

To whom it may concern

Barnabas M. Garay, PhD, DSc
garay@math.bme.hu

Referee's report on the Habilitation Thesis of

Ladislav Adamec

"SOME APPLICATIONS OF THE EVOLUTION OPERATOR METHOD TO THE DYNAMIC EQUATIONS"

submitted at the Faculty of Science, Masaryk University, Brno

The habilitation thesis of Dr. Ladislav Adamec consists of a collection of his eleven papers on the subject accompanied by an extended introduction to dynamic equations in general, and to his own findings in particular.

Dr. Adamec worked in mathematical chemistry for fifteen years. It was only in the first years of the last decade when he started with more abstract mathematics, the theory of ordinary differential equations and, above all, the theory of dynamic equations.

This is quite an unusual turn. I appreciate dr. Adamec's courage who, after so many years in applied mathematics, started research in the pure theory afterwards. Moreover, he was able to gain maturity in the new field chosen. This is clear from comparing his 2001 paper "On asymptotic properties of a strongly nonlinear differential equation" to "A theorem of Wazewski and dynamic equations on time scales", an article he published six years later. Both papers are devoted to Wazewski's retract principle in a wedge-shaped region along the time axis of the phase space of a nonautonomous differential equation and of a dynamic equation, respectively. The author of the 2001 paper does not realize the combinatorial nature of the geometric assumption

$$x_i f_{n-i+1}(t, x_1, \dots, x_n) > 0 \quad \text{for } x_i \neq 0, \quad i = 1, \dots, n$$

— correspondingly, he arrives at a d -parametric family of solutions inside the wedge where d is the integer part of $\frac{n}{2}$. It is natural to consider a permutation π of the coordinate indices instead and to assume that

$$x_i f_{\pi(i)}(t, x_1, \dots, x_n) > 0 \quad \text{for } x_i \neq 0, \quad i = 1, \dots, n.$$

Applications to permanence theory of population dynamics are also at hand. He is right in remarking that the $t \rightarrow \infty$ limit of the distinguished solutions is not necessarily zero but does not give any example. (The core of such an example is the solution family $x(t) = x(t_0) \exp(\int_{t_0}^t q(s) ds)$ of the scalar equation $\dot{x} = -xq(t)$.) I am pretty sure the author of the 2001 paper was aware of having found some kind of a topological substitute of the "stable manifold" in nonsmooth and nonautonomous dynamics but it is only in the 2007 follow-up paper where, after a lucid exposition of the various technical difficulties, a definite task into this direction is formulated. Though connections between his condition (9) and the Nagumo-Haddad property of differential inclusions remain unnoticed and α -neighborhoods are redefined as α -dilations, the author of the 2007 paper is fully armoured to such an undertaking. He has proved his skills in a nice series of papers from 2001 onward.

The dynamics of a dynamic equation is not of continuous time. Time lives on a time scale, i.e., on an arbitrary closed subset of the reals. This makes an obstacle to immediate applications of Wazewski's retract principle for the translation operator along solution curves. Actually, there are no solution curves. On the other hand, any reasonable dynamic equation can be represented as a restriction of a nonautonomous differential equation to the time domain (in the sense that solutions of the dynamic equation are nothing but restrictions of solutions of the respective ordinary differential equation). The "covering" equations are not unique and can be constructed by gluing Hermite interpolants on small time gaps of the time scale. Analysing linear dynamic equations of the form $x^\Delta = A(t)x$, Dr. Adamec discovered a beautiful representation via constructing an exponential flow interpolant on the complement of the time scale. As a by-product, he ended up with structured embeddings of linear Hamiltonian systems and proved Liouville formula for general linear dynamic equations on time scales without large time gaps.

Dr. Adamec exploits various techniques of classical mathematical analysis in a solid, resourceful, and sometimes ingenious way. Problems he deals with are important in the qualitative theory of dynamic equations. As part of the problem of well-posedness, continuous dependence on the base time scale—the topic of his (for the time being) last paper on dynamic equations—is a fundamental issue. I am fully agree with his status report on pages 27, and 30-31 of the general introduction. What matters is the inner structure of the equations. Any further development requires more and more methods of global analysis.

All in all, he is a devoted researcher, with a respectable list of papers. I suggest the acceptance of the Habilitation Thesis for public defence and support Dr. Adamec's application for the *venia legendi*.



Barnabas M. Garay

Budapest, February 3rd, 2011.

Předmět: FW: addendum: Report on the habilitation thesis of Ladislav Adamec

Od: Zdenka Rašková <zdenar@sci.muni.cz>

Datum: Thu, 30 Jun 2011 08:26:47 +0200

Komu: "Zalesakova Marketa" <zalesakova@rect.muni.cz>

Dobrý den,
Předám vyjádření prof. Garaye.

Ing. Zdeňka Rašková
Oddělení pro vývoj, výzkum, zahraniční vztahy a doktorské studium PŘF MU
Kotlářská 2 611 37 Brno
zdenar@sci.muni.cz, tel. 54949 1406

-----Original Message-----

From: Garay Barna [<mailto:garay@math.bme.hu>]

Sent: Wednesday, June 29, 2011 4:00 PM

To: zdenar@sci.muni.cz

Cc: Ondrej Dosly

Subject: addendum: Report on the habilitation thesis of Ladislav Adamec

TO WHOM IT MAY CONCERN

Addendum to the report
on the habilitation thesis of Ladislav Adamec:

I declare that the thesis meets all standard requirements for a
habilitation thesis in the field of Mathematics - Mathematical Analysis.

Respectfully,

Barnabas Garay

_____ Informace od ESET Smart Security, verze databaze 6251 (20110630)

Tuto zpravu proveril ESET Smart Security.

<http://www.eset.cz>

_____ Informace od ESET Smart Security, verze databaze 6251 (20110630)

Tuto zpravu proveril ESET Smart Security.

<http://www.eset.cz>