x}}\, \mathrm{d}x$

$$\int_{-\infty}^{\infty} \frac{1}{e^{-x} + e^{x}}\, \mathrm{d}x = \int_{-\infty}^{0} \frac{1}{e^{-x} + e^{x}}\, \mathrm{d}x + \int_{0}^{\infty} \frac{1}{e^{-x} + e^{x}}\, \mathrm{d}x$$

$$= \lim_{u\to-\infty} \int_{u}^{0} \frac{1}{e^{-x} + e^{x}}\, \mathrm{d}x + \lim_{u\to\infty} \int_{0}^{u} \frac{1}{e^{-x} + e^{x}}\, \mathrm{d}x$$

$$\int \frac{1}{e^{-x} + e^{x}}\, \mathrm{d}x = \operatorname{arctg} e^{x} \qquad \int_{-\infty}^{0} \frac{1}{e^{-x} + e^{x}}\, \mathrm{d}x = \frac{\pi}{4}$$

$$\int_{0}^{u} \frac{1}{e^{-x} + e^{x}}\, \mathrm{d}x = [\operatorname{arctg} e^{x}]_0^u = \operatorname{arctg} e^{u} - \operatorname{arctg} e^{0} = \operatorname{arctg} e^{u} - \operatorname{arctg} 1$$

Find $I = \int_{-\infty}^{\infty} \frac{1}{e^{-x}+e^{x}}\,\mathrm{d}x$

$$\int_{-\infty}^{\infty} \frac{1}{e^{-x}+e^{x}}\,\mathrm{d}x = \int_{-\infty}^{0} \frac{1}{e^{-x}+e^{x}}\,\mathrm{d}x + \int_{0}^{\infty} \frac{1}{e^{-x}+e^{x}}\,\mathrm{d}x$$

$$= \lim_{u\to-\infty} \int_{u}^{0} \frac{1}{e^{-x}+e^{x}}\,\mathrm{d}x + \lim_{u\to\infty} \int_{0}^{u} \frac{1}{e^{-x}+e^{x}}\,\mathrm{d}x$$

$$\int \frac{1}{e^{-x}+e^{x}}\,\mathrm{d}x = \operatorname{arctg} e^{x} \qquad \int_{-\infty}^{0} \frac{1}{e^{-x}+e^{x}}\,\mathrm{d}x = \frac{\pi}{4}$$

$$\int_{0}^{u} \frac{1}{e^{-x}+e^{x}}\,\mathrm{d}x = [\operatorname{arctg} e^{x}]_{0}^{u} = \operatorname{arctg} e^{u} - \operatorname{arctg} e^{0} = \operatorname{arctg} e^{u} - \operatorname{arctg} 1$$

$$= \operatorname{arctg} e^{u} - \frac{\pi}{4}$$

Find $I = \int_{-\infty}^{\infty} \frac{1}{e^{-x}+e^{x}}\,dx$

$$\int_{-\infty}^{\infty} \frac{1}{e^{-x}+e^{x}}\,dx = \int_{-\infty}^{0} \frac{1}{e^{-x}+e^{x}}\,dx + \int_{0}^{\infty} \frac{1}{e^{-x}+e^{x}}\,dx$$

$$= \lim_{u\to-\infty} \int_{u}^{0} \frac{1}{e^{-x}+e^{x}}\,dx + \lim_{u\to\infty} \int_{0}^{u} \frac{1}{e^{-x}+e^{x}}\,dx$$

$$\int \frac{1}{e^{-x}+e^{x}}\,dx = \operatorname{arctg} e^{x}$$

$$\int_{-\infty}^{0} \frac{1}{e^{-x}+e^{x}}\,dx = \frac{\pi}{4}$$

$$\int_{0}^{u} \frac{1}{e^{-x}+e^{x}}\,dx = [\operatorname{arctg} e^{x}]_0^u = \operatorname{arctg} e^{u} - \operatorname{arctg} e^{0} = \operatorname{arctg} e^{u} - \operatorname{arctg} 1$$

$$= \operatorname{arctg} e^{u} - \frac{\pi}{4}$$

$$\int_{0}^{\infty} \frac{1}{e^{-x}+e^{x}}\,dx$$

Find $I = \int_{-\infty}^{\infty} \frac{1}{e^{-x}+e^{x}}\,\mathrm{d}x$

$$\int_{-\infty}^{\infty} \frac{1}{e^{-x}+e^{x}}\,\mathrm{d}x = \int_{-\infty}^{0} \frac{1}{e^{-x}+e^{x}}\,\mathrm{d}x + \int_{0}^{\infty} \frac{1}{e^{-x}+e^{x}}\,\mathrm{d}x$$

$$= \lim_{u\to-\infty} \int_{u}^{0} \frac{1}{e^{-x}+e^{x}}\,\mathrm{d}x + \lim_{u\to\infty} \int_{0}^{u} \frac{1}{e^{-x}+e^{x}}\,\mathrm{d}x$$

$$\int \frac{1}{e^{-x}+e^{x}}\,\mathrm{d}x = \operatorname{arctg} e^{x} \qquad \int_{-\infty}^{0} \frac{1}{e^{-x}+e^{x}}\,\mathrm{d}x = \frac{\pi}{4}$$

$$\int_{0}^{u} \frac{1}{e^{-x}+e^{x}}\,\mathrm{d}x = [\operatorname{arctg} e^{x}]_0^u = \operatorname{arctg} e^{u} - \operatorname{arctg} e^{0} = \operatorname{arctg} e^{u} - \operatorname{arctg} 1$$

$$= \operatorname{arctg} e^{u} - \frac{\pi}{4}$$

$$\int_{0}^{\infty} \frac{1}{e^{-x}+e^{x}}\,\mathrm{d}x = \lim_{u\to\infty} \left(\operatorname{arctg} e^{u} - \frac{\pi}{4}\right)$$

Find $I = \int_{-\infty}^{\infty} \frac{1}{e^{-x} + e^{x}} \, dx$

$$\int_{-\infty}^{\infty} \frac{1}{e^{-x}+e^{x}}\,dx = \int_{-\infty}^{0} \frac{1}{e^{-x}+e^{x}}\,dx + \int_{0}^{\infty} \frac{1}{e^{-x}+e^{x}}\,dx$$

$$= \lim_{u\to-\infty} \int_{u}^{0} \frac{1}{e^{-x}+e^{x}}\,dx + \lim_{u\to\infty} \int_{0}^{u} \frac{1}{e^{-x}+e^{x}}\,dx$$

$$\int \frac{1}{e^{-x}+e^{x}}\,dx = \operatorname{arctg} e^{x} \qquad \int_{-\infty}^{0} \frac{1}{e^{-x}+e^{x}}\,dx = \frac{\pi}{4}$$

$$\int_{0}^{u} \frac{1}{e^{-x}+e^{x}}\,dx = [\operatorname{arctg} e^{x}]_{0}^{u} = \operatorname{arctg} e^{u} - \operatorname{arctg} e^{0} = \operatorname{arctg} e^{u} - \operatorname{arctg} 1$$

$$= \operatorname{arctg} e^{u} - \frac{\pi}{4}$$

$$\int_{0}^{\infty} \frac{1}{e^{-x}+e^{x}}\,dx = \lim_{u\to\infty} \left(\operatorname{arctg} e^{u} - \frac{\pi}{4}\right) = \operatorname{arctg} e^{\infty} - \frac{\pi}{4}$$

Find $I = \int_{-\infty}^{\infty} \frac{1}{e^{-x}+e^{x}}\,dx$

$$\int_{-\infty}^{\infty} \frac{1}{e^{-x}+e^{x}}\,dx = \int_{-\infty}^{0} \frac{1}{e^{-x}+e^{x}}\,dx + \int_{0}^{\infty} \frac{1}{e^{-x}+e^{x}}\,dx$$

$$= \lim_{u\to-\infty} \int_{u}^{0} \frac{1}{e^{-x}+e^{x}}\,dx + \lim_{u\to\infty} \int_{0}^{u} \frac{1}{e^{-x}+e^{x}}\,dx$$

$$\int \frac{1}{e^{-x}+e^{x}}\,dx = \operatorname{arctg} e^{x}$$

$$\int_{-\infty}^{0} \frac{1}{e^{-x}+e^{x}}\,dx = \frac{\pi}{4}$$

$$\int_{0}^{u} \frac{1}{e^{-x}+e^{x}}\,dx = [\operatorname{arctg} e^{x}]_{0}^{u} = \operatorname{arctg} e^{u} - \operatorname{arctg} e^{0} = \operatorname{arctg} e^{u} - \operatorname{arctg} 1$$

$$= \operatorname{arctg} e^{u} - \frac{\pi}{4}$$

$$\int_{0}^{\infty} \frac{1}{e^{-x}+e^{x}}\,dx = \lim_{u\to\infty} \left(\operatorname{arctg} e^{u} - \frac{\pi}{4}\right) = \operatorname{arctg} e^{\infty} - \frac{\pi}{4} = \operatorname{arctg} \infty - \frac{\pi}{4}$$

Find $I = \int_{-\infty}^{\infty} \frac{1}{e^{-x} + e^{x}} \, \mathrm{d}x$

$$\int_{-\infty}^{\infty} \frac{1}{e^{-x} + e^{x}} \, \mathrm{d}x = \int_{-\infty}^{0} \frac{1}{e^{-x} + e^{x}} \, \mathrm{d}x + \int_{0}^{\infty} \frac{1}{e^{-x} + e^{x}} \, \mathrm{d}x$$

$$= \lim_{u \to -\infty} \int_{u}^{0} \frac{1}{e^{-x} + e^{x}} \, \mathrm{d}x + \lim_{u \to \infty} \int_{0}^{u} \frac{1}{e^{-x} + e^{x}} \, \mathrm{d}x$$

$$\int \frac{1}{e^{-x} + e^{x}} \, \mathrm{d}x = \operatorname{arctg} e^{x} \qquad \int_{-\infty}^{0} \frac{1}{e^{-x} + e^{x}} \, \mathrm{d}x = \frac{\pi}{4}$$

$$\int_{0}^{u} \frac{1}{e^{-x} + e^{x}} \, \mathrm{d}x = [\operatorname{arctg} e^{x}]_{0}^{u} = \operatorname{arctg} e^{u} - \operatorname{arctg} e^{0} = \operatorname{arctg} e^{u} - \operatorname{arctg} 1$$

$$= \operatorname{arctg} e^{u} - \frac{\pi}{4}$$

$$\int_{0}^{\infty} \frac{1}{e^{-x} + e^{x}} \, \mathrm{d}x = \lim_{u \to \infty} \left(\operatorname{arctg} e^{u} - \frac{\pi}{4} \right) = \operatorname{arctg} e^{\infty} - \frac{\pi}{4} = \operatorname{arctg} \infty - \frac{\pi}{4}$$

$$= \frac{\pi}{2} - \frac{\pi}{4}$$

Find $I = \int_{-\infty}^{\infty} \frac{1}{e^{-x} + e^{x}} \, dx$

$$\int_{-\infty}^{\infty} \frac{1}{e^{-x} + e^{x}} \, dx = \int_{-\infty}^{0} \frac{1}{e^{-x} + e^{x}} \, dx + \int_{0}^{\infty} \frac{1}{e^{-x} + e^{x}} \, dx$$

$$= \lim_{u \to -\infty} \int_{u}^{0} \frac{1}{e^{-x} + e^{x}} \, dx + \lim_{u \to \infty} \int_{0}^{u} \frac{1}{e^{-x} + e^{x}} \, dx$$

$$\int \frac{1}{e^{-x} + e^{x}} \, dx = \operatorname{arctg} e^{x}$$

$$\int_{-\infty}^{0} \frac{1}{e^{-x} + e^{x}} \, dx = \frac{\pi}{4}$$

$$\int_{0}^{u} \frac{1}{e^{-x} + e^{x}} \, dx = [\operatorname{arctg} e^{x}]_{0}^{u} = \operatorname{arctg} e^{u} - \operatorname{arctg} e^{0} = \operatorname{arctg} e^{u} - \operatorname{arctg} 1$$

$$= \operatorname{arctg} e^{u} - \frac{\pi}{4}$$

$$\int_{0}^{\infty} \frac{1}{e^{-x} + e^{x}} \, dx = \lim_{u \to \infty} \left(\operatorname{arctg} e^{u} - \frac{\pi}{4} \right) = \operatorname{arctg} e^{\infty} - \frac{\pi}{4} = \operatorname{arctg} \infty - \frac{\pi}{4}$$

$$= \frac{\pi}{2} - \frac{\pi}{4} = \frac{\pi}{4}$$

Find $I = \int_{-\infty}^{\infty} \frac{1}{e^{-x}+e^{x}}\,\mathrm{d}x$

$$\int_{-\infty}^{\infty} \frac{1}{e^{-x}+e^{x}}\,\mathrm{d}x = \int_{-\infty}^{0} \frac{1}{e^{-x}+e^{x}}\,\mathrm{d}x + \int_{0}^{\infty} \frac{1}{e^{-x}+e^{x}}\,\mathrm{d}x$$

$$= \lim_{u\to-\infty} \int_{u}^{0} \frac{1}{e^{-x}+e^{x}}\,\mathrm{d}x + \lim_{u\to\infty} \int_{0}^{u} \frac{1}{e^{-x}+e^{x}}\,\mathrm{d}x$$

$$\int \frac{1}{e^{-x}+e^{x}}\,\mathrm{d}x = \operatorname{arctg} e^{x} \qquad \int_{-\infty}^{0} \frac{1}{e^{-x}+e^{x}}\,\mathrm{d}x = \frac{\pi}{4}$$

$$\int_{0}^{u} \frac{1}{e^{-x}+e^{x}}\,\mathrm{d}x = [\operatorname{arctg} e^{x}]_0^u = \operatorname{arctg} e^{u} - \operatorname{arctg} e^{0} = \operatorname{arctg} e^{u} - \operatorname{arctg} 1$$

$$= \operatorname{arctg} e^{u} - \frac{\pi}{4}$$

$$\int_{0}^{\infty} \frac{1}{e^{-x}+e^{x}}\,\mathrm{d}x = \lim_{u\to\infty} \left(\operatorname{arctg} e^{u} - \frac{\pi}{4}\right) = \operatorname{arctg} e^{\infty} - \frac{\pi}{4} = \operatorname{arctg} \infty - \frac{\pi}{4}$$

$$= \frac{\pi}{2} - \frac{\pi}{4} = \frac{\pi}{4}$$

$$\int_{-\infty}^{\infty} \frac{1}{e^{x}+e^{-x}}\,\mathrm{d}x$$

Find $I = \int_{-\infty}^{\infty} \frac{1}{e^{-x} + e^{x}} \, dx$

$$\int_{-\infty}^{\infty} \frac{1}{e^{-x} + e^{x}} \, dx = \int_{-\infty}^{0} \frac{1}{e^{-x} + e^{x}} \, dx + \int_{0}^{\infty} \frac{1}{e^{-x} + e^{x}} \, dx$$

$$= \lim_{u \to -\infty} \int_{u}^{0} \frac{1}{e^{-x} + e^{x}} \, dx + \lim_{u \to \infty} \int_{0}^{u} \frac{1}{e^{-x} + e^{x}} \, dx$$

$$\int \frac{1}{e^{-x} + e^{x}} \, dx = \operatorname{arctg} e^{x} \qquad \int_{-\infty}^{0} \frac{1}{e^{-x} + e^{x}} \, dx = \frac{\pi}{4}$$

$$\int_{0}^{u} \frac{1}{e^{-x} + e^{x}} \, dx = [\operatorname{arctg} e^{x}]_{0}^{u} = \operatorname{arctg} e^{u} - \operatorname{arctg} e^{0} = \operatorname{arctg} e^{u} - \operatorname{arctg} 1$$

$$= \operatorname{arctg} e^{u} - \frac{\pi}{4}$$

$$\int_{0}^{\infty} \frac{1}{e^{-x} + e^{x}} \, dx = \lim_{u \to \infty} \left(\operatorname{arctg} e^{u} - \frac{\pi}{4} \right) = \operatorname{arctg} e^{\infty} - \frac{\pi}{4} = \operatorname{arctg} \infty - \frac{\pi}{4}$$

$$= \frac{\pi}{2} - \frac{\pi}{4} = \frac{\pi}{4}$$

$$\int_{-\infty}^{\infty} \frac{1}{e^{x} + e^{-x}} \, dx = \frac{\pi}{4} + \frac{\pi}{4}$$

Find $I = \int_{-\infty}^{\infty} \frac{1}{e^{-x}+e^{x}}\,dx$

$$\int_{-\infty}^{\infty} \frac{1}{e^{-x}+e^{x}}\,dx = \int_{-\infty}^{0} \frac{1}{e^{-x}+e^{x}}\,dx + \int_{0}^{\infty} \frac{1}{e^{-x}+e^{x}}\,dx$$

$$= \lim_{u\to-\infty} \int_{u}^{0} \frac{1}{e^{-x}+e^{x}}\,dx + \lim_{u\to\infty} \int_{0}^{u} \frac{1}{e^{-x}+e^{x}}\,dx$$

$$\int \frac{1}{e^{-x}+e^{x}}\,dx = \operatorname{arctg} e^{x} \qquad \int_{-\infty}^{0} \frac{1}{e^{-x}+e^{x}}\,dx = \frac{\pi}{4}$$

$$\int_{0}^{u} \frac{1}{e^{-x}+e^{x}}\,dx = [\operatorname{arctg} e^{x}]_0^u = \operatorname{arctg} e^{u} - \operatorname{arctg} e^{0} = \operatorname{arctg} e^{u} - \operatorname{arctg} 1$$

$$= \operatorname{arctg} e^{u} - \frac{\pi}{4}$$

$$\int_{0}^{\infty} \frac{1}{e^{-x}+e^{x}}\,dx = \lim_{u\to\infty} \left(\operatorname{arctg} e^{u} - \frac{\pi}{4}\right) = \operatorname{arctg} e^{\infty} - \frac{\pi}{4} = \operatorname{arctg} \infty - \frac{\pi}{4}$$

$$= \frac{\pi}{2} - \frac{\pi}{4} = \frac{\pi}{4}$$

$$\int_{-\infty}^{\infty} \frac{1}{e^{x}+e^{-x}}\,dx = \frac{\pi}{4} + \frac{\pi}{4} = \frac{\pi}{2}$$

Find $I = \int_{-\infty}^{\infty} \frac{1}{e^{-x}+e^{x}}\,\mathrm{d}x$

$$\int_{-\infty}^{\infty} \frac{1}{e^{-x}+e^{x}}\,\mathrm{d}x = \int_{-\infty}^{0} \frac{1}{e^{-x}+e^{x}}\,\mathrm{d}x + \int_{0}^{\infty} \frac{1}{e^{-x}+e^{x}}\,\mathrm{d}x$$

$$= \lim_{u\to-\infty} \int_{u}^{0} \frac{1}{e^{-x}+e^{x}}\,\mathrm{d}x + \lim_{u\to\infty} \int_{0}^{u} \frac{1}{e^{-x}+e^{x}}\,\mathrm{d}x$$

$$\int \frac{1}{e^{-x}+e^{x}}\,\mathrm{d}x = \operatorname{arctg} e^{x} \qquad \int_{-\infty}^{0} \frac{1}{e^{-x}+e^{x}}\,\mathrm{d}x = \frac{\pi}{4}$$

$$\int_{0}^{u} \frac{1}{e^{-x}+e^{x}}\,\mathrm{d}x = [\operatorname{arctg} e^{x}]_0^u = \operatorname{arctg} e^{u} - \operatorname{arctg} e^{0} = \operatorname{arctg} e^{u} - \operatorname{arctg} 1$$

$$= \operatorname{arctg} e^{u} - \frac{\pi}{4}$$

$$\int_{0}^{\infty} \frac{1}{e^{-x}+e^{x}}\,\mathrm{d}x = \lim_{u\to\infty} \left(\operatorname{arctg} e^{u} - \frac{\pi}{4}\right) = \operatorname{arctg} e^{\infty} - \frac{\pi}{4} = \operatorname{arctg} \infty - \frac{\pi}{4}$$

$$= \frac{\pi}{2} - \frac{\pi}{4} = \frac{\pi}{4}$$

$$\int_{-\infty}^{\infty} \frac{1}{e^{x}+e^{-x}}\,\mathrm{d}x = \frac{\pi}{4} + \frac{\pi}{4} = \frac{\pi}{2}$$

Konec