Referee's report on the habilitation thesis

Masaryk University	
Faculty	of Science
Habilitation field	Mathematics – mathematical analysis
Applicant	Petr Zemánek, Ph.D.
Affiliation	Masaryk University, Faculty of Science
Habilitation thesis	Discrete Symplectic Systems and Square Summable Solutions
Referee	Prof. Julia Elyseeva, Dr. Sci.
Affiliation	Moscow State University of Technology, Russia

Text of the report

I refer to the habilitation thesis submitted which consists of 7 chapters, appendix, and an extensive bibliography. These chapters summarize the published work (partly jointly with co-authors) of 23 articles, which appeared in (mostly high class international) journals and as chapters of Doctoral Dissertation. The habilitation thesis presents recent author's contributions to the theory of square summable solutions of discrete symplectic systems. The study of the Weyl–Titchmarsh theory for discrete symplectic systems was initiated in papers of M. Bohner, S. Sun, S. Clark and the author (2010) where symplectic systems was considered in the special case when its first equation does not depend on the spectral parameter λ. The habilitation thesis presents the results of the author for discrete symplectic systems with general linear dependence on the spectral parameter as well as with a polynomial or analytic dependence. These results do not only improve the type of the dependence on the spectral parameter, but they significantly generalize the previous results laying foundations of the "operator theory" for discrete symplectic systems, which is intimately connected with the topic of square summable solutions.
The thesis is organized as follows.
Introduction summarizes the used notation and some important results from linear algebra, defines discrete symplectic systems, and shows some of their special cases.
Chapter 2 of the thesis is devoted to the limit point and limit circle classification for discrete symplectic systems, which depend linearly on the spectral parameter. The author formulates fundamental properties of symplectic systems, investigates the associated eigenvalue problem with separated boundary conditions, the Weyl disks and Weyl circles, their limiting behavior, and properties of square summable solutions including the precise analysis of the number of linearly independent square summable solutions as well as some criteria for the limit point and limit circle cases.
Results of *Chapter 3* extend Chapter 2 to problems with general jointly varying endpoints. These results include several particular cases, such as the periodic and antiperiodic endpoints. The developed method is based on the augmentation of symplectic system into double dimension, which leads to a problem with separated endpoints having the original boundary conditions as one of its constraints. Since such transformed symplectic system no longer satisfies the corresponding strong Atkinson condition, the results of Chapter 3 were achieved under the *weak* Atkinson condition. For this general situation, the author gives a characterization of eigenvalues of the eigenvalue problem with general jointly varying endpoints, constructs the Weyl disks, their centers and matrix radii, and also focuses

on properties of square summable solutions.
Investigation in *Chapter 4* is motivated by Walker's results (1975) derived for a pair of non-hermitian linear Hamiltonian differential systems. Results of this chapter extends the invariance of the limit circle case to two linear discrete systems which depend linearly on spectral parameter. The main result (Theorem 4.2.2) yields a discrete counterpart of the mentioned above continuous time statement.
In *Chapter 5* the author considers discrete symplectic systems with polynomial and analytic dependence on the spectral parameter, derives fundamental properties of these systems, and discusses their connection with systems known in the literature. It is shown that under appropriate Atkinson-type conditions involving the weight matrix, the theory of eigenvalues, Weyl disks, and square summable solutions developed in Chapter 2 remains valid without any change also for system with polynomial and analytic dependence on the spectral parameter.
Chapter 6 is devoted to the time-reversed symplectic systems with general linear dependence on spectral parameter. In particular, the author focuses on definiteness of the time-reversed discrete symplectic, derives some equivalent characterizations , introduces the minimal and maximal linear relations, and proves some fundamental properties of the corresponding deficiency indices, including a relationship between the number of square summable solutions and the dimension of the defect subspace.
The final chapter of the habilitation thesis is devoted to the characterization of all self-adjoint extensions of the minimal linear relation. The main result of *Chapter 7* (Theorem 7.2.) concerns the characterization of self-adjoint extensions of the minimal linear relation associated with the time-reversed sympectic systems. Then this result is applied to a consideration of the 2×2 (scalar) case for a finite discrete interval and an explicite description of the Krein–von Neumann extension.

The habilitation thesis is well-written in compact, clear, and elegant form. The overall list of 24 publications of Petr Zemánek shows that he is a competent mathematician dealing with moden problems on a high international standart. The variety of topics of this thesis clearly indicates that he is a person which has a lot of own ideas, which allows him to contribute within different settings. The results of this thesis are highly important for international mathematical community, in particular, for all specialists in the field of the qualitative theory of discrete symplectic systems.

Conclusion

Habilitation thesis by Petr Zemánek entitled "Discrete Symplectic Systems and Square Summable Solutions" **meets** the standard requirements for habilitation theses in mathematical analysis.

In Brno, date December 9, 2016

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Annotation of the referee report on the habilitation thesis of Ph.D. P. Zemánek

The habilitation thesis of Ph.D. P. Zemánek is devoted to recent author's contributions to the theory of square summable solutions of discrete symplectic systems, in particular thesis presents the results of the author for discrete symplectic systems with general linear dependence on the spectral parameter as well as with a polynomial or analytic dependence. These results do not only improve the type of the dependence on the spectral parameter, but they significantly generalize the previous results laying foundations of the "operator theory" for discrete symplectic systems, which is intimately connected with the topic of square summable solutions.

The habilitation thesis is well-written in compact, clear, and elegant form. The overall list of 24 publications of Petr Zemánek shows that he is a competent mathematician dealing with moden problems on a high international standart. The variety of topics of this thesis clearly indicates that he is a person which has a lot of own ideas, which allows him to contribute within different settings. The results of this thesis are highly important for international mathematical community, in particular, for all specialists in the field of the qualitative theory of discrete symplectic systems.

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Prof. Julia Elyseeva, DrSc.

In Brno, December 9, 2016

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