Subatomic Quantification

DOCTORAL THESIS

Marcin Wągiel

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Declaration

Hereby I declare that this paper is my original authorial work, which I have worked out on my own. All sources, references, and literature used or excerpted during elaboration of this work are properly cited and listed in complete reference to the due source.

Marcin Wągiel

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iv

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Contents

x

List of Tables

List of Figures

xiv

1 Introduction

Since the early days of formal semantics (starting with Montague 1973) issues concerning quantification, pluralities, and countability have been of central importance for the study of meaning in natural language. A substantial body of research has been dedicated to these topics which led to a number of influential theories (e.g., Barwise and Cooper 1981, Scha 1981, Link 1983, Hoeksema 1983, Gillon 1987, Krifka 1989, Schwarzschild 1996, Landman 2000, Winter 2001). Within this tradition since at least Scha (1981) until, e.g., Champollion (2017), the prevailing view is that denotations of count nouns are atomic or, in other words, involve atoms, i.e., entities that have no proper parts in a mereological sense. Though at first blush this seems unintuitive since we know very well from our common experience that often things do have parts, there seemed to be good reasons to assume that compositional semantics ignores this fact and treats referents of count noun as indivisible units. The achievements of that strand of research are indisputable. At the same time, I believe that the attachment to the notion of atomicity originates from a limited scope of investigation and in the face of novel linguistic evidence to be presented needs to revised. The motivation behind this thesis is to explore what could be gained for semantic theory if we adopted a different perspective.

1.1 What's this all about

This study is about parts of singular concrete things such as apples, walls, and crowns and how we quantify over them. It is also about parts of pluralities of things such as collections of apples, walls, and crowns and why we do not quantify over them. It is about various types of partitives, e.g., structures such as *half of the apple*, multiplier phrases, e.g., expressions like *double crown*, and other natural language expressions, e.g., adjective phrases such as *whole apple*. What all those constructions have in common is that they involve what I will call throughout this study SUBATOMIC QUANTIFICATION. Subatomic quantification is quantification over parts of things that constitute building blocks of denotations of nominal expressions such as *apple, wall*, and *crown*. Though the use of the word 'subatomic' in the title of this thesis

suggests that I will also adopt the concept of an atom, in fact I will argue against approaches to nominal semantics based on the notion of atomicity. I decided for the use of that term since I believe that it intuitively evokes what I intend to focus on. Ironically, however, I believe the evidence concerning subatomic quantification forces us to reconsider what building blocks of denotations of count nouns actually are. The great source of inspiration for this study was the work of Grimm (2012a,b) who proposes a novel view on how to capture the semantic relevance of the mass/count distinction without reference to atoms.

The empirical aim of this thesis is to provide novel linguistic evidence showing that natural language semantics is sensitive to subatomic part-whole structures and to topological relations holding between elements within such structures. Furthermore, I intend to demonstrate the linguistic relevance of subatomic quantification as well as the fact that quantification over parts of a whole is subject to identical restrictions as quantification over wholes. The evidence to be examined indicates that some quantificational operations including counting presuppose particular topological relations.

From a theoretical point of view, this thesis is intended to provide a novel argument for adopting a theory of wholes called mereotopology, i.e., a system in which standard mereology based on parthood is extended with the primitive relation of connectedness as well as nuanced derived topological notions (Casati and Varzi 1999).

Some of the key questions concerning natural language semantics I want to address in this study are the following.

1. Are there different types of parts of entities similar to different types of wholes?

2. How do we count parts of an object?

3. Why cannot we count parts of a plurality?

At first blush, the answers at which I arrive might seem somewhat surprising.

1. Yes, there are continuous and discontinuous parts. The first resemble referents of singular count nouns, whereas the latter are more like pluralities.

2

2. We count parts of an object in the very same way how we count whole objects. The mechanism is unified, thus identical restrictions apply.

3. For the same reason we cannot count pluralities. Only objects with certain topological characteristics can be assigned a number in numerical quantification.

The most important result of this study is a proposal of an account for subatomic quantification. I will argue not only that it correctly predicts why only some parts are countable, but also that extending it to the level of wholes will enable us to seek for more advantageous explanations for known problems. For instance, it could explain why a natural language expression such as *two apples* refers to a plurality of two apples rather than to two pluralities of apples. Or why object mass nouns such as *furniture* are not countable despite involving reference to discrete entities. There are also a number of other questions I will attempt to tackle, many of which will remain without a definite answer. However, I believe that both the evidence presented here and the proposed account provide a novel exciting perspective to think about partitivity and countability.

Note that though I will sometimes mention them in passing, this study is not about abstract terms (see, e.g., Asher 1993, Tovena 2001, Nicolas 2002, Moltmann 2013), event nominals (see, e.g., Grimshaw 1990, 2011, Bierwisch 1990, Borer 2005b), and collective nouns (see, e.g., Landman 1989a,b, Barker 1992, Schwarzschild 1996, Pearson 2011, de Vries 2015), nor about parts of their referents. Despite the fact that in principle I believe that the approach developed here can be extended to these expressions, for the most part I will not consider part-whole structures of abstract entities. Before I introduce the main claims of this thesis in a bit more detail and then delve into intricacies of linguistic evidence, let us first consider my view on what it intuitively means to be a whole and part.

1.1.1 Intuitive notions of a whole and part

The starting point of this journey concerns an ontological intuition dating back at least to Pre-Socratics that entities are often made up of smaller entities which are related to each other in a particular manner

(see Varzi 2016 for a historical overview). In other words, when thinking about what is, humans often assume that objects are generally configurations of parts. This ontological intuition seems to stem from a cognitive fact that human beings often conceive entities as being made up of smaller entities related to each other in certain ways. In fact, psychological evidence demonstrates that at least from early childhood humans are able to perceive entities simultaneously in a twofold way (e.g., Elkind et al. 1964, Kimchi 1993, Boisvert et al. 1999). On the one hand, we can discriminate parts from a whole making them more salient than the entire object, whereas on the other hand we have an ability to integrate the parts in such a way that the perception of a complete whole emerges in our cognition. In other words, humans are able to see entities as collections of parts and as integrated wholes simultaneously.[1]

At the same time, from the very early considerations on the part-whole relation in Plato's *Parmenides* and *Theaetetus* much concern regards unity. What interests Plato is how to differentiate between a true unity and an arbitrary sum of parts. The crucial property of the first is that it is provided by structure which distinguishes it from the latter. Without going into details of Plato's ontological views (but see, e.g., Priest 2014), it seems that the key intuition behind his deliberations is that the crucial component of what it means to be a whole is given by the manner of arrangement of parts. In other words, a whole is not simply reducible to the sum of its parts (see also Casati and Varzi 1999), e.g., a glass is not an arbitrary sum of shards, but rather a particular configuration of shards. It happens that I also share this intuition.

When considering how objects are conceptualized, I believe it is instructive to make use of categories such as unity, boundedness, and integrity. The first is about capturing an individual as a complete whole in itself, i.e., distinguishing between discrete objects as opposed to fragmentary portions of a continuum of a larger entity. The second category concerns boundaries. Objects are well-defined in space in contrast to scattered fragments of entities or a diffused continuum of matter. Finally, integrity characterizes a mutual bond between entities perceived as being parts of a certain whole. Though this bond can be conceptualized in many different ways, one significant mode of

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1. I will return to the discussion of findings of cognitive psychology in Section 5.1.

integrity is viewed in terms of topology. Specifically, objects involve elements that are connected to each other, i.e., stick together, and thus move along the same trajectories as one unit. The vital question that inspired this study is to what extent these categories are relevant for the architecture of natural language semantics? In the following chapters, I will argue that there are good reasons to believe that it is indeed relevant. Specifically, I will provide linguistic evidence not only that the meaning of nominal expressions involves information concerning integrity but also that grammar is sensitive to such a notion.

Crucially, I argue that the same way of thinking should apply when talking about parts of wholes. When describing parthood with respect to entities extended in space or time, employing the topological notion of contiguity might allow us to capture some non-trivial facts about part-whole structures. In particular, some parts constitute individuated continuous strings of matter within a whole, whereas other parts are just arbitrary sums of portions of substances. To demonstrate the difference let us discuss the following example (cf. Acquaviva 2008, pp. 90–93). Consider two parts of a table, its leg and a splinter. Under ordinary circumstances, we would definitely agree that both these entities are parts of the table; however, each of them in a somewhat different sense. A splinter is just some arbitrary portion of matter making up the table, an element among numerous similar fragments of the whole. Before mentioning it, we probably had no or at least very little expectations with respect to its appearance. We might even question whether it exists as a splinter before being detached from the whole since normally tables are not perceived as collections of splinters. On the other hand, a leg is easily identifiable among a definite number of similar elements. We perceive it as having a clearly specified function and expect certain traits of its appearance. Furthermore, there is also a sense in which two separate disconnected splinters or two disjoint legs are part of the table since they constitute some fragment of its material make-up. Importantly, however, this type of parthood resembles the relationship between a splinter and the whole rather than the case of a leg.

In the philosophical literature, this distinction has been attributed to different modes of partitioning an object. For instance, Krecz (1986) proposes different terms for specific subdivisions of a whole, i.e., parts, as opposed to arbitrary subdivision, i.e., pieces. The first would corre-

spond, e.g., to a leg of a table, whereas the latter would be used with respect, e.g., a splinter. Similar, Markosian (1998) draws an analogous distinction between what are called metaphysical parts, i.e., parts that can be considered as objects in their own right, and conceptual parts, i.e., equivalents of arbitrary parts. Finally, Jennings (2010) proposes to differentiate between parts viewed as arbitrary bits of an object one could cut off and functional parts defined as parts that are considered essential for an object since they fulfill a particular function. Crucially, the distinction is conceptually valid but it is also reflected in grammar. For instance, English distinguishes between the two different flavors of parthood discussed above syntactically as can be witnessed in (1.1) (see also Champollion and Krifka 2016). The use of the bare mass noun *part* in the sentence in (1.1a) indicates arbitrary partitioning of the table. On the other hand, the full DP *a part* in (1.1b) corresponds to a specific division in parts. Notice also that only the latter use is countable.

(1.1) English (Acquaviva 2008, p. 90; adapted)
    a. A splinter is part of the table.
    b. A leg is a part of the table.

It has been acknowledged that the distinction discussed above seems to have some further linguistic implications (Champollion and Krifka 2016). For instance, consider the minimal pairs in (1.2) and (1.3). The contrast suggests that transitivity does not hold for arbitrary parts.[2] In other words, the fact that the thumb is a part of a hand does not guarantee that it would be linguistically treated as a part of what that hand is part of, i.e., an arm.

(1.2) English (Champollion and Krifka 2016)
    a. the thumb of this hand
    b. a hand without thumb

(1.3) English (Champollion and Krifka 2016)
    a. #the thumb of this arm
    b. #an arm without thumb

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2. Champollion and Krifka (2016) use the terms structured and unstructured parthood in order to refer arbitrary and specific subdivisions, respectively.

Despite these facts the role of the distinction between arbitrary and specific parts in natural language semantics is usually either totally neglected or reduced to lexical relations such as meronymy, i.e., relations between meronyms and holonyms, e.g. *wing* and *bird*, and hyponymy, i.e., the relation between hyponyms and hyperonyms, e.g., *swallow* and *bird*. Although it has been recognized that meronymy and hyponymy have some general properties which establish taxonomies and structure large parts of the lexicon in natural language (Cruse 1986), compositional approaches to meaning did not assume that such categories play any role in the proposed models. One of the aims of this thesis is to provide linguistic evidence for the relevance of the two aspects of what it means to be part of something and to propose how first steps towards accommodating a less naive view on parthood into semantic theory could look like.

1.1.2 The claims in a nutshell

In this thesis, I will argue that a proper treatment of countability in natural language should take into account the interaction between partitivity, topology, and quantification. In other words, I believe that exploring subatomic quantification can reveal some non-trivial phenomena that were otherwise obscure, and thus contribute to our understanding of the interaction between nominal semantics and the meaning of numerical expressions in general. The central claims of this thesis are the following.

First, natural language is sensitive to the fact that entities denoted by nominal expressions have parts as well as to topological relations holding between them. The relevance of part-whole structures is typically acknowledged with respect to pluralities. However, my claim is stronger. I postulate that natural language is also sensitive to subatomic part-whole structures. This is in discordance with a mainstream view that utilizes the notion of atomicity in order to capture semantic properties of count nouns. Such an approach postulates that atoms, i.e., building blocks of denotations of expressions like *apple* and *crown*, are entities that from a linguistic perspective are treated as having no proper parts. Consequently, countability can be accounted for in terms of quantification over atoms. Nevertheless, the linguistic evidence I will present in the following chapters will demonstrate that

in fact there are a number of natural language expressions involving quantification over such subatomic parts. Crucially, based on the distribution of partitives I will argue for a unified parthood relation for both singularities and pluralities. Furthermore, I will postulate that the difference between the two is a result of different topological relations encoded in the corresponding part-whole structures. The claim is that prototypical referents of count singulars are conceptualized as integrated wholes, whereas regular plurals require entities in their denotations to comprise such cohesive objects as their parts but impose no topological constraints on a spatial configuration of those parts. As we will see from the data concerning topologically sensitive partitive expressions, the distinction between integrated wholes and scattered entities also applies at the subatomic level. Specifically, the novel evidence shows that the contrast between continuous and discontinuous parts is relevant for interpretation of natural language expressions. Hence, the notion of topological integrity should be accommodated into semantic theory.

The second claim concerns counting. In particular, I will argue that countability is not some syntactic feature nor is it a meaning postulate on a certain lexical item, but rather it follows from what I refer to as the general counting principles. In particular, the principle of non-overlap ensures that entities one quantifies over are disjoint, i.e., they do not share a part. In other words, things can be counted once and once only. This rule excludes a possibility of counting entities involving multiple overlapping parts as denoted by mass nouns and pluralia tantum. Furthermore, the principle of maximality guarantees that when counted entities are associated with numbers in their entirety, i.e., no part is left out. This is especially relevant with respect to homogeneous entities referred to by nouns such as *twig* and *fence*. Given a particular counting situation, what counts as one always needs to be the maximal entity no matter how its part-whole structure is defined in that situation. Finally, the principle of integrity requires that what can be counted needs to be conceptualized as an entity that comes in one piece. This restriction rules out arbitrary sums of entities as well as discontinuous portions of substances. Consequently, count-

ing pluralities is not allowed.[3] All things considered, only predicates denoting entities that satisfy the general counting principles can be modified by cardinals.

The third and final claim extends the general mechanism behind counting to the subatomic level. More specifically, I will postulate that the general counting principles constitute a universal set of constraints regulating what is fit for being counted and what is not irrespective whether it is a whole or part. In other words, I will argue that it follows from the principles of non-overlap, maximality, and integrity that certain parts are countable whereas other are not. The evidence from partitive constructions involving cardinal numerals demonstrates that it is not allowed to count discontinuous parts of entities. Moreover, the claim is further corroborated by the fact that natural language developed expressions dedicated to subatomic quantification.

Linguistic evidence for the three claims introduced above comes from a number of natural language expressions. Specifically, I will examine different types of partitives, *whole*-adjectives as well as multipliers such as English *double*. Some parts of this thesis will also investigate the data considering plurals and cardinal numerals. In order to provide a more general picture, the evidence will be examined from a cross-linguistic perspective. The languages which will be discussed most include Polish, German, Italian, and English but throughout the text I will also address examples from (in the alphabetical order) Basque, Bosnian-Croatian-Serbian (BCS), Czech, Dutch, Finnish, French, Hebrew, Hungarian, Irish, Japanese, Lithuanian, Maltese, Mandarin, Mi'gmaq, Portuguese, Russian, and Yucuna.

1.2   General assumptions

There are several issues concerning conceptual grounds that require a short commentary. In this section, I will present my view on natural language semantics and discuss the ontology assumed for the purpose of developing an account for subatomic quantification.

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3.   Notice that group nouns are more than pluralities. In fact, I assume that they denote abstract singular entities with whom pluralities of members can be associated. I will come back to this issue in Section 2.4.

1.2.1 Cognitive view of meaning

The view on theory of meaning I adopt here is on a par with the position expressed by Krifka (1998) and shared, e.g., by Partee 2018 and Grimm (2012b). Though model-theoretic semantics in the tradition of Montague has commonly been assumed to necessarily endorse some form of semantic externalism (see, e.g., Putnam 1975, Davidson 1987), i.e., a view on which expressions of natural language designate objects in the world, and as such is incompatible with approaches to meaning that seek to develop cognitive models of the world (assuming that meanings are in the head after all), such a sharp distinction does not seem plausible. In fact, as pointed out by Krifka it is possible to combine the use of the methods and techniques developed within the model-theoretic tradition with an assumption that natural language expressions are interpreted by conceptual structures that in turn are associated with external entities by some pragmatic mechanism responsible for how we use language. Therefore, I assume that model-theoretic representations discussed in this study are compatible with a cognitive view of meaning and can be seen as tools one can use to try to grasp human mental reality.

In other words, in the system developed here I make no metaphysical claims, nor do I embrace any form of semantic externalism. Instead, I would like to see the postulated notions and structures as attempts to capture the way we conceptualize certain aspects of external reality. That is to say, the theory of subatomic quantification to follow is assumed to characterize some properties of how human beings perceive things in the world, not to describe them how they are. This means that both primitive objects and more complex conceptual forms are assumed to 'be in the head'. Of course, it is not unlikely that properties of such semantic objects may often correspond to the properties of mind/language-external entities. Consequently, the first could be used to talk about the latter assuming matching by some extra-linguistic mechanism. In any case, I do not presume that meanings are out there in the world. Rather, I will often emphasize the significance for natural language semantics of how entities are conceptualized. In the next section, I will describe the minimal assumptions concerning primitive objects grouped together in the domains of the model.

1.2.2 Ontology

In standard Montague semantics, the domain of discourse is simply a collection of disjoint non-empty sets of entities from which denotations of basic expressions as well as more complex constituents are built. It is usually assumed that the domain of entities is supplemented with the set of truth values as well as sets of events, possible worlds, degrees and others. Since the focus of this study is very specific, I will limit the number of distinct types of primitive objects almost to the minimum. In doing so, I will try to be faithful to the position famously formulated by Link (1983) that the guide in ontological considerations should be natural language itself.

I will use the term ENTITY to talk about anything that can be referred to by proper names such as *Noam Chomsky* and *Nim Chimpsky* and definite descriptions such as *the author of "Syntactic Structures"* and *that chimpanzee* as well as denoted by common nouns such as *apple* and *juice*. I assume that entities can be either well-defined discrete objects, pluralities thereof, or shapeless amorphous substances. Though there are significant differences between them, they all fall into one domain. In general, I will refer to a thing in the denotation of a count noun as INDIVIDUAL or OBJECT. Sometimes, I will use the term SCATTERED ENTITY to indicate referents of mass terms and ARBITRARY SUM to talk about denotations of plurals. The words 'entity' and 'thing' are assumed to be general terms covering both individuals/objects, portions of matter as well as pluralities.

I do not assume that all nouns designate entities. Some classes of nominals such as nominalizations, measure words as well as role nouns arguably make reference to eventualities, degrees, and roles, respectively. For instance, *murder* denotes a set of murdering events (e.g., Grimshaw 1990, 2011), *liter* refers to an interval or a set of degrees on a scale of volume (e.g., von Stechow 1984, Heim 1985), whereas *president* designates a social role, i.e., a function or capacity independent of the individuals that bear it (Sowa 1984, Steimann 2000). Such social constructs can be associated with individuals by a special shifting mechanism (Zobel 2017). Though in this study I will restrict my focus to entities, or specifically concrete entities, I believe that at least some of the ideas introduced here can serve as an inspiration for developing

a new way of thinking of partialness with respect to somewhat more abstract things such as eventualities, degrees, and roles.

Since kinds, or more generally concepts (see Krifka 1995, Mueller-Reichau 2006), do not play any role in this study, I will refrain from the discussion concerning their ontological status as well as their role in nominal semantics. Nevertheless, in principle the approach developed here is compatible with accounts modeling kinds in the spirit of Carlson (1977, 1980) and Krifka (1995) where they are treated as entities in their own right that unlike object-level things are not spatio-temporally bounded.

A considerable amount of attention in this study will be dedicated to numerical expressions including cardinal numerals, fractions, and multipliers. I assume that all those lexical items involve some sort of reference to mathematical entities. I will use the term NUMBER to refer to abstract entities definite descriptions such as *the number two* designate. I adopt here an intuitionist perspective on the epistemology of mathematics (see, e.g., Kitcher 1984). According to this view, mathematical objects are constructions of the human mind and do not exist in the external world. Notice, however, that numbers in this sense do not necessary coincide with objects defined by Peano's axioms. For instance, I do not assume that artificial integers are part of the ontology described here. Rather, numbers are what we refer to when we use our everyday language and what might coincide with human number sense.

1.3 Conventions

Both glossing and notational conventions used in this thesis are more or less standard in academic linguistics. Nonetheless, for the sake of clarity I provide a complete list of what to expect in examples and formulae.

1.3.1 Examples

Since in many parts of this thesis I discuss phrases and sentences from various languages some of which are not very well-known, in order to avoid confusion I will always indicate the language of an example, even

if that language is English.[4] Furthermore, I will provide information on the bibliographical source or a name of a native speaker or native speakers with whom I have consulted a particular example. In the case of evidence from Polish, all examples and judgments are my own though I have often confronted them with other native speakers who confirmed my intuitions. Most of the data come from introspection with some exceptions found in corpora. I use the symbol # to indicate both semantic infelicity, i.e., awkwardness in terms of meaning, and constructions that lack a particular interpretation that is crucial from the perspective of a discussed phenomenon. For the sake of clarity, in the latter case I will always provide relevant readings and mark the one that is non-existent with the symbol #. Unless explicitly stated otherwise, I will always consider uncoerced meanings of nominal expressions, i.e., I will ignore the mass-count as well as count-mass shifts. Occasionally, I will use the symbol * in the examples to indicate ungrammaticality.

Concerning glossing, I will introduce the complete grammatical information only when relevant morpho-syntactic issues are discussed or its lack might cause confusion. Otherwise, for the sake of simplicity I limit it to the minimum. In the examples, I will use the following abbreviations. Case: NOM – nominative, GEN – genitive, DAT – dative, ACC – accusative, INS – instrumental, LOC – locative; gender: M – masculine, F – feminine, N – neuter; number: SG – singular, PL – plural, COLL – Italian irregular plural; others: ADJ – adjectival form, COP – copula, DISTR – distributivity marker, INT – interrogative marker, LNK – linker, IMP – imperative, POSS – possessive, REFL – reflexive marker.

1.3.2 Notation

In the following text, I will use a relatively standard notation.[5] For completeness, I specify the following. The symbols x, y, z, and w are used to represent entity variables, whereas n and P are dedicated for number and predicate variables, respectively. The small letters a, b, c, and d stand for entity constants, whereas Arabic numbers, e.g., 1, 2, and 3, for integers. On the other hand, particular properties are transcribed with

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4. Though most of the evidence will be introduced in Chapter 2, 3, and 4, some crucial data will be also provided in Chapter 5 and 7.
5. Formal expressions will appear mostly in Chapter 6 and 7.

small capitals, e.g., APPLE stands for the property of being an apple. The primitive semantic types of entities, numbers, and truth values are represented by the symbols e, n, and t, respectively. Only complex types are given in angle brackets, e.g., $\langle e, t \rangle$ as opposed to e. Furthermore, the scope of both the λ operator and quantifiers $\exists$ and $\forall$ is indicated by square brackets, e.g., $\lambda x[\text{APPLE}(x)]$ and $\exists x[\text{APPLE}(x)]$. For convenience, in complex formulae I use small and big brackets in order to designate scope of particular operators. Finally, presuppositions are introduced always following a dot immediately after the relevant λ operator, e.g., in the term $\lambda x. \text{ FRUIT}(x) \, \lambda P[^*P(x) \wedge \#(P)(x) = 3]$ the sequence FRUIT(x) is a presupposition.

1.4 The things to come

A substantial body of research has been dedicated to quantification in natural language. The thesis is structured as follows. The next three chapters will provide linguistic evidence on the relevance of subatomic quantification mainly from a cross-linguistic perspective. I will examine a broad range of constructions involving different types of partitives, *whole*-adjectives, and multipliers such as English *double*. In Chapter 2, I will discuss certain patterns observed in partitive constructions cross-linguistically that suggest a unified meaning of partitive words such as *part* and *half* as well as the importance of the topological notion of integrity in subatomic quantification. The main focus will be dedicated to the interaction between numerals, partitive words, and nominals involving count singulars, regular plurals as well as Italian irregular plurals. Chapter 3 will provide further evidence for the significance of integrity in quantification over parts. In particular, I will present novel data concerning topology-sensitive partitive words that require the whole to be an integrated object or yield only integrated parts of the whole. Though the evidence will come mainly from Polish, parallels with other languages will also be drawn including English, German, and Mandarin. In Chapter 4, I will explore the meaning of multipliers, i.e., a neglected class of numerical expressions that is specialized to count parts of entities denoted by the modified noun. The discussed data will come from Slavic where morphological evidence supports semantic complexity of those expressions. The main

purpose of Chapter 5 is to provide the three claims that constitute the conceptual background for the analysis. For that purpose, I will relate the previously introduced linguistic evidence with psychological findings concerning the role integrity and part-whole structures play in human cognition and especially in counting. In Chapter 6, I will introduce mereotopology, i.e., a theory of wholes extending the mereological framework based on the notion of parthood with the topological concept of connectedness. Mereotopology allows us to capture the difference between distinct types of things such as integrated wholes, arbitrary sums of individuals, and scattered entities corresponding to substances and will prove advantageous in modeling quantification over parts. Chapter 7 will be dedicated to spelling out a formal analysis of selected issues in subatomic quantification. The account will be based on the mereotopological framework with the intuitive notion of integrity playing the main role in the semantic representation of countable parts. Finally, Chapter 8 will conclude the thesis and suggest how further research could extend the approach in order to tackle some open questions. For convenience, the most relevant contribution of a given part is summarized at the end of each chapter.

2 Partitives and part-whole structures

In this chapter, I will examine cross-linguistic evidence for the relevance of subatomic quantification in natural language. In particular, I will focus on the distribution and semantic properties of distinct types of partitives some of which have not received much attention in the linguistic literature oriented on the study of meaning. Specifically, I will investigate constructions involving proportional expressions such as *part* and *half* in order to reveal some non-trivial facts about partitivity. I will provide novel data indicating that singulars and plurals share one unified parthood relation. Furthermore, I will show that distinct part-whole structures result from the fact that certain entities are conceptualized as cohesive individuals whereas others are not. Crucially, the distinction turns out to be relevant from the perspective of countability. In particular, partitive constructions indicate that cardinal numerals select only for predicates denoting integrated entities.[1]

2.1 Partitives

As already mentioned in the introduction, the part-whole relation is an important notion in human cognition and natural language. For instance, large parts of the lexicon are structured by means of meronymy, i.e., the relation that holds between expressions denoting entities conceptualized as whole objects (holonyms) and expressions denoting parts of such wholes (meronyms), e.g., the relationship between *cat* and *tail*. However, the relevance of partitivity in natural language is not restricted to lexical associations but it is also reflected in grammar. As noticed by Hoeksema (1996), its importance for the syntax-semantics

interface is corroborated by the fact that the link between a whole and its parts can support definite descriptions, as witnessed in (2.1).

(2.1)   English (Hoeksema 1996, p. 1)
         I still have a jar of gooseberry jam, but I cannot get <u>the lid</u> off.

There are many ways how the part-whole relation can be expressed syntactically and languages differ in what means they employ in order to indicate it. Sometimes even though a particular construction lacks a formal exponent expressing partitivity, the relation between the elements is well understood. For example, English compounds lack morphological means to express partitivity but often the part-whole relation between the referents of the first stem and the referents of the second stem is semantically transparent and can be readily retrieved (Hoeksema 1996).

(2.2)   English (Hoeksema 1996, p. 1)
         a.   chicken-feet
         b.   church-door
         c.   mountain-top
         d.   bike-steer

In this study, however, I will restrict my focus to the structure that expresses the part-whole relation formally, i.e. the so-called partitive construction. The partitive is an expression of natural language that indicates partialness. In English nominal partitives commonly take the form as in (2.3).[2]

(2.3)   Partitive structure (Martí i Girbau 2010)
         a.   [$_{DP}$ Det [$_{PP}$ *of* [$_{DP}$ Det NP]]]
         b.   [three [of [my friends]]]

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2.   In fact, there are at least several proposals concerning the syntax of partitives. For instance Ionin et al. (2006) argue for different structures for distinct types of partitives, see (i). However, for the most part I will ignore all the syntactic intricacies here.

(i)   English (Ionin et al. 2006)
         a.   [$_{NP}$ half [$_{PP}$ of [$_{DP}$ these [$_{NP}$ eight apples]]]]
         b.   [$_{NP}$ two [$_{NP}$ ∅/~~apple~~ [$_{PP}$ of [$_{DP}$ these [$_{NP}$ eight apples]]]]]

The upstairs determiner is a quantifier, whereas the preposition *of* links it with the larger whole denoted by its complement, i.e., the embedded definite or specific DP, from which it is partitioned. A similar frame is found in many other Germanic languages as well as in Romance, as demonstrated in (2.4) and (2.5). On the other hand, in languages such as Estonian and Finnish, a special partitive case can be used instead of the prepositional element, see (2.6) (de Hoop 2003), whereas, e.g., German and many Slavic languages can express the partitive with the genitive (the so-called partitive genitive), see (2.7) (see, e.g., Hoeing 1997 for German and Valkova 1999, Rutkowski 2007 for Slavic).

(2.4)   Dutch
      drie van mijn vrienden
      three of   my   friends

(2.5)   French
      trois de mes amis
      three of my friends

(2.6)   Finnish (de Hoop 2003)
      Anne joi   maitoa.
      Anne drank milk$_{PART}$
      'Anne drank (some) milk.'

(2.7)   Russian
      Oleg vypil polovinu moloka.
      Oleg drank half   milk$_{GEN}$
      'Oleg drank half the milk.'

In terms of meaning, the partitive picks out a part from a whole. The part is associated with the upstairs quantifier, whereas the whole is denoted by the embedded DP and can be either a singular entity or a plurality (e.g., Jackendoff 1977, Selkirk 1977, Ladusaw 1982). Thus, partitives can denote parts of individuals, see (2.8a), as well as subsets of larger sets of individuals, see (2.8b), and portions of substances, see (2.8c).

(2.8)   English

      a.   most of the apple
      b.   most of the apples
      c.   most of the juice

An important observation concerning partitive constructions is that not every kind of DP can be embedded within the structure. A restriction called the Partitive Constraint excludes certain expressions from this position (e.g., Jackendoff 1977, Selkirk 1977, Barwise and Cooper 1981, Ladusaw 1982). In particular, its early formulations posited that only definites can appear in the downstairs DP.[3] Later approaches redefined the restriction in semantic terms postulating that the embedded DP needs to denote an entity, i.e., it needs to either definite or specific (de Hoop 1997).

Before I begin a detailed discussion of the relationship between partitivity and countability it will be useful to make several terminological remarks. First, I will use the terms PART-WORDS and HALF-WORDS to refer to expressions such as English *part* and *half*, respectively. Despite significant syntactic differences between those lexical items suggesting more fine-grained categorial distinctions, they both can appear in the upstairs Det position in partitives like (2.8a) on a par with quantifiers such as *most* and *all*. From a cross-linguistic perspective different types of *half*-words often fall into different syntactic categories. For instance, German *Hälfte* 'half' appears to have nominal properties, whereas *halb* 'half' is a standard adjective. As demonstrated in (2.9a)–(2.9c), while *Hälfte* has a gender value and either assigns the genitive case or combines with the prepositional *von*-phrase assigning the dative (Durrell 1996; see also Hoeing 1997, Asbury 2007 for the discussion of the structures), *halb* agrees with the modified nouns. However, since this study is primarily concerned with semantic aspects of partitivity, I take the terms introduced above to refer to expressions explicitly designating a part of a whole. Therefore, despite the different syntactic structures expressions such as *Hälfte* and *halb* occur in, I will treat them as semantically related linguistic objects. The general term PARTITIVE WORDS will be used to cover *part*-words and *half*-words as

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3.   See Abbott (1996) for a discussion and a proposal of an analysis not involving the Partitive Constraint.

well as different types of proportional quantifiers that can be found in partitives.[4]

(2.9)   German

    a.   eine   Hälfte   des   Apfels
        a$_{\text{NOM.SG.F}}$ half$_{\text{1.NOM.F}}$ the$_{\text{GEN.SG.M}}$ apple$_{\text{GEN.M}}$
        'a half of the apple'

    b.   eine   Hälfte   vom   Apfel
        a$_{\text{NOM.SG.F}}$ half$_{\text{1.NOM.F}}$ of-the$_{\text{DAT.SG.M}}$ apple$_{\text{DAT.M}}$
        'a half of the apple'

    c.   ein   halber   Apfel
        a$_{\text{NOM.SG.M}}$ half$_{\text{2.NOM.M}}$ apple$_{\text{NOM.M}}$
        'a half of the apple'

Another issue concerns the grammatical number of the downstairs DP in the partitive construction. There is a tradition in the literature on partitivity to distinguish between partitive expressions referring to a portion of a singular entity and those referring to a subset of a set of individuals. The first are often called mass partitives and the latter group partitives (e.g., Hoeksema 1996, Abbott 1996). However, despite this tradition I will follow de Hoop (1997, 2003) and refer to phrases such as (2.8a) as ENTITY PARTITIVES and to phrases such as (2.8b) as SET PARTITIVES.[5] Typically, entity partitives involve a singular term in the embedded DP, whereas set partitives comprise an upstairs quantifier c-commanding a plural DP. I will restrict the use of the term MASS PARTITIVES to constructions such as (2.8c) where the upstairs determiner combines with the *of*-phrase taking a mass term as its complement. The main reason is that I found the terms mass and group partitives confusing in that they suggest a correspondence with the mass/count distinction, contrary to fact (see also Martí i Girbau 2010).

Furthermore, I will refer to constructions involving an overt *part*-word such as those in (2.10a) as EXPLICIT PARTITIVES. On the other hand,

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4.   The term more or less corresponds to what Quirk et al. (1985, p. 249) call general partitive nouns excluding measure terms.
5.   Yet another term is used by Korat (2016) to refer to partitives involving regular count singular nouns as well as collective nouns, namely singular quantified terms. However, since I do not focus on group nouns in this study, I will ignore the potential distinction.

I will call phrases containing proportional quantifiers such as *most* or *half*, see (2.10b), PROPORTIONAL PARTITIVES, whereas those involving fractions, see (2.10c), will be referred to as FRACTION PARTITIVES.

(2.10)   English
  a. part of the apple
  b. half of the apple
  c. two thirds of the apple

Finally, I will use the terms COUNT EXPLICIT PARTITIVES and COUNT PROPORTIONAL PARTITIVES to refer to phrases such as (2.11a) and (2.11b), respectively, where the partitive word is modified by the cardinal numeral. These should not be confused with COUNT PARTITIVES, the term sometimes used for phrases such as (2.11c) (e.g., Ionin et al. 2006).[6]

(2.11)   English
  a. two parts of the apple
  b. two halves of the apple
  c. two of the apples

The next sections of this chapter will be dedicated to the interaction between cardinal numerals and partitives involving different types of expressions both in the upstairs DP head as well as in the embedded DP.

2.2   The analogy

At least from Link (1983) on it is commonly assumed that it is necessary to distinguish between two distinct yet corresponding part-whole structures employed in natural language, namely one corresponding to the material constitution of objects and one capturing collections of distinct objects. Hence, the material part relation provides ordering for structures representing the relationship between portions of matter and individuals made out of that matter, whereas the individual part relation concerns particular objects and sums thereof. The distinction is motivated by the fact that things and stuff they consist of are apparently differentiated in natural language. The famous example

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6.   Quirk et al. (1985, p. 249) call them plural partitives.

concerns two golden rings. If they were forged recently out of some old Egyptian gold, then it is definitely true to state that the rings are new although the gold is old.

However, there are some data that might be interpreted as suggesting that at least in some expressions natural language semantics does not distinguish between the two distinct part relations postulated by Link (1983) and instead employs a unified part-whole structure for different types of nominal expressions. Moltmann (1997, 1998) observes an analogy between partitives involving singular and plural terms. For instance, in German the same expression *Teil* 'part' can be used both in entity and set partitive constructions in order to quantify over parts of singular individuals and subsets of groups of individuals, see (2.12).[7] In particular, when used in the entity partitive, *Teil* quantifies over material parts of singular objects, i.e., functional parts or portions of a substance making up the whole individual. For instance, (2.12a) denotes a set of parts of the relevant apple. It could be used to refer, e.g., to the stem end of the apple in question or its blossom end as well as its pulp or pips. On the other hand, when *Teil* takes the plural DP as its complement, it quantifies over whole individuals, e.g., (2.12b) makes reference to a part of a particular collection, i.e., a subset of the apples rather than to a set of arbitrary parts of singular apples. For instance, given a situation in which there are three relevant apples a_1, a_2, and a_3 (2.12b) could not be used to refer to a set involving the stem end of a_1 and a pip of a_2. Instead, it would be true of a set of some of the apples in question, e.g., a set comprising a_1 and a_2.

7. In fact, Moltmann (1997, 1998) provides English examples to argue for her point, see (i). However, since the reported felicity judgments for English contrast with Moltmann's claim (see Schwarzschild 1996), I take Moltmann's data not to be reliable and use uncontroversial German examples instead.

(i)   English (Moltmann 1997, p. 11)
        a.   all of / some of / part of the book
        b.   all of / some of / part of the wine
        c.   all of / some of / part of the books

(2.12)   German (Nina Haslinger, p.c.)

    a.   Teil des   Apfels
         part the$_{\text{GEN}}$ apple$_{\text{GEN}}$
         'part of the apple'

    b.   Teil der   Äpfel
         part the$_{\text{GEN}}$ apples$_{\text{GEN}}$
         'some of the apples'

Notice that in German there is yet another interpretation an expression such as (2.12b) can get. For instance, consider the sentence in (2.13). It would definitely be true in a situation where there are ten apples and two of them are completely moldy. Yet, there is also a different scenario where uttering (2.13) would be truthful, namely if there were ten apples and each apple had a small patch of mold. At first blush, this behavior might appear to contradict what I claimed in the previous paragraph. However, there are good reasons to assume that this reading is independent from the two discussed above and results from a silent distributivity operator. Notice that the sentence would be judged false in a scenario where there were ten apples seven of which had a small patch of mold, whereas the others were not moldy. This shows that German explicit set partitives such as (2.12b) have another interpretation in addition to the subset reading discussed above. Crucially, this interpretation does not involve quantification over arbitrary material parts. Rather, the definite seems to have a distributive interpretation with wide scope over the indefinite. Though it was important to draw the distinction between the two possible interpretations, I will ignore this additional distributive reading in the remaining part of this chapter.[8]

(2.13)   German (Nina Haslinger, p.c.)
    Ein kleiner Teil von den   zehn Äpfeln   ist schimmlig.
    a   small   part of the$_{\text{DAT}}$ ten   apples$_{\text{DAT}}$ is moldy
    'A minority of the ten apples are moldy/A small part of each of the ten apples is moldy.'

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8.   In fact, this interpretation might turn out to be a German idiosyncrasy since it seems to be unavailable in other languages, e.g., in Polish. Though it is an interesting issue on its own, this study is about something different.

Thus, it appears that the same expression can form either an entity partitive or a set partitive depending on the number value on its complement. This phenomenon seems intriguing and might imply a puzzle with respect to a proper treatment of German explicit partitives. But is it just a German idiosyncrasy? Or does the data in (2.12) actually reveal something deep about the way the human mind conceptualizes the difference between objects and collections of objects? And if yes, to what extent is such a difference relevant for the semantic distinction between singularities and pluralities outside the domain of partitivity? I believe that, if approached in a systematic manner, an attempt to provide answers to these questions can shed new light on what it means to be an individuated entity and, as we will see, what it means to be countable.

Prima facie, it seems that there are two ways how to account for the phenomenon in question. A radical solution is to conclude that the data in (2.12) suggest a unified parthood structure for both singular and plural individuals across the board (Moltmann 1997, 1998). In other words, it is to posit that singularities and pluralities are in fact much less distinct than standardly assumed. On such an approach, the relationship between an object and its parts would be the same relationship as the one holding between a collection of objects and objects being part of that collection. Another way to explain the analogy would be to assume that partitive expressions such as *Teil* employ a derived notion of parthood which generalizes over two distinct yet corresponding part-structures (Barker 1998, Ionin et al. 2006). However, before I elaborate more on this issue, let us examine to what extent the analogy between entity and set partitives is valid from a wider cross-linguistic perspective.

2.2.1 Entity and set partitives across languages

For some reason, the analogy between explicit entity and set partitives does not hold in English. As observed by Schwarzschild (1996, pp. 165–166), there is a contrast between (2.14a) and (2.14b), see also (2.15).[9] The phrase in (2.15b) is not a well-formed English expression and in order to refer to a subset of the relevant apples one needs to use a

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9. Similar contrasts have been reported by Chierchia (2010).

partitive construction such as (2.15c) where the existential quantifier *some* is employed as the upstairs determiner. Thus, if English were like German, it would be possible to use the sentence in (2.14b) to describe a situation where some of the boys were in Texas. But the sentence is odd. Schwarzschild concludes that since English *part of* cannot co-occur with count plurals, it selects for singularity-denoting complements exclusively.

(2.14)   English (Schwarzschild 1996)

      a.   Part of the car was painted.
      b. #Part of the boys were in Texas.
      c.   Part of the group was in Texas.

(2.15)   English (Guy Tabachnick, p.c.)

      a.   part of the apple
      b. #part of the apples
      c.   some of the apples

Schwarzschild (1996) examines explicit entity and set partitives for a particular reason. On the basis of the contrast between (2.14b) and (2.14c) and other facts concerning predicate non-sharing, he argues against analyses positing that group nouns denote impure atoms.[10] However, what is interesting from the perspective of this study are some possible consequences of the data in (2.14) and (2.15) for the ontology encoded in that part of natural language semantics that deals with parthood. Specifically, what is implied by the argument put forward by Schwarzschild is that singularities and pluralities involve two distinct part-whole structures or, in other words, there is a fundamental difference between how we conceptualize the relationship between parts of an object and that object and members of a set of objects and that set. In fact, that is the received view in theories of parthood (e.g., Link 1983, Simons 1987). On the discussed approach, English *part of* is modeled as an existential 'pieces' quantifier in a sense reverse of *each*. Simplifying a bit, *part of* only takes arguments denoting singularities and ensures that the intersection of the sets of parts of a singularity with the set corresponding to the verbal predicate is non-empty.

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10. See also de Vries (2015) for a different approach building on the same conclusion.

Whatever the reason English does not allow for explicit set partitives, i.e., partitives headed by *part* with embedded plural DPs, it is not an ordinary behavior from a typological point of view. On the contrary, the analogy introduced in (2.12) holds in multiple languages displaying the singular/plural distinction including at least representatives of such diverse language families as Germanic, Slavic, Romance, Celtic, Ugro-Finnic, Semitic, and Basque, see (2.16)–(2.23). In all the languages listed below, explicit partitives allow for both singular and plural DPs. For instance, (2.16a) is an example of the Dutch explicit entity partitive on a par with the English construction in (2.15a). Similar to German, see (2.12b), in order to quantify over parts of a plurality, i.e., particular apples, the same partitive word *deel* 'part' can be used with the plural DP, as witnessed by the felicity of (2.16b). The same holds in other Indo-European languages such as Polish, see (2.17), Russian, see (2.18), Italian, see (2.19), Portuguese, see (2.20), and Irish, see (2.20). Moreover, the same phenomenon can be observed outside the Indo-European language family. For instance, in an agglutinative language such as Hungarian both explicit entity partitives and explicit set partitives are possible, see (2.22). Similar, the Hebrew explicit entity partitive construction in (2.23a) refers to a set of parts of the relevant boy, i.e., his body parts, whereas the phrase with the same *xelek* 'part' expression in (2.23b) would be true of some of the boys. Finally, as attested in (2.24) the analogy holds in the isolate Basque language.[11]

(2.16)   Dutch (Erlinde Meertens, p.c.)

    a.   deel van de appel
         part of   the apple
         'part of the apple'

    b.   deel van de appels
         part of   the apples
         'some of the apples'

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11. That seems to be the case at least in the Biscayan dialect. The speakers of the Gipuzkoan dialect I questioned disagree. Many thanks to Aitor Lizardi Ituarte, Gillen Martinez de la Hidalga, and Itziar Orbegozo Arrizabalaga for their judgments and to Katie Frasier for putting me in touch with them.

(2.17)   Polish

    a.   część jabłka
        part   apple_{GEN}
        'part of the apple'

    b.   część jabłek
        part   apples_{GEN}
        'some of the apples'

(2.18)   Russian (Tetiana Kamyshanova, p.c.)

    a.   čast' jabloka
        part apple_{GEN}
        'part of the apple'

    b.   čast' jablok
        part apples_{GEN}
        'some of the apples'

(2.19)   Italian (Schwarzschild 1996, p. 186; adapted)

    a.   parte del   muro
        part   of-the wall
        'three parts of the wall'

    b.   parte dei   muri
        part   of-the walls
        'some of the walls'

(2.20)   Portuguese (Muriel Assmann, p.c.)

    a.   parte da maçã
        part   the apple
        'part of the apple'

    b.   parte das maçãs
        part   the apples
        'some of the apples'

(2.21)   Irish (Radvan Markus, p.c.)

    a.   cuid den   úll
        part from-the apple
        'part of the apple'

    b.   cuid de   na húlla
        part from the apples
        'some of the apples'

(2.22) Hungarian (Balázs Surányi, Lilla Pintér, p.c.)

    a. az alma egy része
       the apple a   part$_{\text{POSS}}$
       'part of the apple'

    b. az almák egy része
       the apples a   part$_{\text{POSS}}$
       'some of the apples'

(2.23) Hebrew (Schwarzschild 1996, p. 184; adapted)

    a. xelek me-ha-baxur
       part   from-the-boy
       'part of the boy'

    b. xelek me-ha-baxur-im
       part   from-the-boy-s
       'some of the boys'

(2.24) Basque (Aitor Lizardi Ituarte, p.c.)

    a. sagarraren zati bat
       apple$_{\text{GEN}}$ part a
       'part of the apple'

    b. sagarren   zati bat
       apples$_{\text{GEN}}$part a
       'some of the apples'

A similar phenomenon can be observed in partitives involving proportional expressions including *half*-words and fractions. This time also English does show the discussed pattern. For instance, *half*, *most*, and *two thirds* in (2.25a), (2.26a), and (2.27a), respectively, quantify over parts of a singular entity, i.e., yield a portion of stuff constituting the whole, whereas in (2.25b), (2.26b), and (2.27b) they quantify over parts of a plural individual, i.e., indicate a subset of individuals making up the total plurality.

(2.25) English

    a. half of the apple
    b. half of the apples

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(2.26) English
   a. most of the apple
   b. most of the apples

(2.27) English
   a. two thirds of the apple
   b. two thirds of the apples

Again, this kind of pattern is attested cross-linguistically, as witnessed in the examples from Polish in (2.28)–(2.29) and German in (2.31)–(2.33). In all the phrases provided below, the proportional quantifier can quantify either over material parts of the relevant object, i.e., apple, or over individual objects, i.e., apples, making up the whole plurality.

(2.28) Polish

   a. połowa jabłka
       half   apple.gen
       'half of the apple'
   b. połowa jabłek
       half   apples.gen
       'half of the apples'

(2.29) Polish

   a. większość jabłka
       most   apple$_{GEN}$
       'most of the apple'
   b. większość jabłek
       most apples$_{GEN}$
       'most of the apples'

(2.30) Polish

   a. dwie trzecie jabłka
       two   thirds apple$_{GEN}$
       'two thirds of the apple'
   b. dwie trzecie jabłek
       two   thirds apples$_{GEN}$
       'two thirds of the apples'

(2.31)   German (Nina Haslinger, p.c.)

    a.   Hälfte des   Apfels
       half   the$_{\text{GEN}}$ apple$_{\text{GEN}}$
       'half of the apple'

    b.   Hälfte der   Äpfel
       half   the$_{\text{GEN}}$ apples$_{\text{GEN}}$
       'half of the apples'

(2.32)   German (Viola Schmitt, p.c.)

    a.   ein Großteil des   Apfels
       a   most   the$_{\text{GEN}}$ apple$_{\text{GEN}}$
       'most of the apple'

    b.   ein Großteil der   Äpfel
       a   most   the$_{\text{GEN}}$ apples$_{\text{GEN}}$
       'most of the apples'

(2.33)   German (Viola Schmitt, p.c.)

    a.   zwei Drittel des   Apfels
       two   thirds the$_{\text{GEN}}$ apple$_{\text{GEN}}$
       'two thirds of the apple'

    b.   zwei Drittel der   Äpfel
       two   thirds the$_{\text{GEN}}$ apples$_{\text{GEN}}$
       'two thirds of the apples'

Furthermore, it is well known that in many languages displaying the singular/plural distinction, there are classes of number-neutral nominals referring to individuated discrete entities such as object mass nouns (known also as fake or neat mass nouns; see, e.g., Gillon 1992, Chierchia 1998a, 2010, Barner and Snedeker 2005, Bale and Barner 2009, Landman 2011) and pluralia tantum, i.e., plural nouns without singular counterparts (e.g., Wierzbicka 1988, Corbett 2000, Koptjevskaja-Tamm and Wälchli 2001, Wisniewski 2010; see also Wągiel 2015a, 2017). The examples of the first category in English are *jewelry*, *furniture*, and *mail*, whereas *scissors*, *eyeglasses*, and *pants* fall into the second class. Both types of expressions are systematically ambiguous between a singular and a plural reading, i.e., they can either refer to one individual or to a sum of individuals. For instance, *furniture* can denote either one or many pieces of furniture. Consequently, (2.34a) would be true if there were, say, one wardrobe in the room as well as if there were

several chairs or a chair, a table, and a wardrobe in the room. Similar, since *scissors* is number-neutral, (2.34b) would be true either if there were one or many utensils on the table.

(2.34) English
      a.   There is furniture in the room.
      b.   There are scissors on the table.

Interestingly, in many languages partitives including number-neutral nominals such as object mass nouns and pluralia tantum are ambiguous between a part-of-a-singularity and part-of-a-plurality reading. For instance, the Czech phrase in (2.35a) would be true of a part of a singular shoe in question as well as of a subset of the relevant shoes. Similar, (2.35b) can either refer to a part of one utensil or it can mean a subset of the total number of the relevant scissors. The same ambiguity appears in proportional partitives such as (2.36a) and (2.36b).

(2.35) Czech (Hana Strachoňová, p.c.)
      a.   část obuvi
           part footwear$_{\text{GEN}}$
           'part of the footwear/some of the footwear'
           (i)   part-of-a-singularity reading
           (ii)   part-of-a-plurality reading
      b.   část nůžek
           part scissors$_{\text{GEN}}$
           'part of the scissors/some of the scissors'
           (i)   part-of-a-singularity reading
           (ii)   part-of-a-plurality reading

(2.36) Czech (Hana Strachoňová, p.c.)
      a.   polovina obuvi
           half   footwear$_{\text{GEN}}$
           'half of the footwear'
           (i)   part-of-a-singularity reading
           (ii)   part-of-a-plurality reading

b.   polovina nůžek
     half   scissors$_{\text{GEN}}$
     'half of the scissors'
     (i)   part-of-a-singularity reading
     (ii)   part-of-a-plurality reading

A similar effect is reported to appear in some constructions in languages with general number such as Japanese, as originally observed for proportional partitives by Sauerland and Yatsushiro (2004). For instance, (2.37) would be true both in a scenario where John read a number of books which is greater than a half of the total number of books and in a scenario where John read more than a half of one book.

(2.37)   Japanese (based on Sauerland and Yatsushiro 2004)
         John-wa hotondo hon-o   yomi-oeta
         John-TOP most   book-ACC read-finished
         'John finished reading most of the books/most parts of the book(s).'
         a.   part-of-a-singularity reading
         b.   part-of-a-plurality reading

Likewise, Watanabe (2013) discusses pairs of constructions involving both explicit and proportional partitives such as those provided in (2.38)–(2.40). Different linearizations in the doublets correlate with different interpretations. Constructions in which partitive words follow nominals are ambiguous between a part-of-a-singularity and a part-of-a-plurality reading, whereas sentences with partitive words preceding the noun unambiguously refer to parts of a singular object. In particular, (2.38a) can either mean that a part of a single apple in question is rotten or that a subset of all the relevant apples is rotten. Similar, (2.39a) and (2.40a) mean that either more than half of one apple is rotten or that the number of rotten apples exceeds a half of the total number. On the other hand, the partitives with the reversed order in the sentences in (2.38b), (2.39b), and (2.40b) do not give rise to such an ambiguity and can only make reference to subsets of a larger set, i.e., some of the apples and most of the apples, respectively.

(2.38)   Japanese (Watanabe 2013; adapted)
    a.   Ringo-no ichibu-ga kusatteiru.
        apple-GEN part-NOM is-rotten
        'Part of the apple is rotten/Some of the apples are rotten.'
        (i)   part-of-a-singularity reading
        (ii)   part-of-a-plurality reading
    b.   Ichibu-no ringo-ga   kusatteiru.
        part-LNK   apple-NOM is-rotten
        'Some of the apples are rotten.'
        (i)   part-of-a-singularity reading
        (ii) #part-of-a-plurality reading

(2.39)   Japanese (Watanabe 2013)
    a.   Ringo-no hotondo-ga kusatteiru.
        apple-GEN most-NOM   is-rotten
        'Most of the apple(s) is/are rotten.'
        (i)   part-of-a-singularity reading
        (ii)   part-of-a-plurality reading
    b.   Hotondo-no ringo-ga   kusatteiru.
        most-LNK   apple-NOM is-rotten
        'Most of the apples are rotten.'
        (i)   part-of-a-singularity reading
        (ii) #part-of-a-plurality reading

(2.40)   Japanese (Watanabe 2013)
    a.   Ringo-no dai-bubun-ga   kusatteiru.
        apple-GEN large-part-NOM is.rotten
        'Most of the apple(s) is/are rotten.'
        (i)   part-of-a-singularity reading
        (ii)   part-of-a-plurality reading
    b.   dai-bubun-no ringo-ga   kusatteiru.
        large-part-LNK apple-NOM is.rotten
        'Most of the apples are rotten.'
        (i)   part-of-a-singularity reading
        (ii) #part-of-a-plurality reading

The data discussed above suggest that in many languages *part*-words are able to simultaneously access elements making up pluralities of individuals as well as subatomic part-whole structures. Of course,

there is no reason to believe that *part*-words are simply reducible to the mereological partood relation, just as there is no reason to believe that the noun *member* is the natural language exponent of the set-theoretic membership relation (see Champollion 2010, 2017). However, it is tacit to assume that such expressions do employ a notion of parthood which might be modeled in terms of some primitive formal relation. For theories that sort domains, and thus distinguish strictly between individual and material parthood (e.g., Link 1983), the cross-linguistic behavior of partitive words might be seen as problematic to account for in a systematic manner. On the other hand, as will be discussed in Chapter 7 postulating a unified ontology for singular objects and pluralities thereof may encounter a number of problems to account for.

I believe that at this point an empirical conclusion can be drawn. Though the sample of languages and constructions discussed in this section definitely does not exhaust the cross-linguistic diversity in the domain of explicit and proportional partitives (nor was it intended to), I conclude that the pattern involving analogous structures for explicit and proportional entity and set partitives is robust. In multiple typologically and genetically different languages the same partitive word can be used to designate both parts of a single individual and subsets of objects constituting a larger plurality. And though the question whether this fact tells us something non-trivial about the way discrete objects and sums thereof are conceptualized or rather something interesting about partitive words themselves remains somewhat open, I believe that systematic patterns should not be neglected but instead guide development of a proper approach. The next section will provide a potentially problematic observation concerning different properties of *part*-words in explicit entity and set partitives.

2.2.2 Partitivity and countability

At first blush, the cross-linguistically attested distribution of *part*-words as well as other proportional expressions in entity and set partitives discussed in the previous section suggests at least a strong correspondence between subatomic and superatomic part-whole structures. The fact that the very same expression can be used to quantify over parts of a singular individual as well as over parts of a plural-

ity in multiple typologically different and often unrelated languages suggests that an ambiguity approach postulating two different lexical entries for the relevant proportional expressions is quite improbable. To the contrary, it seems that something more essential is happening here and a more detailed examination of the phenomenon might shed new light on the properties of that component of the language faculty that is dedicated to the interpretation of part-whole relations.

However, Schwarzschild (1996) presents data that might seem to be a counterargument for an approach postulating a semantics operating on a unified part-whole structure that would allow for both quantification over wholes as well as subatomic quantification (cf. Moltmann 1997, 1998). The crucial observation concerns the fact that *part*-words in explicit set partitives have different properties than *part*-words occurring in entity partitive constructions. Specifically, such expressions are countable only if they refer to material parts of a singular object. For instance, consider the contrast between the phrases in (2.41). The count explicit entity partitive in (2.41a) comprises the Italian pluralized *part*-word *parti* 'parts' modified by the cardinal numeral *tre* 'three'. What the phrase can only denote is a set of pluralities consisting of three parts of different individual walls, e.g., a set including a sum of a part of the wall w_1, a part of the wall w_2, and a part of the wall w_3. Crucially though, (2.41a) cannot refer to parts of a plurality, i.e., three individual walls or three pluralities of walls. To illustrate the distinction, let us assume that there are 10 relevant walls: $w_1, \ldots, w_10$. In such a scenario, (2.41b) is not felicitous on the reading where one is counting parts of a plurality, e.g., it would not be true of the set of three walls $\{w_1, w_2, w_3\}$ or the set of three subsets of the walls $\{\{w_1, w_2\}, \{w_3, w_4, w_5\}, \{w_6, w_7\}\}$.

(2.41)   Italian (Schwarzschild 1996, p. 186; adapted)

    a.   tre   parti del   muro
           three parts of-the wall
           'three parts of the wall'
    b.   tre   parti dei   muri
           three parts of-the walls
           (i)   part-of-a-singularity reading
           (ii) #part-of-a-plurality reading

This phenomenon is by no means an Italian idiosyncrasy. To the contrary, it is attested cross-linguistically, as will be demonstrated in the the following examples. For instance, the very same pattern can be found in Portuguese. Similar to the interpretation of (2.41b), the sentence (2.42b) would be true if, say, a brick of the wall w_1 were red, two bricks of the wall w_2 were red, and three bricks of the wall w_3 were red. However, given the scenario with the eight relevant walls $w_1, \ldots, w_8$ introduced above it would not be considered true if, e.g., w_1, w_2, and w_3 were all painted red, nor if three subsets of the walls, e.g., $\{w_1, w_2, w_3\}, \{w_4, w_5\}, and \{w_6, w_7\}$, were all painted red.

(2.42)   Portuguese (Muriel Assmann, p.c.)

    a.   Três   partes do muro são vermelhas.
            three parts   the wall   are red
            'Three parts of the wall are red.'
    b.   Três   partes dos muros são vermelhas.
            three parts   the walls   are red
            (i)   part-of-a-singularity reading
            (ii) #part-of-a-plurality reading

Such a pattern is also attested outside Romance. For instance, the German sentence in (2.43a) is true if there are three green spots on the relevant apple. However, (2.43b) cannot mean that three subsets of the apples are green. Rather, the only possible interpretation of the sentence is that three parts of different apples are green.

(2.43)   German (Nina Haslinger, p.c.)

    a.   Drei Teile des   Apfels   sind grün.
            three parts the_GEN apple_GEN are   green
            'Three parts of the apple are green.'
    b.   Drei Teile der   Äpfel   sind grün
            three parts the_GEN apples_GEN are   green
            (i)   part-of-a-singularity reading
            (ii) #part-of-a-plurality reading

A similar effect is found in Slavic, e.g., (2.44) illustrates the relevant contrast in Polish. Again, the count explicit set partitive does not quantify over sub-pluralities of the apples in question, but rather over parts

of individual apples. The sentence in (2.44b) does not get a part-of-a-plurality interpretation.[12]

(2.44) Polish

    a. Trzy części jabłka   są zielone.
        three parts apple$_{GEN}$ are green
        'Three parts of the apple are green.'

    b. Trzy części jabłek   są zielone.
        three parts apples$_{GEN}$ are green
        (i)   part-of-a-singularity reading
        (ii) #part-of-a-plurality reading

Furthermore, there two distinct constructions in Japanese that display the same patter, see (2.45). Despite the fact that the noun *ringo* 'apple' is number-neutral and as such it can either refer to a single apple or to a collection of fruit, the count explicit partitive in (2.45a) where the *part*-word *bubun* is modified by a numeral forces a part-of-a-singularity reading.[13] The sentence simply cannot mean that three different subsets of whole apples are damaged. Similar, the use of the classifier construction involving a numeral and the classifier *kasyo* in oder to quantify over parts of referents of a number-neutral noun excludes a part-of-a-plurality reading. As witnessed in (2.45b), the only possible interpretation involves subatomic quantification, i.e., three parts of a singularity are damaged.

(2.45) Japanese (Yasutada Sudo, p.c.)

    a. Ringo-no mit-tsu-no   bubun-ga itandeiru.
        apple-GEN three-CL-GEN part-NOM damaged
        'Three parts of the apple are damaged.'
        (i)   part-of-a-singularity reading
        (ii) #part-of-a-plurality reading

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12. Notice that there is yet another reading of modified set partitives with DPs denoting food terms in Polish. It involves the count-mass shift and indicates a fraction of volume, e.g., three-fourths of the total volume of the apples. I ignore this interpretation here, but I will return to it in Section 3.3.

13. Again, for the sake of simplicity I ignore here the attested distributive reading on which there are multiple apples and three parts of each of them are damaged.

b. Ringo-no san-kasyo-ga itandeiru.
apple-GEN three-CL-NOM damaged
'Three parts of the apple are damaged.'
(i)   part-of-a-singularity reading
(ii) #part-of-a-plurality reading

The effect is even stronger if we consider partitives with animate DPs. For instance, notice the contrast between (2.46a) and (2.46b) in Italian. The first is felicitous if a subset of the relevant boys were in Texas. However, the second sentence implies an organ trafficking context since the only possible interpretation one can get is that three body parts belonging to the boys were in Texas. A similar effect holds in Polish. While the sentence involving the explicit set partitive in (2.47a) is perfectly fine and simply means that some of the relevant boys sleep, its modified counterpart in (2.47b) is nonsensical since it asserts that three body parts sleep. Unless one considers a macabre scenario in the spirit of the Addams Family with sentient animate disembodied hands, body parts cannot be experiencers of a sleeping state, and thus the awkwardness of the sentence.

(2.46)   Italian (Schwarzschild 1996, p. 186; adapted)

a. Parte dei   ragazzi erano in Texas.
part of-the boys   were in Texas
'Some of the boys were in Texas.'

b. #Tre   parti dei   ragazzi erano in Texas.
three parts of-the boys   were in Texas

(2.47)   Polish

a. Część chłopców śpi.
part boys$_{\text{GEN}}$ sleeps
'Some of the boys sleep.'

b. #Trzy części chłopców śpią.
three parts boys$_{\text{GEN}}$ sleep

Moreover, the phenomenon is not restricted to explicit partitives modified by cardinal numerals but holds also for constructions involving other types of quantifiers. For instance, let us consider the examples in (2.48) where the referents of proportional partitives are arguments of a predicate involving the total adjective *czysty* 'clean' (Yoon 1996, Rot-

stein and Winter 2004). Similar to adjectives such as *full*, *flat*, and *closed*, *clean* is an absolute gradable adjective that indicates the maximal degree of the property in question (Unger 1975, Kennedy and McNally 2005). In other words, *clean* is true of an entity that is completely clean or at least the degree of cleanliness falls near the maximal value on the scale. Let us now contemplate the contrast between (2.48a) and (2.48c) on the one hand and (2.48b) and (2.48d) on the other. The first two sentences involve regular proportional partitives which quantify over whole individuals. Therefore, if there are six relevant toys $t_1, \ldots,$ t_6 in total, (2.48a) and (2.48c) would be true if three of the six toys, e.g., t_1, t_2, and t_3, or at least four of the six toys, e.g., t_1, t_2, t_3, and t_4, were clean, respectively. However, when an explicit set partitive is modified by proportional quantifiers such as *połowa* 'half' and *większość* 'most' the whole phrase does not quantify over wholes anymore but rather over parts of singular individuals. Hence, (2.48b) and (2.48d) can be true even if it is not the case that a half or most of the toys are clean, respectively. In fact, they might be true if none of the toys is actually clean. Imagine a scenario where there are six teddy bears and one paw of each of them is dirty. It follows that none of the teddy bears is clean though most of their parts are clean.

(2.48)   Polish

  a. Połowa zabawek jest czysta.
    half toys$_{\text{GEN}}$ is clean
    'A half of the toys are clean.'

  b. Połowa części zabawek jest czysta.
    half parts$_{\text{GEN}}$ toys$_{\text{GEN}}$ is clean
    'A half of the parts of the toys are clean.'
    (i) part-of-a-singularity reading
    (ii) #part-of-a-plurality reading

  c. Większość zabawek jest czysta.
    most toys$_{\text{GEN}}$ is clean
    'Most of the toys are clean.'

  d. Większość części zabawek jest czysta.
    most parts$_{\text{GEN}}$ toys$_{\text{GEN}}$ is clean
    'Most of the parts of the toys are clean.'
    (i) part-of-a-singularity reading
    (ii) #part-of-a-plurality reading

Similar, the same distinction can be detected in contexts regarding universal quantification. Though in such environments it is hard to distinguish between quantification over parts of singularities and quantification over parts of pluralities, the difference becomes evident if an adverbial modifier such as *almost* is used. Both (2.49a) and (2.49b) can be true even if none of the toys is completely clean. Since it is sufficient for an object be considered dirty if only one of its parts is dirty, one can imagine a scenario where almost every part of the relevant six teddy bears is clean and yet the teddy bears are dirty. And again, modified explicit partitives do not quantify over wholes but rather over parts of wholes despite the fact that the downstairs DP is plural.

(2.49)   Polish

    a.   Prawie każda część zabawek jest czysta.
        almost every part toys$_{\text{GEN}}$ is clean
        'Almost every part of the toys is clean.'
        (i)   part-of-a-singularity reading
        (ii) #part-of-a-plurality reading

    b.   Prawie wszystkie części zabawek są czyste.
        almost all parts toys$_{\text{GEN}}$ are clean
        'Almost all parts of the toys are clean.'
        (i)   part-of-a-singularity reading
        (ii) #part-of-a-plurality reading

A brief investigation into the cross-linguistic behavior of partitive words suggests that the discussed pattern is robust. Table 2.1 summarizes the observed semantic behavior. Both unmodified and modified partitive words c-commanding singular DPs quantify over material parts of the relevant entity, whereas the domain of quantification for partitive words c-commanding plural DPs shifts when they combine with quantifiers. Specifically, although explicit partitives with unmodified partitive words quantify over parts of a plurality, in the presence of a quantifier they are no longer able to count whole individuals and instead operate on the subatomic level. In fact, it appears that there are no count explicit set partitives since apparently no partitives quantify over subsets of the set denoted by the downstairs DP in the first place. Rather, constructions such as (2.41b) in Italian are a special type of

entity partitive with the only difference that material parts do not belong to one object as in (2.41a) but are distributed onto a plurality of individuals. This is puzzling because it suggests that despite the apparent homophony *part*-words in explicit entity partitives and count explicit partitives as well as *half*-words in corresponding proportional partitives are in fact different semantic expressions.

	SINGULARS		PLURALS	
	bare	count	bare	count
subatomic quantification	✓	✓	*	✓
quantification over wholes	*	*	✓	*

Table 2.1: Properties of partitive words

Based on the discussed contrasts in Italian, Schwarzschild (1996) concludes that the uncountability of *part*-words in set partitives reveals that Italian and English do not disagree with respect to their ontologies. Since English *part of* selects for singularity-denoting complements and cannot co-occur with count plurals, it is modeled as an existential 'pieces' quantifier in his system. Presumably Italian *parte* 'part' in entity partitives and count explicit partitives, see (2.19a) and (2.41a), respectively, is an expression of the same type, whereas *parte* in set partitives, see (2.19b) is a quantifier of a different sort.

As mentioned before, Schwarzschild (1996)'s argument concerns primarily extensions of group nouns. However, what his novel observation further corroborated in this section seems to imply is that in diverse languages singular individuals and pluralities are associated with distinct mereological structures. This can be seen as a valid counterargument for an approach attempting to unify part-whole relations corresponding to extensions of singulars and plurals such as the theory of Moltmann (1997, 1998) if the main empirical motivation concerns the morphological parallelisms in quantification in entity, set, and mass partitives.

However, there are two interrelated issues concerning an approach that posits multiple lexical entries for *part*-words to explain their nontrivial quantificational behavior. One problem has to do with cross-linguistic correspondences. Since the same *part*-word is often used both in entity and set partitives cross-linguistically as we have seen

in the previous section, a homophony would have to be postulated in multiple diverse languages. Though of course it is not a logical impossibility, it seems very implausible that languages in general do not differentiate morphologically between the two expressions in question.

Another issue concerns intralinguistic systematicity. As witnessed by the well-formedness of English proportional and fraction partitives such as those in (2.25)–(2.27), partitive words such as *half* and *most* as well as fractions such as *two thirds* can be used both in entity and set partitives where they get a part-of-a-singularity and part-of-a-plurality reading, respectively. Again, as we have seen such distribution holds cross-linguistically. Interestingly, proportional quantifiers pattern with *part*-words when modified by numerals. For instance, consider the Polish sentences in (2.50a) and (2.50b). In the first the count explicit partitive quantifies over material parts of the relevant wall and the sentence is truth-conditionally equivalent with a statement that the whole wall is red. Nonetheless, the count explicit partitive in the second sentence does not mean that all the walls are red but rather that among the walls there two such that they are half red-painted. Despite the downstairs plural DP, the whole phrase is not interpreted as a set partitive and similar to (2.50a) it quantifies over parts of distinct wholes.

(2.50)   Polish

      a.   Dwie połowy muru są czerwone.
           two   halves   wall$_{\text{GEN}}$ are red
           'Two halves of the wall are red.'
      b.   Dwie połowy murów są czerwone.
           two   halves   walls$_{\text{GEN}}$ are red
           'Two halves of the walls are red.'
           (i)   part-of-a-singularity reading
           (ii) #part-of-a-plurality reading

Moreover, the data in (2.51) and (2.52) further corroborate the contrast. Since the quantifier *both* combines universal quantification with an inference that the cardinality of counted objects equals two (Barwise and Cooper 1981, Schwarzschild 1996, pp. 139–145), in most cases

it can be paraphrased by the definite numeral expression *the two*.[14]
Hence, the example in (2.51a) simply states that the two halves of the
wall are red, i.e., that all the halves are red and that there are only two
halves. However, when one tries to process (2.51b), it gets confusing.
Of course, if the modified partitive quantified over whole walls, it
would not but it is not what the phrase means. Instead, *obie* 'both'
implies that the total number of halves is two, whereas at the same
time the plural DP infers that there are at least two walls, thus at least
four halves, which results in a contradictory statement.

(2.51)   Polish

  a. Obie połowy muru są czerwone.
    both halves wall$_{GEN}$ are red
    'Both halves of the wall are red.'

  b. #Obie połowy murów są czerwone.
    both halves walls$_{GEN}$ are red

Similar, since it is only possible for an object to have two halves, the
sentence in (2.52a) is strange because it asserts that the wall has more
than two parts forming 50% of its material constitution each. On the
other hand, there is nothing awkward about (2.52b). The sentence
does not state that there are more walls than there are walls but rather
that among all the walls in question there are three walls half-covered
in red paint.

(2.52)   Polish

  a. #Trzy połowy muru są czerwone.
    three halves wall$_{GEN}$ are red

  b. Trzy połowy murów są czerwone.
    three halves walls$_{GEN}$ are red
    'Three halves of the walls are red.'

At this point, I conclude that if one pursues an approach explaining
the discussed behavior in terms of ambiguity, one is forced to assume
the cross-linguistic as well as intra-linguistic patterns in question are
just a remarkable coincidence. Nevertheless, my own view is that

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14. But see Ladusaw (1982) for discussion of the asymmetry between the two quantifiers in partitives.

systematic parallelism is something that should be approached with serious consideration since it may reveal some deep aspects of the nature of natural language semantics. For instance, the distinction between collective, distributive, and cumulative understandings of sentences involving semantically plural DPs does hold in multiple environments in multiple languages and the research on its roots has lead to many prominent theories in the study of pluralities (e.g., Scha 1981, Link 1983, 1984, Hoeksema 1983, Gillon 1987, Landman 1989a,b, 2000, Schwarzschild 1991, 1996, Schein 1993, Lasersohn 1995, Beck and Sauerland 2000, Winter 2001, Champollion 2010, 2017, Dotlačil 2010, Schmitt 2013, Zweig 2008). I would like to suggest that the discussed behavior of *part*-words modified by cardinal numerals and other quantifiers can shed new light on the matter of countability. Therefore, I will not dismiss hastily the morphological evidence concerning the analogy between entity and set partitives but rather I will argue that the parallelism is not coincidental.

Crucially, there is an intriguing twist concerning count explicit partitives suggesting that the phenomenon discussed in this section results from an independent factor, specifically from the type of entities plural expressions denote. In particular, the extensions of regular plurals comprise arbitrary sums of individuals, i.e., scattered entities which bear no topological commitments with respect to the configuration of the parts of a plurality and it seems that natural language does not consider such entities to be units one could quantify over. But what if there were a language with a plural that similar to, say, the German plural denotes sums of individuals but in addition asserts a specific spatial relation holding between individuals making up a plurality that guarantees that such a sum has object-like properties? If the claim relating countability with being an integrated entity is on the right track, one would expect that when *part*-words heading partitives involving such untypical plural expressions are modified by numerals the quantificational behavior of the whole phrase should differ from what we observed so far. The next section will examine the interaction between cardinals and partitives involving so-called irregular plurals in Italian.

2.3 Italian irregular plurals

There is a relatively small set of irregular nouns in Italian that display both morpho-syntactic and semantic peculiarities with respect to the singular/plural distinction. These nouns form an inflectional class whose defining characteristic is that it exhibits an idiosyncratic agreement pattern involving a gender shift in the plural. For instance, let us consider a representative of the class, e.g., the masculine noun *dito* 'finger'. As witnessed by the well-formed phrases in (2.53), the singular form *dito* is masculine, whereas the plural *dita* not only shows the irregular inflectional marker *-a* (regular masculine nouns end in *-i* in the plural), but also triggers feminine agreement (Acquaviva 2008, p. 125), as opposed to the regular pattern in (2.54). In particular, while *zii* 'uncles' takes the regular plural marker *-i* and combines with the masculine plural definite article *i* and masculine plural possessive pronoun *tuoi*, *dita* can only be modified by their feminine equivalents, i.e., *le* and *tue* respectively.

(2.53) Italian (Acquaviva 2008, p. 125)

 a. il   tuo   dito
    the$_{SG.M}$ your$_{SG.M}$ finger$_M$
    'your finger'

 b. le   tue   dita
    the$_{PL.F}$ your$_{PL.F}$ fingers$_F$
    'your fingers'

(2.54) Italian (Acquaviva 2008, p. 125)

 a. il   tuo   zio
    the$_{SG.M}$ your$_{SG.M}$ uncle$_M$
    'your uncle'

 b. i   tuoi   zii
    the$_{SG.M}$ your$_{SG.M}$ uncles$_M$
    'your uncles'

Forms such as *dita* are sometimes referred to in the literature as Italian irregular plurals in *-a* (see Acquaviva 2008 and references therein) or collective plurals, as opposed to regular distributive plurals (see Cor-

bett 2000 and references therein).[15] The morpho-syntax and semantics of such forms have been a subject of intense investigations both within traditional Italo-Romance linguistics (Hall 1956, Brunet 1978, pp. 30–90) as well as formally oriented approaches (Ojeda 1995, Acquaviva 2008, Kučerová 2015). Historically, the class arose as a result of the disappearance of Latin neuters in Italo-Romance (see Löfstedt 1956, pp. 46–51, Lausberg 1963, Rohlfs 1968), and thus all the nouns that belong to it are inanimates usually referring to body parts, series of events, measurement units, and mass concepts.[16] In the contemporary Italian it is restricted to approximately twenty nouns, but the pattern used to involve many more lexemes in the past (Santangelo 1981). Interestingly, the marker *-a* is nowhere else in the language the exponent of the plural and in at least some cases it seems to be responsible for an additional semantic effect to be discussed below.

The class of Italian irregular nouns can be divided into two groups. The first group consists of nouns that take irregular plural forms exclusively, i.e., do not pluralize in *-i* altogether. Table 2.2 lists some of the examples. Although plural forms such as *uova* 'eggs' and *centinaia* 'hundreds' are morphologically irregular, they do not seem to display any semantic idiosyncrasy. They simply do what regular plurals do, i.e., refer to pluralities of entities satisfying the property introduced by the nominal stem. With this respect, there seems to be no difference between irregular plurals and regular forms such as *pomodoro* 'tomato' ∼ *pomodori* 'tomatoes' and *miliardo* 'billion' ∼ *miliardi* 'billions'. In both cases, the pluralized noun merely denotes a collection of eggs or fruits and a number of hundreds or billions, respectively.

More interestingly, however, there are also a number of nouns with both regular and irregular plural counterparts such as those listed in Table 2.3.[17]

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15. Kučerová (2015) uses the terms the relational pattern and atomic pattern.

16. I will not discuss irregular measure nouns and mass terms here. Though I acknowledge that they might constitute a challenge for a unified semantic interpretation of the class (cf. Ojeda 1995 and Acquaviva 2008), this fact does not affect the core observation to be presented in this section.

17. Acquaviva (2008, pp. 124–129) introduces a more subtle classification by distinguishing also a class which does have the regular *-i* alternant but it is dialectally restricted, and thus not available for all speakers of Italian, e.g., *lenzuolo* 'sheet' ∼ *%lenzuoli* 'sheets' ∼ *lenzuola* 'sheets (bed linen)'. Moreover, he lists irregular pluralia tantum, i.e., feminine plural *-a* forms with no singular nor regular plural counter-

SINGULAR	REGULAR PLURAL	IRREGULAR PLURAL
uovo 'egg'	*uovi	uova 'eggs'
riso 'laughter'	*risi	risa 'pearls of laughter'
paio 'pair'	*paii	paia 'pairs'
centinaio 'hundred'	*centinai	centinaia 'hundreds'
miglio 'mile'	*migli	miglia 'miles'

Table 2.2: Italian nouns with irregular plurals exclusively (based on Acquaviva 2008, pp. 126–127)

SINGULAR	REGULAR PLURAL	IRREGULAR PLURAL
muro 'wall'	murri 'walls'	murra 'walls (in a complex)'
osso 'bone'	ossi 'bones'	ossa 'bones (in a skeleton)'
filo 'thread'	fili 'threads'	fila 'threads (in a fabric)'
fondamento 'basis'	fondamenti 'bases'	fondamenta 'foundations'
urlo 'shout'	urli 'shouts'	urla 'shouts (in a series)'

Table 2.3: Italian nouns with both regular and irregular plural counterparts

Though at least to some extent the effect seems to depend on a particular idiolect and register,[18] in the cases of doublets concurrent irregular feminine -*a* plurals tend to differ from run-of-the-mill masculine -*i* forms in meaning. As witnessed by the translations of irregular forms given in Table 2.3, there is often a sense of collectivity or cohesion in addition to the ordinary plural interpretation. For instance, the irregular form *ossa* 'bones' evokes an interpretation that the bones belong together as in a skeleton, whereas regular *ossi* 'bones' simply refers to a plurality of unrelated bones, see (2.55b) (Corbett 2000, p. 153). Similar, the only way to interpret *mura* 'walls' is by picturing a walled complex, e.g., a perimeter of city walls, see (2.55a) (Acquaviva 2008, p. 150). In both cases, there is a strong intuition that the referents

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parts, e.g., *vestigia* 'relics', as well as fossilized irregular plural forms found only in idiomatic expressions. For sake of clarity of the main argument, I will not discuss all the nuanced morphological and semantic intricacies here and I refer the reader to Acquaviva's study for more details.

18. See Acquaviva 2008 for a detailed discussion.

of the irregular plural forms are naturally related and due to this kind of semantic flavor Italian *-a* plurals have been analyzed as collectivizers of a particular sort (Ojeda 1995) or expressions inherently encoding the cohesion of referents (Acquaviva 2008).

Aware of all the subtleties within the class, I will focus my attention only on expressions following the second pattern in Table 2.4 (Acquaviva 2008). Specifically, plural forms of nouns that do not pluralize in *-i* at all get a regular semantic interpretation, i.e., merely refer to pluralities of individuals. On the other hand, the meaning of *-a* plural forms of nouns having doublets differ from the meaning of those in *-i*. In such cases, regular plurals are semantically regular, whereas irregular plurals denote pluralities of objects conceptualized as constituents of a larger cohesive entity.[19]

| SINGULAR | PLURAL | |
	regular	cohesive
-o	-a	*
-o	-i	-a

Table 2.4: Semantic interpretation of Italian plurals

In the first formal semantic treatment of Italian irregular plurals, Ojeda (1995) proposes an account on which expressions such as *braccia* 'arms' and *ginocchia* 'knees' do not denote mere sets of body parts but instead make reference to naturally occurring sets of arms or knees, i.e., pairs of arms or knees of one body.[20] The core idea of the approach is that nouns denote with a certain indeterminacy of individuation and that the plural is the unmarked number (Ojeda 1993). Hence, plural markers are mere identity functions which do not affect the reference of the stem. On the other hand, the singular restricts the denotation of the stem to the domain of atomic individuals. Within this approach, nominal stems in regular/irregular doublets are in fact homophonous

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19. Acquaviva (2008, pp. 151–152) argues against separating the two classes due to the fact that there exist irregular plurals without an *-i* counterpart that unlike singulars get a mass interpretation, e.g., *midolla* 'marrow'. For the sake of clarity, I will ignore such intricacies here.
20. In a sense, irregular plurals somewhat resemble duals in that they both tend to denote natural aggregates (see also Acquaviva 2008, p. 152 for discussion).

expressions having distinct denotations and the distribution of the plural markers *-i* and *-a* is constrained in that each of them can only combine with a semantic object of a particular type. In other words, *-a* does not have a different semantic value than *-i* but they differ in their input requirements. Specifically, the regular marker *-i* selects for stems denoting sets of all sets of singular referents including singletons, e.g., the set of all sets of individual arms. On the other hand, the irregular ending *-a* combines with stems that attribute a collective meaning associated with the plurality the whole noun refers to. In particular, *-a* selects for a homophonous stem denoting the set of natural aggregates of objects within the universe of discourse, e.g., the set of pairs of knees of each individual and pluralities formed by such pairs rather than arbitrary sums of knees.

However, Acquaviva (2008) rejects the view that the collectivizing effect is the characteristic for the whole class. Instead, he proposes that the common denominator of irregular plurals is that they denote entities conceptualized as weakly individual undifferentiated objects and argues for a lexicalist approach. In fact, Ojeda (1995) himself admits that the proper analysis of irregular plurals referring to mass concepts such as *cervello* 'brain' $\sim$ *cervelli* 'brains' $\sim$ *cervella* 'brains (mass)' is challenging for his account. Furthermore, as pointed out by Acquaviva a straightforward compositional analysis of the relationship between *-a* and the stem is problematic. One reason is that irregular plurals that lack regular *-i* alternants do not show any semantic peculiarity and a mechanism explaining a different selectional requirement would have to be postulated.

In order to avoid controversy I will focus on examples that indisputably invoke the sense of collectivity, specifically I will test the behavior of the irregular forms *mura* 'walls (in a complex)' and *ossa* 'bones (in a skeleton)', as demonstrated in the series of alternations in (2.55).[21]

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21. In (2.55), I use morpho-syntactic glosses. However, in the following part of the text I will gloss irregular plurals following the pattern as COLL.

(2.55)   Italian

    a.   muro $\sim$ muri $\sim$ mura
        wall_{SG.M} wall_{PL.M} wall_{PL.F}
        'wall $\sim$ walls $\sim$ walls (in a complex)'

    b.   osso $\sim$ ossi $\sim$ ossa
        bone_{SG.M} bone_{PL.M} bone_{PL.F}
        'bone $\sim$ bones $\sim$ bones (in a skeleton)'

In this study, I refrain from taking a stand regarding the issue of the internal syntactic and semantic structure of Italian irregular plurals and I remain agnostic with respect to the exact compositional contribution of the irregular marker *-a*, if any. However, I would like to suggest another perspective on reflecting the meaning of irregular forms falling into the pattern when they compete with regular *-i* plurals. In the next section, I will discuss novel data suggesting that at least a subset of irregular plurals is conceptualized as aggregates of entities related to each other by means of spatial connectedness.

2.3.1   Aggregate meaning

Given the discussed semantic evidence and previous accounts by Ojeda (1995) and Acquaviva (2008), it is plausible to posit that when an Italian plural in *-a* alternates with a regular *-i* form, its extension does not simply comprise arbitrary sums of individuals but rather a more complex type of entities. Specifically, I propose that the topological notion of connectedness or stable proximity of parts making up a plurality denoted by the irregular plural is involved in the way how the referents of such an expression are conceptualized. As already indicated in the previous section, at least in some cases there is a good reason to assume that the reported collective flavor is due to the fact that Italian irregular plurals encode integrated plural individuals, i.e., pluralities that unlike referents of regular plurals involve particular spatial relations holding between individual parts, and thus have the potential to form cohesive aggregates and even individuated wholes.

    For instance, Kučerová (2015) gives an example of the alternation between *dito* 'finger' $\sim$ *diti* 'fingers' $\sim$ *dita* 'fingers (of a hand)'.[22] If one

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22. In some varieties of Italian, the regular plural *diti* 'fingers' is either unavailable or available only marginally as a stigmatized form (Acquaviva 2008, p. 126).

refers to a plurality of fingers attached to a hand, then the irregular form *dita* would be used. On the other hand, when talking about detached fingers, e.g., in an anatomy class setting, the regular plural *diti* is the appropriate form to use. It seems that in patterns such as *dito* ~ *diti* ~ *dita* the irregular form indicates a particular type of relation holding between individual singular objects referred to by the noun, namely a topological relation giving rise to a cluster consisting of fingers.

To illustrate that, let us imagine a military context where a siege is led by the famous general Garibaldi who attempts to destroy the defensive fortification of the attacked city. Now, consider the sentences in (2.56). If (2.56a) is true, then there is no wall any more, just a pile of stones. Similar, uttering (2.56b) implies that there are no walls any more, just a number of piles of stones. However, (2.56c) can also mean something else, namely that the walls are not completely dismantled and though they probably received some damage during the siege, they are still recognizable as such, but importantly they are no longer connected. In other words, there is no complex of walls anymore and what is left is only a discontiguous collection of dissociated objects, i.e., independent walls.

(2.56)   Italian (Enrico Flor, p.c.)
a.   Garibaldi ha   smantellato il   muro.
     Garibaldi has dismantled   the wall
     'Garibaldi has dismantled the wall.'
b.   Garibaldi ha   smantellato i   muri.
     Garibaldi has dismantled   the walls
     'Garibaldi has dismantled the walls.'
c.   Garibaldi ha   smantellato le   mura.
     Garibaldi has dismantled   the walls$_{\text{COLL}}$
     'Garibaldi has dismantled the walled complex.'

The predicate *smantellare* 'dismantle' is what I call a verb of separation, i.e., a transitive expression sensitive to the part-whole structure of its first argument. When applied to a DP, it implies that the spatial connection between the parts of its referents has been undone or, in other words, the denoted individual has been disintegrated. Since referents of count singular nouns are arguably conceptualized as clusters,

i.e., pluralities of interrelated objects, if the complement of *smantellare* in a true sentence is a singular count DP, it follows that the entity denoted by that noun has ceased to exist as an integrated whole. To put it another way, the connections holding between its parts have been unbound and what is left is merely an arbitrary collection of pieces which do not form an individuated object anymore. On the other hand, regular plural nouns refer to arbitrary sums of objects, i.e., there is no sense in which parts of a plurality are related to each other other than by being members of a certain set. Denotations of regular plurals are constituted by scattered collections of singular individuals and in that sense plural individuals constitute a very different type of entity than referents of count singulars. Therefore, a verb of separation such as *smantellare* cannot disintegrate a plurality simply because there are no topological relations involved in the first place and instead it operates on the part-whole structures of particular singular individuals making up a plurality. As a result, what is obtained is a number of dismantled singular objects. However, irregular plurals such as *mura* denote cohesive aggregates, and thus arguably involve the notion of spatial connection on top of the plurality meaning. In other words, entities denoted by such expressions are pluralities but at the same time they are similar to singular objects in that their parts, i.e., members of a collection such as individual walls, constitute a cohesive entity. Consequently, verbs of separation can dissolve the topological structure connecting parts of a plurality and as a result another meaning arises, as witnessed by the possible interpretations of a sentence such as (2.56c).

Contrasts such as those in (2.56) show that the difference between *muri* and *mura* lies in that the first denotes a plurality of unrelated walls, whereas the latter makes reference to a cohesive aggregate of walls, i.e., walls making up a particular architectonic unit. In other words, the irregular plural encodes the cohesion of elements and often invokes the meaning of an integrated whole with a complex inner structure of associated parts connected to each other, while the regular *-i* form does not, and thus simply refers to an arbitrary sum of individuals of a particular kind.

This claim seems to be further strengthened by the fact that a significant number of nouns taking the irregular plural are words referring to body parts, i.e., entities naturally occurring in cohesive aggregates and

typically situated in great proximity, involving simultaneous movement, whose collections are often conceptualized as functional units. Another conspicuous subset includes nouns denoting objects typically consisting of multiple elements spatially associated with each other, e.g., city walls, foundations, and fabrics of threads, as well as a genuine group noun *paio* 'pair'. Yet another subclass involves nominals referring to series of events such as *grida* or *urla* 'shouts (in a series)'. Such expressions denote pluralities of events associated with each other, and thus making up a single complex eventuality. For instance, *urla* would be true of a series of cries by a single agent or a plural individual, e.g., audience, which are perceptually contiguous in time and/or space (Acquaviva 2008, pp. 149–150). Though the meaning of this subclass seems to be intuitively closely related to the semantic effect of connectedness discussed here, I will refrain from speculating how exactly the proposed interpretation could be extended to eventualities and simply note the analogy.

Before we move on to the discussion of the interaction between cardinals and partitives in Italian, let us conclude that a subset of irregular plurals refers to cohesive aggregates of objects associated with each other. The evidence concerning verbs of separation suggests that this association could be understood by means of a particular spatial configuration.

2.3.2   Interaction with cardinals in partitives

The core observation to be discussed in this section concerns the contrast between explicit partitives with regular plurals and explicit partitives with irregular plurals displaying the cohesive effect discussed above, e.g., *muri* ∼ *mura* and *ossi* ∼ *ossa*. As already discussed in Section 2.2.1, Italian is similar to many other languages in that it allows for both singulars and plurals in explicit partitives. Moreover, Italian displays the same pattern as, e.g., German with respect to the quantificational properties of partitive words in explicit and proportional partitives. Specifically, similar to count explicit entity partitives, see (2.57a), partitives involving plural DPs c-commanded by pluralized *parte* modified by a numeral such as in (2.57b) always quantify over material parts of objects and not over sets of wholes although unmodified set partitives get a part-of-a-plurality reading. Intriguingly, everything

changes in count explicit partitives involving *-a* forms following the discussed alternating pattern, as in (2.55). When an irregular doublet is swapped for a regular one, the range of possible interpretations significantly extends.

(2.57)   Italian (Enrico Flor, p.c.)

    a.   Due parti del   muro sono rosse.
        two parts of-the wall   are   red
        'Two parts of the wall are red.'

    b.   Due parti dei   muri sono rosse.
        two parts of-the walls are   red
        'Two parts of the walls are red.'
        (i)   part-of-a-singularity reading
        (ii) #part-of-a-plurality reading

    c.   Due parti delle   mura   sono rosse.
        two parts of-the walls$_{COLL}$ are   red
        'Two parts of the walled complex are red.'
        (i)   part-of-a-singularity reading
        (ii) part-of-a-plurality reading

For instance, imagine a scenario of a walled city whose fortifications consist of six walls $w_1, \ldots, w_6$ connected to each other in such a way that they form a hexagon, as depicted in Figure 2.1. Given that scenario, as one could expect the sentence in (2.57c) would be true if two parts of two distinct walls making up the complex were red, e.g., a part of w_1 and a part of w_3. But surprisingly unlike (2.57b) it would also be true if two distinct individual walls making up the complex were red, e.g., the wall w_1 and the wall w_3, or if two continuous sections of walls making up the complex were red, e.g., the subsets $\{w_1, w_2\}$ and $\{w_4, w_5\}$. In other words, the meanings that are impossible in explicit set partitives with regular plurals, are easily available with irregular plurals. However, as already mentioned there is a crucial constraint on possible interpretations of (2.57c). Namely, one can count parts of the plurality consisting of the walls in question as long as they form a contiguous section of the whole structure, i.e., (2.57c) would not be true of two arbitrary pluralities constituted by walls that are not in a

tangential relationship with each other. In particular, it would be false if, say, the wall w_1 as well as the walls $\{w_3, w_5\}$ were red.[23]
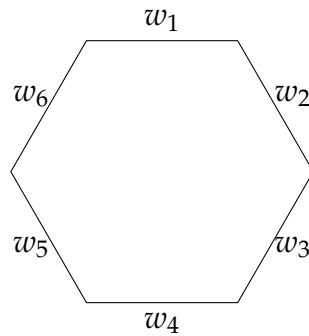
Figure 2.1: Sections of city walls

Similar, assume a bit morbid scenario involving a decaying human bone or skeleton and consider the contrasts between the examples in (2.58). Given that there is one relevant bone, the entity partitive in (2.58a) gets a part-of-a-singularity reading as usual, and thus the sentence would be true if there were three decayed spots on, e.g., a femur. This is not surprising and similar the interpretation of (2.58b) fits the discussed semantic shift regarding modified explicit set partitives. However, the partitive involving the irregular plural *ossa* in (2.58c) behaves differently. Seemingly, the most readily available interpretation of the example concerns three groups of connected bones making up the skeleton, e.g., it would be true of a decayed femur and knee, a decayed ulna and radius, and the decayed neck and skull. The sentence would be felicitous if three separate bones decayed though, e.g., a femur, ulna, and the skull. Furthermore, similar to (2.58b) it would be judged true if three were decayed parts on three distinct bones, e.g., a part of the femur, a part of the ulna, and a part of the skull. But again, the reading that (2.58c) cannot get involves three collections of disconnected bones. For instance, it would be infelicitous in a scenario where, say, a femur and the skull, a radius and a knee, and a rib and an ankle bone are decayed. That is because these pairs of bones do

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23. I assume here the 'exact *n*' semantics for numerals (e.g., Horn 1992, Geurts 1998, Breheny 2005; see also Geurts 2006 for an overview). Notice that the judgments do not change if the numeral is modified by overt *exactly*.

not form cohesive wholes, i.e., topologically continuous entities of a particular sort, and consequently cannot be conceptualized as units one could quantify over.

(2.58)   Italian (Enrico Flor, p.c.)
        a.   Tre   parti del   osso sono cariate.
             three parts of-the bone are   decayed
             'Three parts of the wall are decayed.'
        b.   Tre   parti degli ossi   sono cariate.
             three parts of-the bones are   decayed
             'Three parts of the bones are decayed.'
             (i)   part-of-a-singularity reading
             (ii) #part-of-a-plurality reading
        c.   Tre   parti delle ossa   sono cariate.
             three parts of-the bones$_{COLL}$ are   decayed
             'Three parts of the skeleton are decayed.'
             (i)   part-of-a-singularity reading
             (ii)   part-of-a-plurality reading

Table 2.5 summarizes the patterns we have observed so far. Specifically, the quantificational behavior of the Italian *part*-word *parte* depends on certain denotational properties of expressions it combines with.

	SINGULARS		REGULAR PL		IRREGULAR PL	
	bare	count	bare	count	bare	count
subatomic quantification	✓	✓	*	✓	✓	✓
quantification over wholes	*	*	✓	*	✓	✓

Table 2.5: Properties of Italian *parte* 'part'

The investigation into the interaction between cardinal numerals and partitives involving Italian irregular plurals leads to a somewhat surprising conclusion. It turns out that the cross-linguistic observation concerning different properties of partitive words in entity and set partitives as discussed in Section 2.2.2 actually does not provide evidence for distinct part-whole structures for singulars and plurals, as suggested by Schwarzschild (1996). That is not to say that there is no difference in how constitution of objects as opposed to sums thereof

is conceptualized. That is to say that the Italian data suggest that the difference does not regard the relation between parts and a whole, but rather that it regards the relation governing how parts are organized in space with respect to each other. In particular, regular plurals encode no topological relations between entities making up pluralities or, in other words, simply denote arbitrary sums of individuals. Consequently, parts of a plurality do not form an integrated entity that could be considered a unit one could assign a number which seems to be a prerequisite for quantification. On the other hand, despite the plurality inference Italian irregular forms in *-a* differ significantly from regular plurals and somewhat resemble singulars in that they do encode how parts are organized with respect to each other. Specifically, their extensions consist of cohesive aggregates, i.e., pluralities arranged in a particular spatial configurations such that individual objects making up a plurality are either connected or at least stay in a relatively stable proximity. Given that the domain of quantification consists of entities conceptualized as such clusters, counting parts consisting of several singularities is possible as long as they form integrated fragments or continuous sections of the whole. Therefore, I posit that the explanation of the patterns summarized in Table 2.1 and 2.5 should be contingent on the manner how we conceptualize topological relations holding between parts of different types of entities as well as on the sensitivity of quantificational operations employed in natural language to such relations.

2.4 Parts vs. subsets

The Italian data provide intriguing evidence but let us hold on for a minute and reconsider the claim that being countable is a property of only those entities that are integrated wholes. One could wonder to what extent the fact that count explicit partitives involving regular plural DPs cannot quantify over sums and can only refer to parts of singular objects tells us anything about the nature of countability. After all, to the extent they are pragmatically admissible sentences in which the cardinal modifies an expression such as *subset* instead of a *part*-word are definitely compatible with an interpretation where the domain of quantification consists of collections of individuals, i.e.,

felicitous when counting subsets of a larger set. For instance, unlike (2.59b) the sentence in (2.59a) would be true in a situation where there are, say, ten relevant apples $\{a_1, \ldots, a_{10}\}$ and three arbitrary subsets of the apples got spoiled, e.g., $\{a_1, a_2\}$, $\{a_3, a_4, a_5\}$ and $\{a_6, a_7\}$.[24] In other words, is it really the case that counting requires units that are integrated objects? Or maybe there is simply something weird about partitive words and we should not jump to a conclusion concerning countability based on the partitive data?

(2.59) Polish

    a. Trzy podzbiory jabłek zgniły.
        three subsets apples$_{\text{GEN}}$ rot
        'Three subsets of the apples got spoiled.'

    b. Trzy części jabłek zgniły.
        three parts apples$_{\text{GEN}}$ rot
        'Three parts of the apples got spoiled.'
        (i)   part-of-a-singularity reading
        (ii) #part-of-a-plurality reading

At first blush, this seems to be a justified objection and I think it should be taken seriously. However, there are a number of empirical facts suggesting that what happens in (2.59a) is very different from what happens in (2.59b). First of all, it is crucial to emphasize that natural language expressions such as *set* or *subset* are not barely linguistic reflections of set-theoretic concepts involving notions such as $\{\ldots\}$ or relations $\subseteq$ and $\subset$. On the contrary, such expressions have a number of properties that classify them on a par with group nouns such as *committee*, *band*, or *pile*.[25] On the other hand, similar to bare plurals set partitives refer to pluralities but *part*-words are not collectives, as attested by a number of tests involving predicate non-sharing.

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24. This reading is different from a type of reading discussed in Section 3.3 where reference to three-fourths of the total volume of the apples would be involved. What is crucial for the discussion of (2.59b) is that the quantified subsets of individuals cannot be arbitrary.

25. In the following text, I will ignore the evidence for distinguishing between different classes of group nouns (see Pearson 2011, Henderson 2017, Dočekal and Wągiel 2018) and assume that for our purposes there are no relevant differences between nouns such as *band* and *pile*.

It has been proposed in the literature that group nouns and plural NPs do not co-refer since one can find multiple contrasts concerning (in)compatibility with certain predicates (Barker 1992, Schwarzschild 1996). For instance, the sentence with a collective DP as a subject is perfectly fine with the predicate *have five members*, see (2.60a), but its altered version involving a plural DP as in (2.60b) is not.

(2.60) English (Lønning 1987, p. 153)
  a. The committee has five members.
  b. #The men have five members.

Now, let us test the behavior of the Polish *part*-word *część* 'part' with respect to plurals, regular collective nouns, and the expression *podzbiór* 'subset'.[26] The contrasts in (2.61) show that plurals and explicit set partitives differ from the typical group noun *stos* 'pile' as well as *podzbiór* in that they are not compatible with predicates of the type *have ten elements* which are assumed to indicate bunches (Schwarzschild 1996).

(2.61) Polish
  a. #Jabłka mają dziesięć elementów.
      apples have ten elements$_{GEN}$
  b. #Część jabłek ma dziesięć elementów.
      part apples$_{GEN}$ has ten elements$_{GEN}$
  c. Stos jabłek ma dziesięć elementów.
      pile apples$_{GEN}$ has ten elements$_{GEN}$
      'The pile of apples has ten elements.'
  d. Podzbiór jabłek ma dziesięć elementów.
      subset apples$_{GEN}$ has ten elements$_{GEN}$
      'The subset of apples has ten elements.'

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26. It needs to be admitted that due to the restricted distribution of the noun *podzbiór* (often limited to mathematical and computational contexts) the following sentences involving this expression is not how people usually talk. Rather, these examples sound nerdy or somewhat like clumsily worded math exercises rather than something a regular person would say. Nevertheless, despite the slight lexical oddity the judgments concerning compatibility with different predicates, interactions in co-variational contexts and with overt distributive operators seem to be very clear.

The same pattern can be observed when the examined expressions appear as first arguments of the two-place predicate *należy do* 'belongs to' which arguably designates assignment of objects to some abstract entity representing a collection. Though both (2.62c) and (2.62d) are felicitous, the sentences including plurals and explicit set partitives in (2.62a) and (2.62b), respectively, cannot saturate the predicate.

(2.62)   Polish
      a. #Ta papierówka   należy do jabłek.
           this white-transparent belongs to apples$_{GEN}$
      b. #Ta papierówka   należy do części jabłek.
           this white-transparent belongs to part$_{GEN}$ apples$_{GEN}$
      c. Ta papierówka   należy do stosu jabłek.
           this white-transparent belongs to pile$_{GEN}$ apples$_{GEN}$
           'This white transparent apple belongs to the pile of apples.'
      d. Ta papierówka   należy do podzbioru
           this white-transparent belongs to subset$_{GEN}$
           jabłek.
           apples$_{GEN}$
           'This white transparent apple belongs to the subset of apples.'

Another contrast between plurals and collective nouns concerns distribution with reciprocals, as demonstrated in (2.63).

(2.63)   English (Schwarzschild 1996, p. 168)
      a. The rocks in that pile are touching each other.
      b. #That pile is touching each other.

Polish examples such as those in (2.64) show that there is a contrast between plurals and explicit set partitives on the one hand and typical collectives such as *stos* and *podzbiór* on the other. The first combine felicitously with reciprocal expressions, whereas the latter do not. In particular, (2.61a) and (2.61b) indicate either strong or intermediate reciprocity (Fiengo and Lasnik 1973, Dalrymple et al. 1998, Beck 2001), i.e., either each of the apples in question is touching all the other apples or any two of the apples are connected by a chain of apples that stand

in the reciprocal relation. Nevertheless, it is impossible to define such truth conditions for (2.61c) and (2.61d).

(2.64) Polish
    a. Jabłka dotykają się nawzajem.
        apples touch   REFL each-other
        'The apples are touching each other.'
    b. Część jabłek   dotyka się nawzajem.
        part apples$_{GEN}$ touches REFL each-other
        'Some of the apples are touching each other.'
    c. #Stos jabłek   dotyka się nawzajem.
        pile apples$_{GEN}$ touches REFL each-other
    d. #Podzbiór jabłek   dotyka się nawzajem.
        subset   apples$_{GEN}$ touches REFL each-other

A similar distribution is attested in contexts where the reciprocal is covert, see (2.65), including co-variational environments such as (2.66). Specifically, plurals do combine with predicates triggering covariation, whereas group nouns do not.

(2.65) English (Dougherty 1970)
    #The trio collided.

(2.66) English (Schwarzschild 1996, p. 168)
    a. The members of group A live in different cities.
    b. #Group A lives in different cities.

Let us then consider Polish examples involving plurals, explicit partitives, group nouns, and phrases c-commanded by *podzbiór* in subject position, as provided in (2.67) and (2.68). In (2.67a), the sentence-internal *different* expression *w różnych miastach* 'in different cities' is bound within a clause to express covariation with a plural argument due to a built-in distributive operator (Carlson 1987, Beck 2000, Dotlačil 2010, Brasoveanu 2008). As a result, the sentence would be true if for each of the musicians it was the case that they lived in a city that is different than the cities other musicians live in. Due to this kind of meaning, sentence-internal *different* expressions are good indicators of distributivity.

(2.67)   Polish

    a.   Muzycy   mieszkają w różnych   miastach.
         musicians live   in different$_{LOC}$ cities$_{LOC}$
         'The musicians live in different cities.'

    b.   Część muzyków   mieszka w różnych   miastach.
         part   musicians$_{GEN}$ lives   in different$_{LOC}$ cities$_{LOC}$
         'Some of the musicians live in different cities.'

    c.   #Zespół mieszka w różnych   miastach.
         band   live   in different$_{LOC}$ cities$_{LOC}$

    d.   #Podzbiór muzyków   mieszka w różnych
         subset   musicians$_{GEN}$ lives   in different$_{LOC}$
         miastach.
         cities$_{LOC}$

Similar, the sentences in (2.68) include the distributive preposition *po* which is a marker for distant distributivity in Polish (e.g., Przepiórkowski 2008, 2010, 2014, Przepiórkowski and Patejuk 2013).[27] As such, similar to binominal *each* (e.g., Safir and Stowell 1988, Zimmermann 2002, Dotlačil 2012, Champollion 2012a) and distributive numerals (e.g., Gil 2002, Oh 2005, Cable 2014, Hofherr and Etxeberria 2017) *po* marks the distributive share and distributes it over the distributive key denoted by a different phrase, here the subject DP. As a result, the possibility of a collective reading is ruled out and a sentence in which *po* occurs is obligatorily interpreted distributively, e.g., in (2.68a) each of the musicians played a song. This conflicts with the semantics of group nouns which in general do not allow for the access to individual members of a collection, hence (2.68c) is an awkward sentence. As witnessed by the contrast betwwen (2.68b) and (2.68d), explicit set partitives pattern with plurals, whereas *podzbiór* behaves as typical collectives.

(2.68)   Polish

    a.   Muzycy   zagrali po   piosence.
         musicians played DISTR song$_{LOC}$
         'The musicians played a song each.'

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27. For detailed studies on distant distributivity and distributive *po* in Slavic see also, e.g., Franks (1994, 1995), Harves (2003), Knežević (2015).

b. Część muzyków   zagrała po   piosence.
part   musicians$_{GEN}$ played DISTR song$_{LOC}$
'Some of the musicians played a song each.'

c. #Zespół zagrał po   piosence.
band   played DISTR song$_{LOC}$

d. #Podzbiór muzyków   zagrał po   piosence.
subset   musicians$_{GEN}$ played DISTR song$_{LOC}$

Yet another diagnostic involves the VP disjunction test proposed by (de Vries, 2015, pp. 30–31). When the test is applied, *część* does not exhibit the kind of behavior associated with group nouns. To illustrate this, let us consider (2.69a) where the definite plural DP *the children* serves as a subject of the disjunctive sentence. There are two possible interpretations of (2.69a). On the collective interpretation, see (2.69a-i), the disjunction *singing or dancing* holds of the plurality as a whole. In other words, it is either the case that all the relevant children are singing or that all the relevant children are dancing. However, there is yet another reading of (2.69a), namely the distributive interpretation, see (2.69a-ii), according to which the disjoined VPs hold not of the plurality of children as a whole but rather of each individual child. Thus, some of the children might be singing while others might be dancing. Crucially, when the definite plural is replaced with a group noun such as *team* as in (2.69b), the distributive interpretation is no longer available. The only reading such a sentence can get is that the whole plurality was either involved in singing or in dancing.

(2.69)   English (de Vries 2015, p. 31; adapted)
a.   The children are singing or dancing.
(i)   collective reading
(ii)   distributive reading
b.   The team is singing or dancing.
(i)   collective reading
(ii) #distributive reading

Now, let us examine the behavior of Polish nouns *część* and *podzbiór* compared to plurals and group nominals in sentences involving VP disjunction such as those in (2.70). The example (2.70a) can have either a collective or a distributive reading, i.e., it would be true either when

all the children are dancing or singing or when some dancing whereas others are singing. On the other hand, a sentence with a group noun as a subject like (2.70b) can only be interpreted collectively. That is also the case in (2.70d), i.e., all the children in the relevant subset need to be either dancing or singing. However, (2.70c) is felicitous in both collective and distributive scenarios. Once again, the *part* word *część* patterns with plurals rather than collectives which definitely proves that it differs from expressions such as *subset*.

(2.70)   Polish

      a.   Dzieci   tańczą lub śpiewają.
            children dance or   sing
            'The children are dancing or singing.'
            (i)   collective reading
            (ii)   distributive reading

      b.   Grupa dzieci   tańczy lub śpiewa.
            group children$_{GEN}$ dances or   sings
            'The group of children is dancing or singing.'
            (i)   collective reading
            (ii) #distributive reading

      c.   Część dzieci   tańczy lub śpiewa.
            part   children$_{GEN}$ dances or   sings
            'Some children are dancing or singing.'
            (i)   collective reading
            (ii)   distributive reading

      d.   Pozbiór dzieci   tańczy lub śpiewa.
            subset   children$_{GEN}$ dances or   sings
            'A subset of children is dancing or dancing.'
            (i)   collective reading
            (ii) #distributive reading

Notice that whatever theory of the relationship between agreement and collectivity/distributivity one might have (e.g., de Vries 2015, see also Schwarzschild 1996 and Pearson 2011 for the discussion of group nouns in British English as well as Bosnić 2016 for experimental data concerning distributivity and agreement mismatches in BCS), it cannot explain the semantic difference between *part* expressions and group nouns. In Polish, *część* similar to *zespół* (and any other group

noun) triggers singular agreement on the verb. As witnessed by the glosses in the examples provided above, in sentences with both explicit partitives and group nouns, see (2.67b)–(2.67c) and (2.70c)–(2.70b), the verbs agree in number with the subjects. Furthermore, both in (2.68b) and (2.68c) the past participles agree with *część* and *zespół* in gender by taking the feminine and masculine form, respectively. Therefore, it is not the agreement pattern but rather lexical properties of *część* that are accountable for the semantic difference.

Finally, despite being distinct syntactic categories partitive words pattern with cardinal numerals in that in general they allow for modification by class A/B modifiers such as *more than* and *at least* (Nouwen 2010, 2016, Brasoveanu 2012), see (2.71).[28] This again suggests that their nature is purely quantificational.

(2.71)   Polish

    a. Co najmniej część murów jest czerwona.
        at-least   part walls$_\text{GEN}$ is   red
        'At least some of the walls are red.'

    b. Co najmniej część murów jest czerwona.
        at-most   part walls$_\text{GEN}$ is   red
        'At most some of the walls are red.'

    c. Co najmniej połowa murów jest czerwona.
        at-least   half   walls$_\text{GEN}$ is   red
        'At least half of the walls are red.'

    d. Co najwyżej połowa murów jest czerwona.
        at-most   half   walls$_\text{GEN}$ is   red
        'At most half of the walls are red.'

    e. Co najwyżej pięć murów jest czerwonych.
        at-least   five walls$_\text{GEN}$ is   red
        'At least five of the walls are red.'

    f. Co najwyżej pięć murów jest czerwonych.
        at-most   five walls$_\text{GEN}$ is   red
        'At most five of the walls are red.'

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28. Since there are some restrictions with this respect which I believe result from the indefinite nature of some partitive words, I present here only parallel examples with class B modifiers.

The results of a battery of tests applied to Polish nouns *część* 'part' and *podzbiór* 'subset' unequivocally indicate that *podzbiór* is a group noun,[29] and thus triggers obligatory collective interpretations of sentences in which it occurs, whereas *część* patterns with plurals including being ambiguous between collective and distributive readings. This is a significant contrast between the expressions in question suggesting that their extensions differ in a significant way. In particular, it has been commonly advocated in the literature that group nouns do not refer to pluralities but rather to atomic individuals, i.e., somewhat abstract singularities[30] that might be associated with their members (e.g., Barker 1992, Schwarzschild 1996, Winter 2001, Champollion 2017).[31] Whatever extraordinary properties such atoms have, it is plausible to assume that they are conceptualized as something distinct from mere sums of objects.

Therefore, I conclude that explicit partitives refer to genuine pluralities whereas group nouns including expressions such as *set* and *subset* denote abstract entities that are associated with pluralities of their members but are ontologically distinct from them. Having this in mind, there is nothing surprising about the fact that count explicit set partitives do not quantify over sums of individuals and in fact, it turns out that *part*-words actually pattern with regular nominals, as will be shown below.

It has been observed for a long time that the semantics of numeral phrases in languages such as English poses a compositional puzzle. On the one hand, cardinal numerals higher than *one* combine only with plural nominals, on the other, plural NPs modified by cardinals are interpreted as singular expressions. Specifically, although plurals denote sums of entities, the domain of quantification is always a set of atomic individuals (Kratzer 1989, Chierchia 1998b, Landman 2000; see also Kobuchi-Philip 2006). Contrary to what one would expect if

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29. Schwarzschild (1996, p. 168) actually lists *set* among bunch nouns.
30. Accidentally, this corresponds to the fact that in set theory sets are considered as objects in their own right.
31. But see, e.g., Link (1984, 1998), Landman (1989a,b, 2000), de Vries (2015) for an alternative view on which a group noun denotes a special type of plurality shifted to an impure atom or group-atom depending on a particular approach. Pearson (2011), on the other hand, proposes that group nouns are predicates of individual concepts.

numerals simply counted elements in a given set (e.g., Barwise and Cooper 1981) and combined with plural nouns in a straightforward manner, cardinals ignore sums of objects and assign numbers only to singular individuals. Actually, this fact has been already realized in the system of Link (1983) where the extension of a phrase such as *three children* is supposed to contain only special types of elements, namely exactly three atoms. To illustrate why such a restriction is necessary, let us consider (2.72). Let us assume that there are four relevant children in the universe of discourse, say, Anne, Betty, Carl, and Danny, i.e., a, b, c, and d respectively. Then the meaning of the plural noun *children* is the set involving both atomic children and all the sums generated by joining the atoms, see (2.72b). If *three* simply counted the elements in the set denoted by the modified noun, among the possible verifications of the sentence in (2.72a) would be (2.72c) and (2.72d) since in both cases the set of children that slept consists of three elements.

(2.72)　a.　Three children slept.
　　　　 b.　$[\![\text{children}]\!] = \{a, b, c, d, ab, ac, ad, bc, bd, cd, abc, abd, acd,$
　　　　　　 $bcd, abcd\}$
　　　　 c.　$[\![\text{three children}]\!] \cap [\![\text{slept}]\!] = \{a, ab, abcd\}$
　　　　 d.　$[\![\text{three children}]\!] \cap [\![\text{slept}]\!] = \{a, b, ab\}$

The truth conditions in (2.72c) and (2.72d) predict (2.72a) to be true both in a scenario where there were four individual children sleeping and in a scenario where only two individual children slept. However, this not how the sentence is understood. It seems that what cardinals in a language such as English do is that they restrict possible verifications of sentences including numeral phrases in such a way that sums of individuals are excluded from the domain of quantification.[32]

　　 With this in mind, let us return to count explicit partitives such as the Italian examples in (2.41), repeated here as (2.73). Similar to regular plural DPs unmodified explicit set partitives denote pluralities or, more precisely, sub-pluralities. However, when combined with a numeral they do not allow for quantification over sub-pluralities but only over material parts of individuals making up pluralities. At first

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32. Alternatively, one might assume that there is a null classifier responsible for such a restriction (e.g., Selkirk 1977, Kobuchi-Philip 2006).

blush, it seems surprising but actually in languages such as English the same effect can be found in numeral phrases. Specifically, the numeral combines with the plural noun which denotes a set of pluralities; however, the domain of quantification does not consist of pluralities but rather atomic individuals. Hence, as demonstrated in (2.74) the phrase *three walls* does not mean three pluralities of walls but rather a plurality of three walls. Similar, the Italian partitive phrase in (2.73b) cannot be interpreted as three pluralities of sub-pluralities of walls. Instead, it would be felicitously paraphrased as a plurality of three parts of walls.

(2.73)   Italian (Schwarzschild 1996, p. 186; adapted)
   a.   tre   parti del   muro
        three parts of-the wall
        'three parts of the wall'
   b.   tre   parti dei   muri
        three parts of-the walls
        'three parts of the walls'
        (i)   part-of-a-singularity reading
        (ii)   #part-of-a-plurality reading

(2.74)   Pluralities of wholes and parts
   a.   three walls
        (i)   #three pluralities of walls
        (ii)   plurality of three walls
   b.   tre parti dei muri
        (i)   #three pluralities of parts-of-a-plurality of walls
        (ii)   plurality of three parts of walls

In this section, I have argued that set partitives differ from collectives and discussed issues regarding the domain of quantification in count explicit partitives as well as group nouns and plurals modified by cardinal numerals. An important question is why natural language does not allow for counting arbitrary sums of entities. However, before we discuss more data that might guide us to provide an answer to this puzzle, let us return to the analogy concerning entity and set partitives and its potential implications for the part-whole structures singular and plural expressions employ.

2.5 Ambiguity or indeterminacy

The data we have begun with in Section 2.2 might be interpreted as suggesting that both singulars and plurals employ a unified part-whole structure. Another way of looking at the evidence involves preserving distinct mereologies and at the same time postulating that partitive words are either ambiguous between two different meanings or that they encode a generalized parthood relation which enables them to operate on both part-whole structures. A question I have not raised until this moment concerns to what extent there is an empirical basis broad enough to provide a solid argument in favor of one of the approaches. The evidence for both cross-linguistically and intra-linguistically systematic behavior of partitive words indicates that an ambiguity account seems rather implausible. Nevertheless, before we conclude, let us see whether an independent test can provide an argument for or against such an approach.

A standard diagnostic to detect ambiguity is the so-called zeugma test (Lasersohn 1995, see also Zwicky et al. 1975). The test works as follows. The examined lexical item is put in a syntactic configuration with a coordination structure. If the expression in question is in fact ambiguous, then it must take the same interpretation with respect to both conjuncts. Otherwise the zeugma effect arises, i.e., the sentence takes on the flavor of a joke. On the other hand, if the tested item is unspecific or indeterminate, it can take different interpretations with respect to different conjuncts without any hilarious effect.

To illustrate this, let us consider the two examples in (2.75). The verb *rent* must have the same interpretation with respect to both DPs, *a car* and *a house*, i.e., it is either the case that John owned a car and a house and he rented them out to someone, or it is the case that John rented a car and a house from someone. But crucially it cannot be the case that John rented out a car and rented a house as a tenant or vice versa. Therefore, since *rent* must have the same interpretation with respect to the DPs *a car* and *a house*, it is ambiguous between the two described readings. On the other hand, the verb *describe* in (2.75b) is indeterminate with respect to whether describing is done in speech or in writing. Since it is perfectly natural to understand the sentence in a way implying that John described a car in speech, whereas he

described a house in writing, *describe* is not ambiguous but rather merely unspecific.

(2.75)   English (Lasersohn 1995, p. 94; adapted)
    a.   John rented a car and a house.
    b.   John described a car and a house.

The zeugma test has been used commonly in the study of natural language semantics including the research on pluralities (see, e.g., Dowty 1987, Lasersohn 1995, Nouwen 2016).[33] Let us then examine whether *part*-words are ambiguous between two different readings giving rise to a part-of-a-singularity reading in entity partitives on the one hand and to a part-of-a-singularity reading in set partitives on the other or, alternatively, they are merely indeterminate in this respect. To this end, let us first consider the well-formed and felicitous German sentences in (2.76). In the explicit entity partitive in (2.76a) the partitive word *Teil* 'part' takes a structure involving coordination of two singular DPs as a complement. As witnessed by the translation, the meaning of the sentence involves material partitivity, i.e., quantification over parts of the relevant singular objects, namely the apple and the pear. In other words, (2.76a) would be true if part of the apple and part of the pear got spoiled. On the other hand, *Teil* in (2.76b) combines with a conjoined phrase involving two plural conjuncts. In this case, *Teil* quantifies over parts of pluralities, i.e., the whole sentence designates that a subset of the apples and a subset of the pears got spoiled.

(2.76)   German (Viola Schmitt, Martin Prinzhorn, p.c.)
    a.   Ein Teil des   Apfels   und der   Birne   sind
        a   part the$_{GEN}$ apple$_{GEN}$ and the$_{GEN}$ pear$_{GEN}$ are
        verfault.
        rotten
        'Part of the apple and the pear got spoiled.'

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33. However, Lasersohn (1995, p. 95) points out that in fact it does not detect ambiguity, but rather whether a particular item always makes the same contribution to the proposition. I will leave this issue open though.

b. Ein Teil der Birnen und der Äpfel sind
a part the_GEN pears_GEN and the_GEN apples_GEN are
verfault.
rotten
'Some of the pears and the apples got spoiled.'

Now let us consider the interaction between *Teil* and a coordinate structure conjoining a singular and a plural DP. The assumption behind (2.77) is that if *Teil* were merely indeterminate with respect to a part-of-a-singularity and part-of-a-plurality reading, it should be able take the first interpretation with respect to the first conjunct and the latter interpretation with respect to the second conjunct in (2.77a) and vice versa in (2.77b). On the other hand, if it were ambiguous between two different meanings such behavior should not be possible. Interestingly, *Teil* not only can combine felicitously with a construction involving coordination of a singular and plural DP, but also irrespective of the linear order of the conjuncts, see (2.77a) and (2.77b), it can take simultaneously two different readings.[34]

(2.77) German (Viola Schmitt, Martin Prinzhorn, p.c.)

a. Ein Teil des Apfels und der Birnen sind
a part the_GEN apple_GEN and the_GEN pears_GEN are
verfault.
rotten
'Part of the apple and some of the pears got spoiled.'

b. Ein Teil der Birnen und des Apfels sind
a part the_GEN pears_GEN and the_GEN apple_GEN are
verfault.
rotten
'Some of the pears and part of the apple got spoiled.'

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34. Notice, however, that this is not the case in Slavic. For instance, an exact translation of the sentences in (2.77) in Polish or in Czech would be considered awkward. I hypothesize that the reason for the infelicity of such examples has to do with the fact that these are conjoined NPs rather than DPs. Assuming that the silent determiner sits above the coordinate structure would explain why such partitives are impossible since DP-internal coordination does not allow for mixing of singulars and plurals (Heycock and Zamparelli 2003).

The very same effect arises in (2.78), where one of the conjuncts is a singulare tantum noun, specifically the object mass noun *Geschirr* 'tableware' can refer either to one piece of tableware or to a collection thereof. Both (2.78a) and (2.78b) are felicitous and they both can get the intended meaning, i.e., *Teil* can simultaneously quantify over material parts of the kitchen sink and over individual items making up the tableware. Hence, the mismatch between the values of grammatical number on conjunct nominals in (2.77) definitely does not play a role here and the effect seems to be purely semantic.

(2.78)   German (Viola Schmitt, Martin Prinzhorn, p.c.)

a.   Ein Teil des   Geschirrs   und des   Spülbeckens
a   part the_GEN tableware_GEN and the_GEN sink_GEN
sind verrostet.
are   rusted
'Some of the tableware and part of the kitchen sink got rusted.'

b.   Ein Teil des   Spülbeckens und des   Geschirrs sind
a   part the_GEN tableware_GEN and the_GEN sink_GEN   are
verrostet.
rusted
'Part of the kitchen sink and some of the tableware got rusted.'

At this point, a conclusion can be drawn. Given the results of the zeugma test, I conclude that *part*-words in languages such as German are not ambiguous between two different interpretations. Rather, they are semantically unspecific in the sense that their meaning is general enough to cover configurations involving both material and individual parthood. As a result a part-of-a-singularity understanding is derived in entity partitives whereas a part-of-a-plurality understanding arises in set partitives.

2.6   Summary

The data discussed in this section provide compelling evidence that cross-linguistically it is common for the same partitive word including proportional quantifiers such as *half*-words to appear both in entity

and set partitives. This fact suggests that such expressions can operate both at the superatomic and subatomic level depending on what structure is provided by the embedded DP. However, at the same time explicit partitives modified by a cardinal numeral can only quantify over material parts of singular individuals irrespective of whether the complement DP is singular or plural. At first sight, this fact seems to be at variance with the claim that there is unified parthood utilized by partitive words in both entity and set partitives. Nonetheless, an important empirical finding concerning Italian irregular plurals showed that if a plural expression encodes a certain topological configuration, i.e., denotes cohesive pluralities, count explicit partitives can get a part-of-a-plurality reading in such a case. This fact demonstrates that the uncountability of explicit partitives with plurals does not indicate per se different part-whole structures for singularities and pluralities. Rather, countability results from the interplay between the meaning of a partitive word and the extension of a singular or plural DP it combines with and topological constraints play a crucial role in this interaction.

Furthermore, on the basis of the zeugma test involving German partitives with conjoined singular and plural DPs I have shown that it is empirically inadequate to account for the parallelism between explicit and proportional entity and set partitives in terms of semantic ambiguity. In addition, given the cross-linguistic and intra-linguistic evidence the two remaining possibilities are the following. It is either the case that singulars and plurals employ a unified part-whole structure or that their mereologies do differ but at the same time partitive words involve a derived indeterminate notion of parthood which allows them to quantify over elements of whatever part-whole structure they are applied to. I argue for the first option. However, by advocating this claim I do not advance a view that singulars and plurals employ the same structures. What I claim is that it is not the part-whole relation that differs but rather that there is another notion involved which is responsible for how parts are topologically arranged with respect to each other. Intuitively, this seems correct since we conceptually distinguish between integrated wholes and arbitrary sums of parts and quantification in natural language is sensitive to the distinction. Specifically, only integrated parts of integrated wholes can be assigned numbers when counting. On the other hand, scattered entities such

as pluralities are restricted from the domain of quantification cardinal numerals establish. In addition, Italian irregular plurals provide evidence that there are natural language expressions involving yet another type of structure. In particular, such nominals designate entities that are similar to plurals in that they consist of multiple integrated objects but at the same time the sum thereof is arranged in a particular way, i.e., it constitutes a cluster.

So far, I have argued that explicit and proportional partitives provide evidence that countability is restricted to entities that form an integrated whole as typically denoted by concrete singular count nouns, while the arbitrary sums of parts regular plural nouns refer to cannot be counted. Furthermore, the data concerning subatomic quantification suggests that this constraint is also applicable at the level of material parts of individuals. If that is correct, one would expect that there are natural language expressions dedicated exclusively to quantification over integrated parts similar to cardinal numerals that count integrated wholes. In the next section, I will introduce novel data concerning distinct types of *half*-words in Polish as well as other partitive expressions.

3 Exploring topological sensitivity

In the previous chapter, I have discussed the data concerning quantification over parts in explicit entity and set partitives. Specifically, I have presented evidence suggesting that only integrated parts of integrated wholes can be subject to counting. As I have already indicated in the introduction, in English the difference can be captured by the contrast between bare *part* and *a part*. I have argued that expressions corresponding to the first refer to arbitrary parts of a whole including discontinuous portions of a substance making up a whole, and thus are uncountable. On the other hand, expressions corresponding to the latter refer to continuous integrated parts of a whole and as such can pluralize and combine with cardinals. In this chapter, I explore to what extent natural language is sensitive to this distinction. To this end, I will provide novel evidence from different types of Polish partitive words that encode the contrast formally. It will turn out that this phenomenon is not something idiosyncratic but rather cross-linguistic evidence suggests that it is relatively widespread with different languages using different means to express it. Moreover, based on the data concerning *whole*-adjectives I will demonstrate the linguistic relevance of two aspects of being whole, namely maximality and integrity.[1]

3.1   Continuous and discontinuous parts

Given the meaning of the plural in languages such as English, it follows that expressions such as *a part* are incompatible with predicates denoting pluralities. Intuitively, the reason is that the extension of a phrase headed by *a part* comprises integrated portions, i.e., parts that come in one piece, whereas plurals denote arbitrary sums, i.e., scattered entities consisting of discontinuous elements. However, if

the plural were augmented with an additional semantic feature guaranteeing that a plurality of integrated objects is itself an integrated object, as is arguably the case in at least some Italian irregular plurals, then an expression corresponding to English *a part* is able to combine with such a plural expression. In such a case, it would yield a part of a plurality whose constituents are in a particular topological relation, namely they are connected to each other.

But is there any additional reason to assume that natural language expressions are sensitive to topological relations operating alongside part-whole structures? And if so, to what extent do the discussed phenomena really tell us something important about countability? The claim advanced here is that what counts as one needs to be an integrated part (proper or improper) of an integrated whole. Though this might seem intuitively correct, a question arises where the property of 'coming in one piece' comes from. After all, countability is the ability of an expression to appear in morpho-syntactic environments related to counting, but counting is a semantic operation of assigning numbers to units of a particular type. Hence, one could wonder whether there is evidence that there is something in an expression's meaning that corresponds to the notion of being an integrated entity, and thus being countable. One prominent view holds that countability is not a property of a particular class of nouns but rather of full DPs (e.g., Borer 2005a, see also Allan 1980 and Pelletier and Schubert 1989 and the following work), and thus arises within a complex nominal syntactic structure. On the other hand, other approaches posit that it is lexical meaning that determines countability patterns (e.g., Wierzbicka 1988, Wisniewski et al. 2003). However, evidence discussed in the literature is mainly restricted to cases of quantification over wholes and almost totally ignores constructions in which what is counted are bits of objects rather than whole individuals.

In this section, I will provide additional evidence for the significance of the role of spatial integrity, i.e., compactness, of parts in subatomic quantification. The core data come from the distribution and semantic properties of distinct classes of Polish *half*-words.

3.1.1 Polish *half*-words

Polish distinguishes lexically between three distinct *half*-words, as presented in (3.1). They are morphologically derived from one another with *pół* being morphologically the least marked form with no derivational affixes. As presented in (3.1a), *pół* consists only of a root and null inflectional marker. On the other hand, *połowa* and *połówka* are derived forms involving in addition the morpheme *-ow-/-ów-* as well as the diminutive suffix *-k-* in the latter case, see (3.1b)–(3.1c).

(3.1)   Polish

        a.   pół-$\varnothing$
              root-inflectional.marker
              'half$_1$'

        b.   poł-ow-a
              root-derivational.suffix-inflectional.marker
              'half$_2$'

        c.   poł-ów-k-a
              root-deriv.suffix$_1$-deriv.suffix$_2$-infl.marker
              'half$_3$'

In terms of ϕ-features, *pół* shows neuter agreement whereas *połowa* and *połówka* are feminine, as demonstrated in (3.2). Furthermore, similar to vague quantifiers such as *dużo* 'many/much' *pół* is defective in that it has only the singular nominative and accusative forms. On the other hand, *połowa* and *połówka* are both well-behaved nouns exhibiting the full feminine declension including plural forms.

(3.2)   Polish

        a.   To   czerwone pół   jabłka   zgniło.
              this$_{NOM.N}$ red$_{NOM.SG.N}$ half$_{1.NOM.N}$ apple$_{GEN.N}$ rot$_{SG.N}$
              'This red half of the apple got spoiled.'

        b.   Ta   czerwona połowa jabłka   zgniła.
              this$_{NOM.F}$ red$_{NOM.SG.F}$ half$_{2.NOM.F}$ apple$_{GEN.N}$ rot$_{SG.F}$
              'This red half of the apple got spoiled.'

        c.   Ta   czerwona połówka jabłka   zgniła.
              this$_{NOM.F}$ red$_{NOM.SG.F}$ half$_{3.NOM.F}$ apple$_{GEN.N}$ rot$_{SG.F}$
              'This red half of the apple got spoiled.'

At first blush, the Polish *half*-words in (3.2) are mere synonyms making the same contribution to the interpretation of the discussed sentences. However, closer investigation reveals intriguing differences in their distribution and meaning. Let us first consider the most frequent and semantically least marked *half* expression, i.e., *połowa*. As witnessed in (3.3), there are no constraints on its distribution and it can felicitously combine with all types of entity-denoting predicates including count singulars, collectives, plurals, and mass terms.[2]

(3.3) Polish

    a. połowa jabłka
        half$_2$ apple$_{\text{GEN}}$
        'half of the apple'

    b. połowa stosu (jabłek)
        half$_2$ pile$_{\text{GEN}}$ (apples$_{\text{GEN}}$)
        'half of the pile (of apples)'

    c. połowa jabłek
        half$_2$ apples$_{\text{GEN}}$
        'half of the apples'

    d. połowa soku
        half$_2$ juice$_{\text{GEN}}$
        'half of the juice'

On the other hand, *pół* has a more restricted distribution since it is incompatible with cumulative predicates such as plurals and mass nouns, see (3.4). Thus, it can only combine with regular count singulars and collective nouns.

(3.4) Polish

    a. pół jabłka
        half$_1$ apple$_{\text{GEN}}$
        'half of the apple'

    b. pół stosu (jabłek)
        half$_1$ pile$_{\text{GEN}}$ (apples$_{\text{GEN}}$)
        'half of the pile (of apples)'

---

2. I provide the examples with collective nouns only for sake of completeness. In the following part of the study, I will not investigate the interaction between and partitives and collectives.

    c. #pół jabłek
       half$_1$ apples$_{\text{GEN}}$
    d. #pół soku
       half$_1$ juice$_{\text{GEN}}$

Notice that the constraint on the distribution of *pół* is not morpho-syntactic. The phrases in (3.5) show that it can felicitously combine with pluralia tantum. However, in such a case the whole partitive phrase gets a part-of-a-singularity interpretation. Specifically, (3.5a) cannot refer to 50% of the relevant utensils but only to a half of one pair of scissors. Similar, (3.5b) involves the noun *bakalie* 'dried fruit' which is a plurale tantum expression in Polish and as such it can either denote one piece of dried fruit or a plurality thereof. The *half*-word *pół* can combine with *bakalie* as long as it quantifies over material parts of a single foodstuff. For instance, it would be true of a half of a raisin but not of, say, five out of ten raisins.

(3.5)   Polish

    a.   pół nożyczek
        half$_1$ scissors$_{\text{GEN}}$
        'half of the scissors'
    b.   pół bakalii
        half$_1$ dried-fruit$_{\text{GEN.PL}}$
        'half of the dried fruit'

Furthermore, the claim that *pół* is incompatible with mass nouns is somewhat imprecise and requires elaboration. In fact, when it can combine with mass terms, it enforces a mass-count shift via the Universal Packager (see, e.g., Bach 1986, Jackendoff 1991, Landman 1991). In other words, the phrases in (3.6) are only felicitous on the portion reading, i.e., (3.6a) would only be true of half a glass of juice whereas (3.6b) refers to half a pint of beer or some other standardized or contextually salient measure of volume. This shows that *pół* is sensitive to the semantics of DPs it c-commands. In particular, cumulative predicates are disallowed because they denote scattered entities such as arbitrary sums or amorphous substances.

(3.6)   Polish

      a.   pół   soku
          half$_1$ juice$_\text{GEN}$
          'half of the juice'
          (i)   #substance reading
          (ii)   portion reading
      b.   pół   piwa
          half$_1$ beer$_\text{GEN}$
          'half of the beer'
          (i)   #substance reading
          (ii)   portion reading

Finally, the distribution of the third Polish *half*-word, i.e., *połówka*, is even further constrained since it is compatible only with regular concrete singular nouns, as witnessed by the contrasts in (3.7). Prototypically, it takes nominals denoting solid objects one could easily cut or divide into separate parts such as food terms or building materials like bricks. Similar to *pół*, it is distinctively odd with plurals and mass terms. Moreover, it does not work with collective nouns.

(3.7)   Polish

      a.   połówka jabłka
          half$_3$ apple$_\text{GEN}$
          'half of the apple'
      b.   #połówka stosu   (jabłek)
          half$_3$ pile$_\text{GEN}$ (apples$_\text{GEN}$)
      c.   #połówka jabłek
          half$_3$ apples$_\text{GEN}$
      d.   #połówka soku
          half$_3$ juice$_\text{GEN}$

The distributional facts concerning particular Polish *half*-words are summarized in Table 3.1. While the distribution of *połowa* is unconstrained, i.e., it can appear in all types of proportional partitives including entity, set, and mass partitives, the distributional potential of *pół* and *połówka* is significantly restricted in that neither of them can modify cumulative predicates such as plurals and mass terms, and thus they can only occur in entity partitives. The different distribution

of particular expressions with respect to different types of nominals suggests distinct semantic properties of those lexical items. Given that the only type of nominals that is compatible with all the Polish *half*-words is singular count nouns, let us discuss in detail intuitions about the meanings of entity partitive phrases in (3.4a), (3.3a), and (3.7a).

	SINGULARS	COLLECTIVES	PLURALS	MASS NOUNS
połowa	✓	✓	✓	✓
pół	✓	✓	*	*
połówka	✓	*	*	*

Table 3.1: Distribution of Polish *half*-words

In order to do so, consider two different subdivisions of an entity, as depicted in Figure 3.1 and 3.2. In both cases, the dashed lines mark an area designating a portion of the apple that constitutes approximately 50% of the whole. However, there is a crucial difference between the two depicted situations in terms of topological configuration of the stuff making up the halves. The share represented in Figure 3.1 constitutes a continuous half of the apple and as such it can be considered a solid object in its own right within the whole, whereas Figure 3.2 illustrates just an arbitrary portion that is not contiguous, i.e., the apple stuff does not form an integrated part of the apple. In other words, the marked quantity in Figure 3.1 is an integrated part, whereas the portion in Figure 3.2 is not.
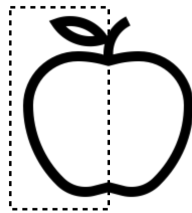
Figure 3.1: Continuous half   Figure 3.2: Discontinuous half
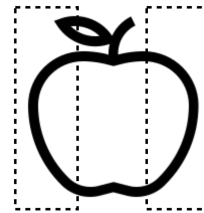
Let us now discuss possible extensions of proportional entity partitives involving *pół*, *połowa*, and *połówka*. The phrases in (3.4a) and

(3.3a) are true of both the object depicted in Figure 3.1 and the entity in Figure 3.2. Out of the blue, the continuous half reading is definitely the dominant one, however the discontinuous half interpretation can become salient in multiple contexts. For instance, it gets extremely relevant in a cooking scenario where pragmatically halves might be considered in terms of volume rather than individuated parts. Crucially, the ambiguity is systematic in entity partitives involving both *połowa* and *pół* and the sentences in (3.8a) and (3.8b) are true both in a situation where the marked half in Figure 3.1 is red and when the designated portion in Figure 3.2 is red. Now, it is quite intriguing that the partitive construction in (3.7a) does not denote entities such as the one illustrated in Figure 3.2. Therefore, entity partitives with *połówka* refer only to continuous integrated halves of an object denoted by the 'downstairs' DP. Consequently, the sentence in (3.8c) would be judged true if the part indicated in Figure 3.1 were red and false with respect to the portion in Figure 3.2.

(3.8) Polish

    a. Pół jabłka jest czerwone.
       half$_1$ apple$_{\text{GEN}}$ is red
       'Half the apple is red.'

    b. Połowa jabłka jest czerwona.
       half$_2$ apple$_{\text{GEN}}$ is red
       'Half the apple is red.'

    c. Połówka jabłka jest czerwona.
       half$_3$ apple$_{\text{GEN}}$ is red
       'A half of the apple is red.'

A similar phenomenon can be observed in object position. In the sentences in (3.9), the partitives appear as arguments of a verb of consumption and as such denote an incremental theme (see, e.g., Dowty 1991, Krifka 1998, Filip 1999, Rothstein 2003). Again, there is a contrast between (3.9a) and (3.9b) on the one hand and (3.9c) on the other in terms of truth conditions. The first two sentences denote weaker propositions since for them to be judged true it would be enough for Marysia to eat a quantity of the apple in question corresponding approximately to 50% of the apple's total volume. However, in many scenarios in which (3.9a) and (3.9b) would be true the sentence in

(3.9c) would not. This is because the proportional partitive headed by *połówka* denotes a contiguous half, i.e., an integrated object within the whole as opposed to arbitrary portions of the apple mass. This stronger meaning results in that (3.9c) is true of a scenario such as the one illustrated in Figure 3.1 and false with respect to Figure 3.2.

(3.9)   Polish

    a.   Marysia zjadła pół   jabłka.
          Marysia ate half_1 $\text{apple}_{\text{GEN}}$
          'Marysia ate half the apple.'

    b.   Marysia zjadła połowę jabłka.
          Marysia ate half_2 $\text{apple}_{\text{GEN}}$
          'Marysia ate half the apple.'

    c.   Marysia zjadła połówkę jabłka.
          Marysia ate half_3 $\text{apple}_{\text{GEN}}$
          'Marysia ate a half of the apple.'

Furthermore, similar to *pół* proportional partitives involving *połówka* and pluralia tantum get a part-of-a-singularity reading. The phrases in (3.10a) and (3.10b) refer to halves of one object.[3] In addition, similar to the previously discussed examples a part yielded as a result of subatomic quantification has to be integrated. Thus, (3.10a) and (3.10b) would be true of one scissor blade and, say, one continuous half of a raisin, respectively.

(3.10)   Polish

    a. %połówka nożyczek
       half_3 $\text{scissors}_{\text{GEN}}$
       'half of scissors'

    b. połówka bakalii
       half_3 $\text{dried-fruit}_{\text{GEN.PL}}$
       'half of dried fruit'

Given the non-trivial truth conditions of the sentence in (3.8c), let us consider several naturally occurring examples involving entity

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3.   Some speakers report that (3.10a) is somewhat odd. I suspect that the reason is that scissors are not a kind of thing one normally divides into separate parts. My own intuition, however, is that though the phrase is certainly unusual, it is definitely interpretable.

partitives headed by the *half*-word *połówka* from the National Corpus of Polish (NCP)[4] as given in (3.11).

(3.11) Polish (NCP)

  a. …zjadł (…) połówkę sztokfisza.
    he-ate half$_{3.ACC}$ dried-cod$_{GEN}$
    '…he ate (…) a half of a dried cod.'

  b. …masz ochotę na (…) drink w połówce
    you-have desire$_{ACC}$ on cocktail$_{ACC}$ in half$_{LOC}$
    kokosa?
    coconut$_{GEN}$
    '…would you like (…) a cocktail in a half of a coconut?'

  c. …jesteśmy jak te dwie połówki jajka.
    we-are like these two halves$_3$ egg$_{GEN}$
    '…we're like those two halves of an egg.'

  d. …połówki okna (…) trzasnęły o ścianę.
    halves window$_{GEN}$ banged about wall$_{ACC}$
    '…the halves of a window (…) banged on a wall.'

  e. Otworzył portfel i wyjął (…) połówkę karty.
    he-opened wallet$_{ACC}$ and he-took-out half$_{3.ACC}$ card$_{GEN}$
    'He opened his wallet and took (…) a half of a card out of it.'

The first three examples in the sample, see (3.11a)–(3.11c), involve reference to foodstuffs. The sentence in (3.11a) would be true if the man in question consumed half a dried cod. Importantly, however, it would not be true in a situation where he ate, say, the tail and took some bites of the front left side and middle right side of the fish even if the total amount consumed equaled 50% of the whole issue. The man had to eat either the left side or the right side of the cod or alternatively the half starting from the head or the half starting from the tail. Similar, the question in (3.11b) makes reference to a half of a coconut coming in one piece, i.e., one would feel deceived if after answering "yes" they got several pieces of a shell filled with portions of a cocktail they ordered. On the other hand, the use of *połówka* in (3.11c) is somewhat

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4. The National Corpus of Polish is a representative digital corpus of the contemporary Polish language consisting of over 1 billion words with a balanced subcorpus of 300-million tokens.

metaphorical. The inference here is that the speaker asserts that they and the addressee are like two individuated halves of an egg that fit well together. Again, the halves need to be continuous parts one receives after a clear horizontal or vertical cut. Yet another example is provided in (3.11d). Here, however, the downstairs DP in the partitive does not refer to food but to a solid object of another type, i.e., a window consisting of two casements. The partitive phrase clearly implies that the halves in question are individuated and easily distinguishable parts, hence the casement window interpretation. Finally, the sentence in (3.11e) would be judged true only if the man in question took out a half of a card in one piece, i.e., it would necessarily be false if he took out several torn scraps of a card.

The discussion of (3.8c) as well as the genuine examples from the National Corpus of Polish in (3.11) leaves no doubt that *połówka* requires the referents of a partitive it occurs in to constitute an integrated part. Such a finding is quite remarkable and forces us to abandon the initial intuition that Polish *half*-words are semantically indifferent. The recapitulation of referential properties of explicit partitives involving the expressions in question is given in Table 3.2. The main contrast between less marked *połowa* and *pół* on the one hand and more marked *połówka* on the other lies in that the first two can designate discontinuous portions of stuff making up a whole, whereas the latter denotes only such divisions that constitute continuous, i.e., integrated, halves of an object.

	CONTINUOUS PART	DISCONTINOUS PART
połowa	✓	✓
pół	✓	✓
połówka	✓	*

Table 3.2: Denotations of Polish *half*-words

The novelty of the data introduced here is twofold. First of all, Polish *half*-words provide strong evidence that natural language is sensitive to the topological arrangement of parts of an entity. Second, the data imply that individuation is also possible at the subatomic level, i.e., quantification over parts in natural language reflects the fact that some parts might be assigned the status of an individual in its own

right. In general, the distinction between different *half*-words in Polish can be described as follows. The least semantically marked expression *połowa* is topologically neutral, i.e., the spatial make-up of an entity denoted by the downstairs DP in a partitive is irrelevant as long as it constitutes approximately 50% of a whole. That is why *połowa* does not discriminate between count singulars, plurals, and mass terms and can felicitously appear in entity, set, and mass partitives. It simply separates out a half of whatever entity it is applied to. Furthermore, in this case subatomic quantification does not encode any constraints on what kind of entity is yielded. Hence, the result might be either a scattered entity such as a plurality or a discontinuous portion of matter. On the other hand, *pół* and *połówka* are topologically sensitive. They both require a DP they c-command to denote an integrated object, i.e., a cohesive whole. That explains why they cannot combine with expressions referring to scattered entities such as arbitrary sums of individuals or portions of a substance as typically denoted by plurals and mass nouns. The difference between the two is that *pół* is similar to *połowa* in that it does not impose any topological constraints on the resulting entity, whereas *połówka* does. In other words, while *pół* selects for an integrated object and returns either its continuous or discontinuous half, *połówka* yields an integrated half of an integrated whole.

3.1.2 Inherent vagueness

Similar to other proportional quantifiers expressions such as English *half* appear to be inherently vague. Though disregarded by prescriptive grammars as an oxymoron the concept of a bigger and smaller half is present in lexicons of many languages, as attested in the English, German, and Polish example in (3.12)–(3.14). Interestingly, such phrases cannot be explained as bleached expressions meaning simply something like *part*. For instance, for Polish speakers who would use or at least accept the NP in (3.14a), i.e., speakers who have not internalized the linguistic prescription in question, it would be definitely true of a share constituting 55% of a whole. However, it is unlikely it would be judged true of 75% of a whole and intuitively it seems definitely false of 90% of a whole.

(3.12)   English
    a. %bigger half
    b. %smaller half

(3.13)   German
    a. %größere Hälfte
       bigger   half
       'bigger half'
    b. %kleinere Hälfte
       smaller half
       'smaller half'

(3.14)   Polish
    a. %większa połowa
       bigger $half_2$
       'bigger half'
    b. %mniejsza połowa
       smaller $half_2$
       'smaller half'

Given the possible interpretations discussed above, it seems plausible to assume that in natural language proportional quantifiers such as English *half* denote a relation of constituting approximately 50% share of a whole. In other words, their meaning is fuzzy in the sense it allows for unequal parts as long as the disproportion falls within a particular range defined contextually.

3.2   More topology-sensitive partitive words

The observations made in Section 3.1 provide novel evidence for certain semantic properties of natural language expressions that have not been recognized so far in the formal study of meaning. Nevertheless, a question might arise whether topological sensitivity is a marginal issue or whether it constitutes a broader phenomenon associated to a number of expressions in a language. Interestingly, it appears that *half*-words are not the sole category in Polish that displays the distinction between topological neutrality and topological sensitivity. In

this section, I will examine more data from Polish partitive words indicating that the pattern is robust.

3.2.1 *Quarter*-words

Let us first consider the contrasts between two other proportional partitives involving what I will refer to as *quarter*-words *ćwierć* and *ćwiartka* 'quarter', as provided in (3.15). Similar, to *połówka*, see (3.1c), *ćwiartka* is a morphologically complex expression derived from *ćwierć* by the suffix *-k-*.

(3.15)   Polish

      a.   ćwierć-∅
            root-inflectional.marker
            'quarter$_1$'

      b.   ćwiart-k-a
            root-derivational.suffix$_1$-infl.marker
            'quarter$_2$'

In terms of agreement, similar to the distinction between *pół* and *połówka*, see (3.2), *ćwierć* is neuter, whereas *ćwiartka* triggers feminine agreement on adjectives and pronouns, as demonstrated in (3.16).

(3.16)   Polish

      a.   To   czerwone ćwierć   jabłka   zgniło.
            this$_{NOM.N}$ red$_{NOM.SG.N}$ quarter$_{1.NOM.N}$ apple$_{GEN.N}$ rot$_{SG.N}$
            'This red quarter of the apple got spoiled.'

      b.   Ta   czerwona ćwiartka   jabłka   zgniła.
            this$_{NOM.F}$ red$_{NOM.SG.F}$ quarter$_{2.NOM.F}$ apple$_{GEN.N}$ rot$_{SG.F}$
            'This red quarter of the apple got spoiled.'

The distribution of *ćwierć* and *ćwiartka* mimics the pattern observed in the *pół* and *połówka* alternation, i.e., both *quarter*-words appear to be sensitive to the type of nominal they combine with, see (3.17) and (3.18), respectively. In particular, as witnessed by the infelicity of the phrases in (3.17c)–(3.17d) as well as (3.18c)–(3.18d) neither of them occurs in set and mass partitives and only *ćwierć* combines with collective nouns. Similar to *połówka*, *ćwiartka* typically combines with food terms or nominal denoting objects people usually cut into comparable pieces.

(3.17)   Polish

    a.   ćwierć   jabłka
         quarter$_1$ apple$_{GEN}$
         'quarter of the apple'

    b.   ćwierć   stosu   (jabłek)
         quarter$_1$ pile$_{GEN}$ (apples$_{GEN}$)
         'quarter of the pile (of apples)'

    c.   #ćwierć   jabłek
         quarter$_1$ apples$_{GEN}$

    d.   #ćwierć   soku
         quarter$_1$ juice$_{GEN}$

(3.18)   Polish

    a.   ćwiartka jabłka
         quarter$_2$ apple$_{GEN}$
         'quarter of the apple'

    b.   #ćwiartka stosu   (jabłek)
         quarter$_2$ pile$_{GEN}$ (apples$_{GEN}$)

    c.   #ćwiartka jabłek
         quarter$_2$ apples$_{GEN}$

    d.   #ćwiartka soku
         quarter$_2$ juice$_{GEN}$

This kind of behavior can be contrasted with a regular fraction expression such as *jedna czwarta* 'one-fourth', see (3.19). Unlike *quarter*-words or the *half*-words *pół* and *połówka*, fractions show no distributional constraints and can freely combine both with quantized predicates involving count singulars and collectives and cumulative predicates such as plural nouns and mass terms.

(3.19)   Polish

    a.   jedna czwarta jabłka
         one-fourth   apple$_{GEN}$
         'one-fourth of the apple'

    b.   jedna czwarta stosu   (jabłek)
         one-fourth   pile$_{GEN}$ (apples$_{GEN}$)
         'one-fourth of the pile (of apples)'

    c.   jedna czwarta jabłek
          one-fourth   apples$_{\text{GEN}}$
          'one-fourth of the apples'
    d.   jedna czwarta soku
          one-fourth   juice$_{\text{GEN}}$
          'one-fourth of the juice'

Based on the distributional evidence, I posit that similar to *pół* and *połówka* the *quarter*-words *ćwierć* and *ćwiartka* are topology-sensitive. Specifically, they quantify only over parts of integrated wholes. To put it differently, expressions having scattered entities such as arbitrary portions of matter or sums of individuals are disallowed as their input. On the other hand, fractions do not show such requirements and can yield a proportion of an entity of any kind. Table 3.3 summarizes the observations, namely the possibility of fraction entity, set, and mass partitives as opposed to the non-existence of proportional set and mass partitives headed by the *quarter*-words in Polish.

	SINGULARS	COLLECTIVES	PLURALS	MASS NOUNS
jedna czwarta	✓	✓	✓	✓
ćwierć	✓	✓	*	*
ćwiartka	✓	*	*	*

Table 3.3: Distribution of Polish *quarter*-words

Moreover, the pattern is further corroborated by the differences in truth conditions of sentences involving fraction partitives and proportional partitives headed by *ćwierć* on the one hand and proportional partitives headed by *ćwiartka* on the other. Similar, to the distinction discussed with respect to *half*-words, see (3.8) and (3.9), there is a contrast in terms of possible verifications of (3.20a) and (3.20b) and (3.20c). The first two sentences are true if any proportion of the apple constituting approximately 25% of the whole apple is red regardless of whether it forms a discontinuous or continuous part. On the other hand, (3.20c) would only be true if an integrated quarter of the surface of the apple were red, i.e., similar to sentences involving *połówka* the partitive headed by *ćwiartka* would not be true of a discontinuous part.

(3.20)   Polish

    a. Jedna czwarta jabłka   jest czerwona.
        one-fourth   apple_{GEN} is   red
        'One-fourth of the apple is red.'

    b. Ćwierć   jabłka   jest czerwone.
        one-fourth apple_{GEN} is   red
        'A quarter of the apple is red.'

    c. Ćwiartka jabłka   jest czerwona.
        one-fourth apple_{GEN} is   red
        'A quarter of the apple is red.'

Again, the same effect appears in object position. The incremental themes expressed by fraction partitives and proportional partitives with *ćwierć* can refer either to continuous or discontinuous entities. Hence, the sentences (3.21a) and (3.21b) would be true in a scenario where Marysia ate one piece of the apple as well as in a scenario where she took several unconnected bites constituting approximately 25% of the total volume. In contrast, (3.21c) requires the theme of the verb of consumption to be an integrated object, i.e., it would be judged false in the second scenario.

(3.21)   Polish

    a. Marysia zjadła jedną czwartą jabłka.
        Marysia ate   one-fourth   apple_{GEN}
        'Marysia ate one-fourth of the apple.'

    b. Marysia zjadła ćwierć   jabłka.
        Marysia ate   quarter_1 apple_{GEN}
        'Marysia ate a quarter of the apple.'

    c. Marysia zjadła ćwiartkę jabłka.
        Marysia ate   quarter_2 apple_{GEN}
        'Marysia ate a quarter of the apple.'

Furthermore, examples naturally occurring in the NCP corpus seem to corroborate the intuitions described above. The sentences in (3.22a) and (3.22b) involve reference to foodstuffs. The first does not report that the best meal in the place in question consisted of, say, fried chicken strips, but rather that the portion of meat was processed in one piece. Similar, (3.22b) means that there was a continuous part of a

baguette in a bag and not, e.g., several slices. Finally, (3.22c) would be judged as false if what slided out were some scraps of a paper sheet.

(3.22)   Polish (NCP)
    a.   Najlepsza tu   kolacja: ćwiartka kurczaka…
          best   here dinner quarter$_2$ chicken
          'The best dinner here was a quarter of a chicken…'
    b.   Biorę (…) torebkę, w której   leży ćwiartka (…)
          I-take   bag$_{ACC}$ in which$_{LOC}$ lies quarter$_2$
          bagietki…
          baguette$_{GEN}$
          'I'm taking a bag in which there is a quarter of a baguette.'
    c.   Ze   środka   wysunęła się ćwiartka papieru.
          from interior$_{GEN}$ slided-out REFL quarter$_2$ paper$_{GEN}$
          'A quarter of a paper sheet slided out from inside.'

The comparison of extensional properties of partitives involving fractions, *ćwierć*, and *ćwiartka* with regard to topological relations is given in Table 3.4. The results show the very same pattern as discussed with regard to Polish *half*-words, see 3.2.

	CONTINUOUS PART	DISCONTINOUS PART
jedna czwarta	✓	✓
ćwierć	✓	✓
ćwiartka	✓	*

Table 3.4: Denotations of Polish *quarter*-words

The comparison of Polish *quarter*-words with fractions mirrors the pattern observed for the distinction between topology-neutral and topology-sensitive *half*-words. In the next section, I will discuss even more evidence from partitive words.

3.2.2 *Piece*-words

Another class of expressions displaying topological sensitivity consists of *piece*-words. Polish distinguishes between two such expressions, namely *cząstka* and *kawałek* 'piece'. The first is derived from the par-

titive word *część* 'part' by the suffix *-k-*, compare (3.23a) and (3.23b), whereas the second appears to be a basic form.[5] Similar to English *piece*, in many contexts *kawałek* and *cząstka* can be used interchangeably with the *part*-word *część* though the first is a more frequent expression than the latter.[6] Nonetheless, similar to other Polish partitive words also in this case there are some interesting topological properties worth discussing.

(3.23) Polish

    a. część-∅
       root-inflectional.marker
       'part'

    b. cząst-k-a
       root-derivational.suffix-infl.marker
       '$piece_1$'

    c. kawałek-∅
       root-inflectional.marker
       '$piece_2$'

The distribution of the two expressions in question shows that they both resist combining with plural DPs, see (3.24b) and (3.25b). Furthermore, the infelicity of phrases such as (3.25c) and (3.25c) suggests that both *kawałek* and *cząstka* impose the same topological restrictions on DPs they combine with as the partitive words *pół*, *połówka*, *ćwierć*, and *ćwiartka*, i.e., disallow cumulative predicates denoting scattered entities.

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5. In fact, *kawałek* is also morphologically complex and from a diachronic point of view it has been formed from *kawał* 'large share'. However, from the perspective of contemporary Polish, the relationship between the two is rather obscure and since the latter is semantically marked, it appears at it were actually derived by means of the deletion of *-ek-* from the first. This seems to be further corroborated by the fact that it is only possible to derive verbs from *kawałek* and not from *kawał*, e.g., *kawałkować* 'to portion' ∼ **kawałować*.

6. Another meaning of the noun *cząstka* is 'physical particle, corpuscule'. This meaning, however, will not be discussed here.

(3.24)   Polish

    a.   cząstka jabłka
       piece$_1$ apple$_{\text{GEN}}$
       'piece of an apple'

    b.   #cząstka jabłek
       piece$_1$ apples$_{\text{GEN}}$

    c.   #cząstka soku
       piece$_1$ juice$_{\text{GEN}}$

(3.25)   Polish

    a.   kawałek jabłka
       piece$_2$ apple$_{\text{GEN}}$
       'piece of an apple'

    b.   #kawałek jabłek
       piece$_2$ apples$_{\text{GEN}}$

    c.   #kawałek soku
       piece$_2$ juice$_{\text{GEN}}$

Nevertheless, under closer inspection it turns out that in fact the *piece*-word *kawałek* is not incompatible with all mass nouns but rather only with one particular class of such expressions. As witnessed in (3.26), mass partitives headed by *kawałek* are possible as long as the downstairs DP does not involve a liquid term, see (3.26a). In particular, *kawałek* can felicitously combine with mass expressions denoting solid substances as well as granular and artifactual aggregates.[7]

(3.26)   Polish

    a.   #kawałek wody
       piece$_1$ water$_{\text{GEN}}$

    b.   kawałek złota
       piece$_1$ gold$_{\text{GEN}}$
       'piece of gold'

---

7.   Although examples such as (3.26c) and (3.26d) are not frequent and seem quite unusual, they are definitely interpretable and the intuitions regarding their meaning are very clear. Notice also that a similar pattern is observed with respect to German *Stück* 'piece' as well as the diminutive forms such as *Stückchen* and *Stückerl* (Nina Haslinger, p.c.).

    c.   kawałek ryżu
          piece$_1$ rice$_{\text{GEN}}$
          'piece of rice'

    d.   kawałek obuwia
          piece$_1$ footwear$_{\text{GEN}}$
          'piece of footwear'

Interestingly, the partitive in (3.26b) does not simply refer to any portion of gold but rather to to an integrated (though most probably amorphous) lump, i.e., to a nugget. Similar, the phrases in (3.26c) and (3.26d) have contiguous entities in their extension. Granular mass terms denote aggregates of objects such as grains of rice or granules of gravel that seem too small or not significant enough to be perceived as individuals in their own right (Grimm 2012b, Sutton and Filip 2016). Instead, they are conceptualized as clusters of elements. As it is well-known, those elements cannot be accessed be regular quantificational expressions such as distributive quantifiers or cardinal numerals, hence the uncountability of granulars. However, the piece-word *kawałek* in (3.26c) can access the part-whole structure of particular granules making up the extension of a mass noun and trigger subatomic quantification, i.e., quantification over parts of a single granule. As a result, the whole partitive can get a part-of-a-singularity interpretation. In particular, (3.26c) is true of a piece of one grain of rice. Similar, object mass nouns refer to stable discrete entities such as pieces of furniture or items of tableware (Gillon 1992, Chierchia 1998a, 2010, Barner and Snedeker 2005, Bale and Barner 2009, Rothstein 2010, Landman 2011). Despite their denotations, object mass nouns are uncountable, i.e., do not allow for quantification over individual objects they refer to. However, *kawałek* yet again quantifies over parts of such discrete objects and as a result (3.26d) is interpreted as referring to a piece of a single shoe.

The findings concerning the distribution of Polish *piece*-words in comparison to the *part*-word *część* are summarized in Table 3.5. Both *cząstka* and *kawałek* have a selectional restriction prohibiting them from combining with expressions denoting arbitrary sums of individuals, i.e., plurals. However, the main difference concerns the distribution with uncountable nouns. Unlike *cząstka* which is generally incompatible with mass terms, *kawałek* is sensitive to the properties of referents

of a particular uncountable noun. In particular, it seems to require a certain stability of form so that parts singled out from a whole can sustain a fixed constant shape. In other words, *kawałek* accepts entities displaying any kind of topological arrangement as long as it guarantees that the spatial form of pieces is relatively stable. This is way it rejects liquid terms but can combine with expressions denoting solid substances, granular aggregates as well as artifacts, i.e., discrete entities related in terms of similar origin and functionality.

| | SINGULARS | PLURALS | MASS NOUNS | |
			liquids	others
część	✓	✓	✓	✓
cząstka	✓	*	*	*
kawałek	✓	*	*	✓

Table 3.5: Distribution of Polish *piece*-words

Another issue concerns the internal structure of objects denoted by partitives headed by *piece*-words. As provided in Table 3.6, similar to the partitive words *połówka* and *ćwiartka*, both *cząstka* and *kawałek* yield parts that can be recognized as contiguous integrated entities within a whole. This contrasts with the standard *part*-word *część* which similar to, e.g., the *half*-word *połowa*, can deliver both continuous and scattered parts. What is especially interesting about *kawałek* is that unlike *połówka* and *ćwiartka* it does not require the whole to be an integrated object. As long as it is not an arbitrary sum of individuals or a constantly deforming substance, *kawałek* yields a portion of the whole that is one piece.

	CONTINUOUS PART	DISCONTINOUS PART
część	✓	✓
cząstka	✓	*
kawałek	✓	*

Table 3.6: Denotations of Polish *piece*-words

The properties of the *piece*-word *kawałek* seem to relate to the scale of individuation proposed by Grimm (2012b). However, there seems to

be an interesting discrepancy. In Grimm's original proposal, see (3.27), liquids and solid substances are grouped together with no ordering relation between them, i.e., *water* falls into the same category as *gold*. Yet the data concerning the distribution of the Polish *piece*-word *kawałek* seem to suggest that an even more fine-grained distinction might be necessary to account for some grammatical phenomena in natural language. Since partitive phrases headed by *kawałek* can only denote stable objects that come in one piece and are able to sustain their shape, they cannot combine with liquid terms such as *water* simply because fluids continually deform, i.e., lack a given shape. Thus, it is possible that distinguishing between liquids and solid substances in terms of ordering on the scale of individuation, as proposed in (3.28), might appear to be required.[8]

(3.27)   Fragment of the scale of individuation (Grimm 2012b, p. 80; adapted)
          liquid/solid substance < granular aggregate < artifactual aggregate…

(3.28)   Modified fragment of the scale of individuation
          liquid < solid substance < granular aggregate < artifactual aggregate…

Before we move on to discussing how the interplay of quantification and topology looks like from a cross-linguistic perspective and whether other languages can shed new light on the issues related to this phenomenon, let us briefly discuss the somewhat surprising fact that mass partitives are countable. In the next section, I will consider the relationship between substances and portions thereof.

3.3   Mass, parts, quantities, and pieces

One of the very few attempts to link the issue of countability with partitivity and pseudo-partitivity (see, e.g., Selkirk 1977) has been pur-

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8.   It would be an interesting enterprise to explore whether there are more natural language expressions sensitive to the distinction between liquid and solid substance terms and if so whether that fact provides any interesting insights on the semantics of mass nouns. However, such a research project lies far beyond the scope of this study.

sued in Chierchia (2010) who raises an interesting question concerning how it can be that one cannot count mass, whereas portions of mass are countable.[9] Specifically, how come that expressions such as (3.29a) and (3.29b) can have a different status with respect to the mass/count distinction and yet refer to the very same entity.

(3.29) English (Chierchia 2010; adapted)
   a. That water is contaminated.
   b. Those three quantities of water are contaminated.

Chierchia examines English relational expressions such as *quantity*, *part*, and *piece* in pseudo-partitives and partitives such as those in (3.30)–(3.32) and observes some differences in their distribution including the following. Though *quantity* is slightly degraded with singular count nouns, it combines felicitously with bare plurals and mass terms. On the other hand, as we have already seen in Section 2.2.1 the English expression *a part of* is incompatible with bare plurals. Moreover, it fails to take mass nouns as its complements. Finally, *piece* seems to fall somewhere in between the two categories since it can head partitives involving singular definite DPs and bear mass nouns but cannot combine with bear plurals.

(3.30) English (Chierchia 2010; adapted)
   a. (?) a quantity of that person
   b. a quantity of apples
   c. two quantities of gold

(3.31) English (Chierchia 2010; adapted)
   a. a part of that person
   b. #a part of apples
   c. #a part of gold

(3.32) English (Chierchia 2010; adapted)
   a. a piece of that pizza
   b. #a piece of apples
   c. a piece of gold

---

9. See also Landman (2016) for related considerations.

Chierchia attributes the selectional restrictions of *quantity*, *part*, and *piece* to input requirements of the corresponding functions ranging over singular entities and/or sets of individuals and analyzes them in terms of partitions imposing relative atomicity and non-overlap of members. [10] Although the approach is definitely thought-provoking, it seems that it misses an important point regarding the contrast between the expressions in question for at least two reasons. First, it neglects the role of topological sensitivity as discussed above. But also it seems to ignore the distinction between counting and measuring (see, e.g., Rothstein 2017). To foreshadow, counting is sensitive to topological characteristics such as being an integrated object, whereas measuring is not.[11] For start, consider the count pseudo-partitive in (3.33a) and count explicit partitive in (3.33b). It seems that what happens in both cases is measurement in terms of volume. In (3.33a), the singular count noun *pizza* is apparently shifted to the mass interpretation via the Universal Grinder or a similar operation. Similar, the *part*-word in (3.33b) could be paraphrased as *portion* or *proportion* and it essentially designates a particular volume of the rice in question.

(3.33)   English (Chierchia 2010; adapted)
    a.   two quantities of that pizza
    b.   two parts of that rice

Why the distinction between counting and measuring is of any significance here? Intuitively, quantities are about volume, pieces involve some form of individuation, and parts can shift between these two aspects of quantification. However, to address this question more properly, let us consider the difference between the Polish *part*-word *część* and *piece*-word *kawałek*. Similar to multiple other languages, Polish does not allow mass terms to combine with cardinal numerals (modulo coercion), see (3.34). However, as witnessed in (3.35) mass partitives headed by *część* are countable. In this case, the *part*-word gets a portion interpretation and measures the extension of the mass noun in terms of volume. For instance, the phrase in (3.35a) designates two out of some number of more or less equally large portions the

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10. I will return to Chierchia's proposal in Section 7.1.
11. I will discuss the distinction between the two operations in detail in Section 5.2.2.

total quantity of gold was divided into. Crucially, it implies that in the relevant context there are more than two such portions. Similar, (3.35b) indicates two comparable portions out of a larger number of portions of rice.

(3.34) Polish
    a. #dwa złota
       two golds
    b. #dwa ryże
       two rices

(3.35) Polish
    a. dwie części złota
       two pieces$_1$ gold$_{GEN}$
       'two pieces of gold'
    b. dwie części ryżu
       two parts rice$_{GEN}$
       'two parts of rice'

However, the lack of a measuring reading in the minimal pair examples involving the *piece*-word *kawałek* in (3.36) suggests that this expression resists mere quantification in terms of volume and can only serve as a basis for counting individuated objects. Neither (3.36a) nor (3.36b) can get the volume interpretations available for (3.35a) and (3.35b), respectively. They can only denote pluralities of pieces, i.e., integrated parts, of gold and rice, respectively.

(3.36) Polish
    a. dwa kawałki złota
       two pieces$_1$ gold$_{GEN}$
       'two pieces of gold'
       (i)   quantification in terms of objects
       (ii) #quantification in terms of volume
    b. dwa kawałki ryżu
       two pieces$_1$ rice$_{GEN}$
       'two pieces of rice'
       (i)   quantification in terms of objects
       (ii) #quantification in terms of volume

To better illustrate the contrast, let us consider several natural sentences. Measuring uses of *part*-words are especially frequent in recipes and other cooking-related contexts. For instance, consider the examples in (3.37). Both in (3.37a) and (3.37b), the count explicit partitives make reference to a particular volume of water and rice, respectively.

(3.37)   Polish

    a.   Dolej   do garnka dwie   części   wody,   a   w
        pour$_{\text{IMP}}$ to pot$_{\text{GEN}}$ two$_{\text{ACC}}$ parts$_{\text{ACC}}$ water$_{\text{GEN}}$ and in
        trzeciej namocz fasole.
        third$_{\text{LOC}}$ soak$_{\text{IMP}}$ beans$_{\text{ACC}}$
        'Add two thirds of the water to the pot and soak the beans in the third part.'

    b.   Wymieszaj kaszę   w proporcji   2:1, to
        mix$_{\text{IMP}}$ porridge$_{\text{ACC}}$ in proportion$_{\text{LOC}}$ 2:1 this
        znaczy dwie   części   ryżu   na jedną część
        means two$_{\text{ACC}}$ parts$_{\text{ACC}}$ rice$_{\text{GEN}}$ on one$_{\text{ACC}}$ part$_{\text{ACC}}$
        prosa.
        millet$_{\text{GEN}}$
        'Mix the porridge in the 2:1 ratio, which means two parts of rice for one part of millet.'

A similar effect can be observed in count partitives with plural nouns. For instance, the count plural *jabłka* 'apples' in (3.38a) is shifted to the mass denotation and the *part*-word *część* quantifies over portions of apple mass making up the relevant apples and not over the apples as separate individuated objects. Whether the ingredients in question are stored as wholes or are cut into pieces plays no role for the truth conditions of the sentences. Consequently, if in a response to (3.38a) one apple was put on the baking tray and a half of an apple was blended or, alternatively, four apples were put on the baking tray and two were blended, the instruction would be followed correctly. Crucially, however, if we swap *część* in (3.38a) for the *piece*-word *kawałek*, we get a significant contrast. Specifically, the sentence in (3.38b) cannot be understood as instructing to put four apples on the baking tray and to blend the remaining two. Instead, it says you should put two integrated parts of different apples on the baking tray as well as blend one such part.

(3.38) Polish

    a. Dwie <u>części</u> jabłek   rozłóż równomiernie na
        two$_{ACC}$ <u>parts$_{ACC}$</u> apples$_{GEN}$ put$_{IMP}$ evenly   on
        blasze do pieczenia, a   pozostałą   <u>część</u>
        tray$_{LOC}$ to baking$_{GEN}$ and remaining$_{ACC}$ <u>part$_{ACC}$</u>
        rozmiksuj.
        blend$_{IMP}$
        'Put two thirds of the apples evenly on the baking tray
        and blend the remaining part.'

    b. Dwa   <u>kawałki</u> jabłek   rozłóż równomiernie na
        two$_{ACC}$ <u>pieces$_{ACC}$</u> apples$_{GEN}$ put$_{IMP}$ evenly   on
        blasze do pieczenia, a   pozostały   <u>kawałek</u>
        tray$_{LOC}$ to baking$_{GEN}$ and remaining$_{ACC}$ piece$_{ACC}$
        rozmiksuj.
        blend$_{IMP}$
        'Put two apple pieces evenly on the baking tray and blend
        the remaining piece.'
        (i)   quantification in terms of objects
        (ii) #quantification in terms of volume

At this point, a preliminary conclusion can be drawn. The structure of parts can be such that it is merely a scattered entity, an unconnected configuration of bits related merely by virtue of belonging to the same whole. However, some parts are arranged in such a manner that they form a spatially contiguous entity that can be conceptualized as an individual in its own right. It appears that natural language is sensitive to this contrast and the Polish data show that this semantic distinction can be encoded formally in the lexicon. Although not every partitive word imposes topological restrictions regarding the spatial form of a portion of an entity the whole partitive construction denotes, there are some that do. Furthermore, it appears that counting and measuring differ with respect to topological properties of quantified entities. While the first operation requires integrated objects, the latter does not.

    The facts discussed above suggest that accounts that neglect the distinction in question most probably miss something crucial about how we humans conceptualize part-whole structures and how it relates to

the phenomenon of countability in grammar. In the next section, I will inspect cross-linguistic evidence that further corroborates this claim.

3.4   Cross-linguistic parallels

Polish gives an important insight into the role of topological notions in quantification over wholes and parts. However, one could wonder to what extend the semantic behavior of proportional partitives discussed above is a Polish idiosyncrasy. In this section, I will discuss cross-linguistic evidence suggesting that topology-sensitive expressions are a relatively widespread phenomenon in partitive constructions though not necessarily expressed by distinct lexical items. For instance, it is reported that the English sentences in (3.39b) and (3.39a) have different truth conditions. While the first is simply true in a scenario where 50% of an apple was eaten by Mary, the meaning of the latter is stronger since it is true only if Mary ate a continuous half of an apple such as the one depicted in Figure 3.1.

(3.39)   English (Guy Tabachnick, p.c.)
    a.   Mary ate half an apple.
    b.   Mary ate a half of an apple.

However, the semantic difference between (3.39a) and (3.39b) is very slight and in multiple contexts it might be extremely hard to detect. Therefore, before we discuss novel data from English, German, Dutch, Portuguese, and Mandarin, let us briefly elaborate on the method developed to tell an integrated-part and a scattered-part reading apart. For the sake of clarity, I will limit the discussion of subatomic topological sensitivity to proportional partitives involving *half*-words.

Since topological aspects of partitivity can be very subtle and intuitions with this respect tend to be somewhat obscure, in order to develop a proper diagnostic to detect topology-sensitive partitive expressions it seems necessary to come up with plain test objects readily divisible into comparable continuous and discontinuous parts differentiated by easily distinguishable properties. For instance, consider the flag test, as represented in Figure 3.3 and 3.4. The Maltese-like flag in Figure 3.3 consists of two continuous areas each constituting 50% of the flag's surface. It is easy to distinguish between the halves

since the left one is red whereas the right one is white. Therefore, the spatial arrangement of the parts fit what I will call an AB pattern. On the other hand, the red area in the Canadian-like flag illustrated in Figure 3.4 is discontinuous, i.e., it does not constitute an integrated half. I will call such an arrangement an ABA pattern.
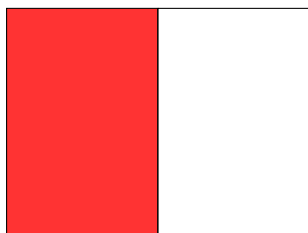
Figure 3.3: Flag AB
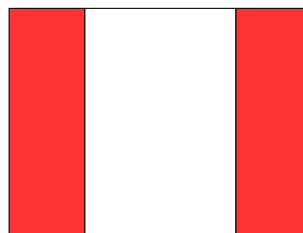
Figure 3.4: Flag ABA

As we have seen in the previous sections, Polish encodes topological sensitivity lexically by differentiating between distinct partitive words. Similar to Polish, Portuguese distinguishes between distinct *half*-words, namely *metade* and *meio* 'half'. The first has nominal features and takes the partitive *da*-phrase, as demonstrated in (3.40), whereas the latter is an adjectival expression agreeing with a modified noun in number and gender. Interestingly, similar to Polish *half*-words *metade* and *meio* differ with respect to both their distributional and referential properties.

Let us first consider the difference in the distribution of the two Portuguese partitive expressions. While *metade* patterns with *połowa* in not being picky about the type of predicate within the *da*-phrase it can combine with, see (3.40), *meio* appears to correspond to *pół* and *połówka*. As demonstrated in (3.41c), it cannot form a partitive with a mass term. Moreover, when combined with a plural noun it lacks a half-of-a-plurality reading, i.e., it cannot be interpreted as a quantifier over individuals making up the denoted plurality. Instead it can only operate on the subatomic level of particular members of the plurality denoted by a modified noun, i.e., it gets a half-of-a-singularity interpretation exclusively. In other words, *meio* can only occur in an entity partitive and not in a set partitive. For instance, (3.40b) cannot refer to a half of the total number of apples. The only interpretation available regards a plurality of halves of apples.

106

(3.40)   Portuguese (Muriel Assmann, p.c.)

    a.   metade da   maça
        half$_1$ the$_{SG.F}$ apple$_F$
        'half of the apple'

    b.   metade das   maças
        half$_1$ the$_{PL.F}$ apples$_F$
        'half of the apples'

    c.   metade do   suco
        half$_1$ the$_{SG.M}$ juice$_M$
        'half of the juice'

(3.41)   Portuguese (Muriel Assmann, p.c.)

    a.   meia   maça
        half$_{2.SG.F}$ apple$_F$
        'half of the apple'

    b.   #meias   maças
        half$_{2.PL.F}$ apples$_F$

    c.   #meio   suco
        half$_{2.SG.M}$ juice$_M$

It seems plausible to posit that the contrasts between the phrases in (3.40) and (3.41) follow from topological restrictions imposed by *meio* and *metade* on what they select, similar to the distinction between Polish *pół* and *połówka* on one hand and *połowa* on the other. In other words, the nominal *half*-word *metade* is topology-neutral, and thus can combine with DPs that denote integrated objects as well as those that have scattered entities in their extensions. On the other hand, adjectival *meio* appears to be sensitive to the type of topological relations holding between parts of the referents of NPs it modifies. In particular, it requires integrated wholes. From the syntactic perspective, *meio* as a nominal modifier sits lower in the structure than *metade* which c-commands the whole DP, i.e., as an adjective it is merged with the NP. Assuming that the NumP is higher than the AP (e.g., Ritter 1991, 1992) this might explain scopal properties of *meio* when it comes to subatomic quantification with respect to regular plurals, i.e., why (3.41b) denotes a plurality of halves rather than a half of a plurality. However, *meio* can also combine with lexical plurals such as pluralia tantum, see (3.42), and it has been argued that the plural in lexical

plurals is significantly lower, i.e., inside the nP (see, e.g., Acquaviva 2008, Alexiadou 2011, Smith 2015, 2017). Notice that (3.42) gets a similar interpretation as the phrase in (3.41b), i.e., denotes a plurality of halves or rather, due to the number-neutrality of *óculos* 'eye-glasses', either a plurality of halves or one half of an object. I argue that this fact alongside the infelicity of phrases where *meio* modifies mass terms, see (3.41c), supports the semantic explanation of the observed behavior in terms of topological sensitivity.

(3.42) Portuguese (Muriel Assmann, p.c.)
        meio   óculos
        half$_{2.M}$ eye-glasses$_M$
        a.   part-of-a-singularity reading
        b. #part-of-a-plurality reading

Even more interestingly, the results of the flag test show that *meio* in fact patterns with Polish *połówka*. As reported by my informant, the sentence with the proportional partitive with *metade* in (3.43a) is true both in the flag AB, see Figure 3.3, and in the flag ABA scenario, see Figure 3.4. However, the sentence involving (3.43b) is judged true only in the first case and false in the latter. This means that *metade* does not impose any topological restrictions on the portion of an entity denoted by the downstairs DP, wheres *meio* yields only integrated parts.

(3.43) Portuguese (Muriel Assmann, p.c.)
        a.   Metade da bandeira é vermelha.
             half$_1$ the flag   is red
             'Half the flag is red.'
             (i)   AB
             (ii)   ABA
        b.   Meia bandeira é vermelha
             half$_2$ flag   is red
             'A half of the flag is red.'
             (i)   AB
             (ii) #ABA

A similar pattern is attested in Dutch where the distinction between the nominal *half*-word *de helft* 'half' on the one hand, see (3.44a), and adjectival *half* 'half' on the other, see (3.44b), is used in order to in-

dicate the discussed contrast. The first expression forms a standard proportional entity partitive by combining with the prepositional *van*-phrase which takes a singular DP as its complement (de Hoop 1997, 2003; see also Cleven 2013, pp. 16–18). The latter, however, resembles *meio* in that it shows agreement with the modified NP. The suffix -*e* on *half* shows up when the noun is either definite or common gender, or both, as in the case of (3.44b). Similar to the Portuguese example, the flag test shows that *de helft* licenses both a continuous-half and a discontinuous-half interpretation, whereas adjectival *half* disallows the latter reading. In other words, while the sentence in (3.44a) is reported to be true of both Figure 3.3 and 3.4, (3.44b) is judged false in the ABA flag scenario. This fact strongly suggests that Dutch lexicalized the distinction between topologically neutral and topologically sensitive partitive words. While *de helft* is similar to Polish *połowa* or Portuguese *metade* in that it simply returns an entity constituting 50% of a whole, truth conditions imposed by *half* are stronger, i.e., the partitive yields a half that needs to be integrated.

(3.44)   Dutch (Erlinde Meertens, Izabela Jordanoska, p.c.)

    a.   De helft van de vlag is rood.
            the half$_1$ of   the flag is red
            'Half the flag is red.'
            (i)   AB
            (ii)   ABA
    b.   De halve vlag is rood.
            the half$_2$ flag is red
            'A half of the flag is red.'
            (i)   AB
            (ii) #ABA

It turns out that similar to Polish Portuguese and Dutch distinguish between topology-neutral and topology-sensitive *half*-words lexically. However, it is not the only way the distinction can be encoded in natural language. Thus, let us now consider a few strategies languages use to differentiate between the two partitive meanings in question I have identified. Interestingly, different structures give rise to similar semantic effects concerning topological relations in subatomic quantification. The sample discussed here is relatively small, but the discussed data

suggest that the observed semantic phenomenon is systematic and cross-linguistically valid.

Let us start with the syntactic distinction occurring in English, as already mentioned in (3.39). From the morphological point of view there is no contrast between the partitive word *half* in (3.45a) and (3.45b). Nevertheless, the syntactic structures in which it appears in each of the two sentences significantly differ (see Vannestål 2004 for discussion). In (3.45a), *half* is a predeterminer since it can occur before regular determiners such articles and demonstratives (Quirk et al. 1985, pp. 257–258). On the other hand, in (3.45b) *half* forms a standard proportional entity partitive by taking the *of*-phrase with a singular DP as its complement. In this environment it seems to have nominal properties since it can be preceded by a determiner and it allows for numeral and adjectival modification (Huddleston and Pullum 2002, p. 434). The syntactic distinction translates into a semantic difference. Specifically, the sentence in (3.45a) is considered appropriate with respect to both the AB and ABA flag in Figure 3.3 and 3.4, respectively, whereas (3.45b) would be judged true only in the first scenario. The contrast shows the relevance of topology in subatomic quantification even in the absence of distinct lexical items differentiating between a continuous-part and a discontinuous-part interpretation. In other words, it appears that natural language can employ purely syntactic means in order to indicate the notion of topological integrity of parts. Notice that the *half*-word in (3.45b) is preceded by by the article. Superficially, the distinction between *half* and *a half of* resembles to some extent the contrast between the *part*-expressions *part of* and *a part of*. The appearance of the indefinite article suggests that the latter should be treated on a par with count nominals and countability seems to imply spatial integrity.

(3.45)   English (Jonathan Bobaljik, Jeffrey Parrott, p.c.)
      a.   Half the flag is red.
          (i)   AB
          (ii)   ABA
      b.   A half of the flag is red.
          (i)   AB
          (ii) #ABA

Another strategy is employed by German. At first blush, it appears to pattern with Portuguese and Dutch since it also distinguishes between the nominal *half*-word *Hälfte* 'half' and adjectival *halb* 'half'. They show similar behavior with respect to count singulars, plurals, and mass terms as the *metade/meio* and *de helft/half* alternations. As witnessed in (3.46a), *Hälfte* combines felicitously with all the types of expressions in question, whereas *halb* is incompatible with mass nouns, see (3.47c), and obligatorily gets a plurality-of-parts interpretation with plurals, as in (3.47b). These facts suggest that *Hälfte* is topology-neutral, whereas *halb* is sensitive to the spatial constitution of referents of the NP it attaches to. Similar to Portuguese *meio* and Dutch *half* it rejects cumulative predicates which denote scattered entities.[12]

(3.46)   German (Nina Haslinger, p.c.)

    a.   Hälfte des   Apfels
        half$_1$ the$_{GEN}$ apple$_{GEN}$
        'half of the apple'

    b.   Hälfte der   Äpfel
        half$_1$ the$_{GEN}$ apples$_{GEN}$
        'apple-halves'

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12. Note that the pattern reported in (3.47) holds only for indefinite DPs. For reasons unclear to me, when the definite article c-commands NPs modified by *halb*, new interpretations emerge in an unexpected way. In particular, (ib) gets a part-of-a-plurality reading, whereas (ic) is absolutely felicitous. Whatever mechanism is responsible for the contrast, I assume it is related with some non-trivial properties of the definite article.

(i)   German (Nina Haslinger, p.c.)

    a.   der   halbe Apfel
        the$_{2.SG.M}$ half$_{SG.M}$ apple$_M$
        'the half of the apple'

    b.   die   halbe Äpfel
        the$_{2.PL}$ half$_{PL}$ apples
        'the half of the apples'

    c.   der   halbe Saft
        the$_{2.SG.M}$ half$_{SG.M}$ juice$_M$
        'the half of the juice'

    c.   Hälfte des   Saftes
          half$_1$ the$_{GEN}$ juice$_{GEN}$
          'half of the juice'

(3.47)   German (Nina Haslinger, p.c.)

    a.   ein halber   Apfel
          a$_M$ half$_{2.SG.M}$ apple$_M$
          'a half of the apple'
    b.   #halbe   Äpfel
          half$_{2.PL}$ apples
          'apple-halves'
    c.   #halber   Saft
          half$_{2.SG.M}$ juice$_M$

However, despite identical selectional restrictions German *halb* differs significantly from the corresponding adjectival *half*-words in Portuguese and Dutch. As the results of the flag test reveal, unlike in the case of *meio* and *half* there is no contrast between *halb* and *Hälfte* with respect to the the spatial make up of the partial entity they yield. As witnessed in (3.48a) and (3.48b) sentence with both types of expressions are judged true in both the AB and the ABA flag scenario. It turns out that *halb* patterns with *pół* rather than with *połówka*, i.e., it requires an integrated object as an input and simply returns its half irrespective whether it is a continuous or discontinuous entity.

(3.48)   German (Nina Haslinger, Maximilan Prüller, p.c.)

    a.   Die Hälfte von der   Fahne ist rot.
          the half$_1$ of   the$_{GEN}$ flag   is red
          'Half the flag is red.'
          (i)   AB
          (ii)   ABA
    b.   Die halbe Fahne ist rot.
          the half$_2$ flag   is red
          'Half the flag is red.'
          (i)   AB
          (ii)   ABA

However, there is yet another construction in German which displays the kind of semantic behavior observed in typologically sensitive ex-

pressions such as Polish *połówka*, Portuguese *meio*, and Dutch *half*. In particular, consider the subject of the sentence in (3.49a) involving the *half*-word *Hälfte* preceded by both what at first blush appears to be an indefinite article and the definite article *die* 'the'. Notice, however, that the element *eine* in this environment actually is not an indefinite article. The mere fact that it can combine with the definite article *die* suggests it is an expression of a different type. Furthermore, unlike regular indefinite articles, it is always stressed. As shown by the ungrammaticality of (3.49b), in this construction one cannot put stress on the *half*-word.

(3.49) German (Maximilian Prüller, p.c.)

    a. Die EINE Hälfte der   Fahne ist rot.
        the a/one half$_1$ the$_{\text{GEN}}$ flag is red.
        'The half of the flag is red.'
    b. *Die eine   HÄLFTE der   Fahne ist rot.
        the a/one half$_1$ the$_{\text{GEN}}$ flag is red.

Interestingly, the truth conditions of sentences with the *die eine Hälfte* partitive expression contrast with sentences involving regular proportional partitives with respect to the flag test. As we have already discussed, (3.50a) is true of both relevant scenarios, i.e., Figure 3.3 and 3.4. However, (3.50b) is consistently judged false in the flag ABA situation. This shows that similar to Polish German developed a tripartite semantic distinction between different proportional partitive expressions, i.e., topologically neutral partitives headed by *die Hälfte* on the one hand and topology sensitive expressions modified by adjectival *halb* as well as *die eine Hälfte* constructions which yield integrated parts on the other. The expressions are distinguished from each other both lexically and structurally with the most marked structurally marked form being at the same time most semantically complex.

(3.50) German (Nina Haslinger, Maximilan Prüller, p.c.)

    a. Die Hälfte von der   Fahne ist rot.
        the half$_1$ of   the$_{\text{GEN}}$ flag   is red
        'Half the flag is red.'
        (i)   AB
        (ii)   ABA

b. Die eine   Hälfte der   Fahne ist rot.
   the a/one half₁   the_GEN flag   is red
   'The half of the flag is red.'
   (i)   AB
   (ii) #ABA

Finally, the examined contrast involving yet another two types of constructions is found in Mandarin. The first kind of structure is provided in (3.51a). Here, the partitive meaning is conveyed by the phrase involving the proportional expression *yí-bàn* 'half' related by the associative marker *de* to the nominal *guó qí* 'national flag' (see, e.g., Jing-Schmidt 2005, p. 294, Jin 2018). In this construction, the *half*-word *bàn* does not follow a classifier and combines directly with the preceding cardinal numeral *yí* 'one' forming a constituent. It is possible that *yí* in *yí-bàn* is in fact a grammaticalized expression interpreted as an indefinite rather than a numeral (cf. Hsieh 2008, p. 127, Jin 2018).[13] In any case, this structure is topologically neutral and yields both integrated and discontinuous halves of entities since sentences such as (3.51a) are reported to be true of the flag in Figure 3.3 as well as the one in Figure 3.4. In (3.51b), however, the partitive morpheme *bàn* behaves more like a quantifier since it needs to combine with the classifier *miàn* dedicated to counting flat and smooth objects such as mirrors in order to quantify over parts of the national flag denoted by the nominal. Therefore, it appears that in this case the syntactic status of the *half*-word is on a par with cardinals. And again, a different structure corresponds to a different semantics. Unlike (3.51a), the sentence in (3.51b) can only mean that an integrated half of the national flag is red, i.e., it would be judged false in the scenario represented in Figure 3.2.

(3.51)   Mandarin (Chang Liu, p.c.)

a.   guó   qí de yí-bàn   shì hóng de.
     national flag LNK one-half COP red   LNK
     'Half the national flag is red.'
     (i)   AB
     (ii) ABA

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13. See also Hsieh (2008, p. 46) and Zhang (2011) for the discussion of issues regarding constituency and other types of environments *bàn* appears in.

    b.   bàn-miàn guó   qí   shì hóng de.
          half-CL   national flag COP red   LNK
          'A half of the national flag is red.'
          (i)   AB
          (ii) #ABA

The Mandarin example discussed above concludes this brief exploration of cross-linguistic strategies of encoding sensitivity to topological notions in partitive expressions. Though the sample of languages examined here was relatively small and most probably more strategies await to be discovered within a future thorough cross-linguistic investigation, the presented evidence proves that natural language encodes subtle topological distinctions in the domain of partitivity. The contrasts examined here are often very slight but a careful examination suggests that the topology-related phenomena in question are not something idiosyncratic but rather occur systematically across languages.

In the next section, I will discuss yet another class of expressions that might shed new light on the issues related to the interplay of quantification and topology. In particular, I will consider some similarities and contrasts concerning two *whole*-adjectives in Polish, namely *cały* 'whole' and *kompletny* 'complete'. To this end, I will explore two different respects of being entire as well as the interaction between *whole*-adjectives and different types of nominals from the perspective of subatomic quantification and topological sensitivity.

3.5   *Whole*-adjectives

Due to contrasts parallel to the one illustrated in (3.52), adjectives such as *whole* and *entire* have been analyzed as universal quantifiers over parts (Moltmann 1997) or maximizing modifiers, i.e., expressions restricting exception tolerance (Morzycki 2002).[14] There is a strong intuition that compared to (3.52a) the sentence in (3.52b) implies that no part of the entity in question are exempt from having to satisfy the main predicate. In particular, (3.52a) would be true if a sufficiently

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14. Morzycki (2002) builds on Brisson (1998)'s analysis of *all* in terms of maximizing modification.

significant proportion of the ferret were under the water, i.e., if part of a paw or a tail were not submerged, under most circumstances the sentence would not be considered false. However, the proposition expressed by (3.52b) is stronger. It states that all the parts of the ferret are submerged.

(3.52)   English (Morzycki 2002)
      a.   The ferret is submerged.
      b.   The whole/entire ferret is submerged.

In Polish, alongside the standard *whole*-adjective *cały* 'whole', there is yet another expression that shares some of its semantic properties yet differs in an interesting respect, namely the adjective *kompletny* 'complete'. Though there is only a partial overlap between the two expressions in question in terms of their distribution, in principle they show the same pattern. Let us first consider the distributional potential of *whole*-adjective *cały*. What is interesting about Polish is that it can be used for universal quantification over parts of stuff denoted both by singular count nouns and mass terms. Specifically, (3.53a) refers to all the material parts making up a particular apple, whereas (3.53b) is true of an entity comprising all the portions of juice in a relevant context. On the other hand, when modifying a plural expression such as (3.53c) *cały* cannot scope over the plural. In other words, (3.53c) is true of a plurality of whole apples but false of a whole plurality of not necessarily whole apples.

(3.53)   Polish
      a.   całe   jabłko
          whole$_{\text{SG.N}}$ apple$_{\text{N}}$
          'whole apple'
      b.   cały   sok
          whole$_{\text{SG.M}}$ juice$_{\text{M}}$
          'all the juice'
      c.   całe   jabłka
          whole$_{\text{PL}}$ apples
          'whole apples'
          (i)   plural > whole
          (ii) #whole > plural

Compared to *cały*, the distribution of *kompletny* is significantly restricted. Intuitively, this is due to the fact that it seems to presuppose certain complexity in terms of a part-whole structure. In particular, it can only modify nominals referring to objects consisting of multiple easily recognizable and distinguishable parts such as utensils and clothes.[15] However, apart from this restriction the whole-adjective *kompletny* mirrors the pattern observed for *cały*. For instance, due to the fact that computers consist of many detachable elements, (3.54a) is a meaningful phrase in Polish. Similar, when combined with an object mass term it implies that all discrete objects comprising the extension of the noun are in place. For example, (3.54b) would be true of a collection of equipment where there is no item missing. Moreover, the *whole*-adjective *kompletny* also appears frequently with collective nouns such as *zestaw* 'set' and *kolekcja* 'collection' which denote groups of artifacts. On the other hand, when modifying a plural expressions as in (3.54c) *kompletny* cannot assert of a plurality as such, but rather of singular individuals. In other words, (3.54c) means that individual computers are complete and not the plurality thereof.

(3.54)   Polish

    a.   kompletny   komputer
        complete$_{\text{SG.M}}$ computer$_{\text{M}}$
        'complete computer'

    b.   kompletny   sprzęt
        complete$_{\text{SG.M}}$ equipment$_{\text{M}}$
        'complete equipment'

    c.   kompletne komputery
        complete$_{\text{PL}}$ computers
        'complete computers'
        (i)   plural > complete
        (ii) #complete > plural

Notice also that both *cały* and *kompletny* can combine with pluralia tantum, see (3.55a)–(3.55b). The plurale tantum nominal *nożyczki laparaskopowe* 'laparoscopic graspers' denotes a specialized object con-

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15. There is yet another meaning of *kompletny* that could be glossed as 'total' which does not impose such a constraint. This use is especially frequent in colloquial Polish; however, it will not be considered here.

sisting of multiple detachable and replaceable elements which makes it a proper expression to be modified by *kompletny*. In both cases, the whole phrase is ambiguous between a singular and a plural reading, i.e., it either refers to one whole or complete object, respectively, or to a plurality of whole or complete objects. However, the reading on which (3.55a) or (3.55b) imposes a semantic restriction on a plurality is not available, i.e., the phrases cannot mean something like a whole or complete plurality of laparoscopic graspers. This is despite the fact that the plural in plurale tantum nouns has been argued to be low in the structure, i.e., inside the nP (see, e.g., Acquaviva 2008, Alexiadou 2011, Smith 2015, 2017). Thus, I assume that the data suggest the reason for the lack of the interpretation in question is not exclusively due to the position in structure but also due to some other factor.

(3.55)   Polish

    a.   całe   nożyczki laparoskopowe
         whole$_{PL}$ scissors   laparoscopic$_{PL}$
         'whole laparoscopic graspers'

    b.   kompletne nożyczki laparoskopowe
         complete$_{PL}$ scissors laparoscopic$_{PL}$
         'complete laparoscopic graspers'

Table 3.7 gives an overview of the distribution of the Polish whole-adjectives *cały* and *kompletny* as discussed above. While both can co-occur with singular count nouns and mass terms, the second requires the modified noun to refer to complex entities consisting of multiple easily individuated parts. On the other hand, when combined with regular plurals none of the Polish *whole*-adjectives can scope over the plural. In other words, what is targeted is always the structure of particular singular individuals and not that of a plural entity.

	SINGULARS	MASS NOUNS	PLURALS
cały	✓	✓	#
kompletny	✓	✓	#

Table 3.7: Distribution of Polish *whole* adjectives

The evidence regarding the distribution of the Polish *whole*-adjectives in question suggests the distinction between integrated objects as prototypically denoted by singular count nouns and scattered entities designated by mass terms on the one hand and arbitrary sums of individuals referred to plural expressions. To some extent this contrast resembles what was observed with respect to the distribution of the piece-word *kawałek*. Before I will move on to discussing some semantic differences between *cały* and *kompletny*, let us briefly examine a cross-linguistic context which at first blush might cast some doubt on the cross-linguistic validity of the pattern observed in Polish. In the next section, I will consider the data concerning a German *whole*-adjective modifying plural nouns.

3.5.1 German *ganz* and plurals

Similar to Polish, in German the *whole*-adjective *ganz* 'whole' can be used as a universal quantifier over parts of matter making up referents of both singular count nouns as well as mass terms, as demonstrated in (3.56a)–(3.56b) (see also Moltmann 1997, pp. 123–127).[16]

(3.56)   German (Nina Haslinger, p.c.)

    a.   der   ganze   Apfel
         the$_{\text{SG.M}}$ whole$_{\text{SG}}$ Apfel$_{\text{M}}$
         'the whole apple'

    b.   der   ganze   Saft
         the$_{\text{SG.M}}$ whole$_{\text{SG}}$ juice$_{\text{M}}$
         'all the juice'

However, it has also been reported that at least in some German dialect the adjective *ganz* 'whole' when modifying plural DPs gives rise to a universal quantification interpretation (see Moltmann 1997, pp. 123–127). This is claimed to be possible with plurals referring to sums constituting natural wholes, i.e., sums whose members are considered as having less individuality, and thus do not appear to be prominent with respect to the whole (Moltmann 1997, p. 125). For instance, in the sentences (3.57a) and (3.57b) children are part of a more or less anonymous group, whereas bees are perceived as constituents of a

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16. Similar to what was discussed in footnote

swarm. In both cases, the modifier *ganz* seemingly quantifies over individuals making up the pluralities and the whole DP yields the total number of children and bees involved in coming and stinging, respectively.

(3.57) German (Moltmann 1997, p. 125; adapted)

    a.   Die ganzen Kinder   sind gekommen.
           the$_{PL}$ whole$_{PL}$ children are   come
           'All the children have come.'

    b.   Die   ganzen Bienen haben Maria gestochen.
           the$_{PL}$ whole$_{PL}$ bees   have   Maria stung
           'All the bees have stung Maria.'

However, under more rigorous investigation it appears that at least in one Austrian dialect showing the discussed behavior there is an important distinct between *ganz* and the universal quantifier *alle* 'all'. Whatever the exact semantic contribution of *ganz*, it seems that since unlike *alle* it allows for exceptions. For instance, consider the contrast given in (3.58). While the sentence in (3.58a) can be supplemented with another clause stating that not all of the children have come, (3.58b) obviously excludes such a continuation.

(3.58) German (Maximilian Prüller, p.c.)
Context: My neighbors have 10 children and I don't like them but have to invite them.

    a.   Dann sind die Nachbarn mit ihren   <u>ganzen</u>
           then are   the neighbors with their$_{DAT}$ whole$_{DAT.PL}$

           Kindern   gekommen. Zwei waren krank zuhause,
           children$_{DAT}$ come   two were ill   at-home

           und es waren immer noch acht!
           and it were   still   eight
           'Then the neighbors have come with their children. Two of them were ill and stayed home, but there were still eight!'

    b. #Dann sind die Nachbarn mit <u>allen</u> ihren
        then are the neighbors with all$_{\text{DAT}}$ their$_{\text{DAT}}$
        Kindern   gekommen. Zwei waren krank zuhause,
        children$_{\text{DAT}}$ come   two were ill   at-home
        und es waren immer noch acht!
        and it were still   eight

Although definitely more systematic research is required to establish what precisely is going on, I take the contrast in (3.58) as evidence suggesting that German does not pose a lethal threat for the generalization regarding the topological sensitivity of *whole*-adjectives based on the Polish data. In the next section, I will return to the alternation between *cały* and *kompletny* and examine two different aspects of wholeness.

3.5.2 Integrity vs. maximality

There is yet another difference between the Polish *whole*-adjectives *cały* and *kompletny* which also intuitively appears to be the most significant one. The difference relates to an ambiguity between what I will call an integrity reading and a maximality reading. The first is topological in nature and indicates that the parts of a whole are connected in such a way that they form an intact integrated object.[17] The latter, on the other hand, simply employs universal quantification over parts or mereological exhaustivity, and thus indicates that no part of a whole is missing.[18] Under ordinary circumstances these two aspects of wholeness are interrelated. Hence, the contrast between the two interpretations might seem somewhat subtle and in many contexts it

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17. See also Moltmann (1997, p. 127) for a remark on the integrity reading of German *ganz* in predicate position as, e.g., in (i).

(i)   German (Moltmann 1997, p. 127; adapted)
        Das Glas ist noch ganz.
        the glass is still whole
        'The glass is still intact.'

18. However, see Morzycki (2002) for arguments against treating *whole*-adjectives as universal quantifiers over parts based on the anaphoric and scope properties they give rise to. Instead, he proposes to treat them as maximizing modifiers. Since the main point here concerns the distinction between quantification over wholes and subatomic quantification, I only signal the problem here and leave it unaddressed.

is probably difficult to detect. However, it is relatively easy to come up with examples where it becomes significant. For instance, consider the scenario described in (3.59a). Notice that there are two kinds of model aircraft Jan might have bought for Marysia, namely either a pre-built model that has already been put together or a construction kit with a collection of separate parts that require gluing. Then, depending on whether Marysia got the first or the latter her brother Piotruś's question can be understood along the lines of topological integrity or mereological exhaustivity. In particular, (3.59b) provides an answer to a question implying a maximality reading, i.e., whether no part is missing, whereas (3.59c) is a reply in terms of an integrity interpretation, i.e., whether the parts are connected. This shows that though the two ways in which being a whole can be understood are frequently related, they are in fact separate semantic phenomena and in some contexts only one of them can become prominent if not the only relevant aspect.

(3.59)   Polish
         Context: Jan bought his daughter Marysia a model aircraft.
         After Marysia unwrapped the gift and opened the box, her
         brother Piotruś asks:
         a.   Czy ten samolot   jest cały?
              INT   this model-aircraft is   whole
              'Is this model aircraft whole?'
         b.   Nie, brakuje śmigła.
              no, it-lacks propeller$_{GEN}$
              'No, it lacks the propeller.'
         c.   Nie, trzeba   go   skleić.
              no, it-needs it$_{ACC}$ stick-together
              'No, you need to stick it together.'

Now, let us compare what has been observed in (3.59a) with what happens in the same context when the question in (3.60a) is slightly modified. Specifically, instead of *cały* what appears in predicate position is the *whole*-adjective *kompletny*. Here, the only meaningful answer is the one in (3.60b). The reply in (3.60c) is just remarkably odd. That is because unlike *cały kompletny* is not ambiguous and implies exclusively the maximality aspect of wholeness. Therefore, the question in (3.60a)

can only be interpreted as whether the box includes all the relevant parts of the model aircraft.

(3.60) Polish
        Context: Jan bought his daughter Marysia a model aircraft. After Marysia unwrapped the gift and opened the box, her brother Piotruś asks:
        a. Czy ten samolot   jest kompletny?
            INT this model-aircraft is   complete
            'Is this model aircraft complete?'
        b. Nie, brakuje śmigła.
            no, it-lacks propeller_GEN
            'No, it lacks the propeller.'
        c. #Nie, trzeba   go   skleić.
            no, it-needs it_ACC stick-together
            'No, you need to stick it together.'

The contrast between the number of possible answers in (3.59a) and (3.60) shows that *cały* is ambiguous between an integrity and maximality reading, whereas *kompletny* can be interpreted exclusively in terms of maximality. Though the distinction is delicate, it is systematic and can be detected in multiple environments. Even out of the blue the intuitions regarding the contrast between the sentence in (3.61a) and (3.61b) are very clear. The first states that all the relevant parts of the toy are in place or that the parts are connected, or both, whereas the latter implies only that no relevant part of the toy is missing.

(3.61) Polish
        a. Ta   zabawka jest cała.
            this toy   is   whole
            'This toy is whole.'
            (i)   maximality reading
            (ii) integrity reading
        b. Ta   zabawka jest kompletna.
            this toy   is   complete
            'This toy is complete.'
            (i)   maximality reading
            (ii) #integrity reading

Table 3.8 recapitulates the findings. Distinguishing between a maximality and integrity interpretation allows us to conclude that while the Polish *whole*-adjective *kompletny* is topology-neutral, *cały* is not.

	MAXIMALITY	INTEGRITY
cały	✓	✓
kompletny	✓	*

Table 3.8: Interpretations of Polish *whole*-adjectives

The contrast concerning possible interpretations of the Polish *whole*-adjectives discussed in this section provides yet another piece of evidence that some natural language expressions are sensitive to topological relations holding between particular entities. Specifically, the *whole*-adjective *cały* can be used to imply that all the parts of an object remain in such a spatial configuration that the object in question constitutes an integrated whole. In the next section, I will focus on the maximality aspect of wholeness. In particular, assuming that the semantics of *cały* involves universal quantification over entities of some sort I will examine what exactly can constitute the domain of such quantification.

3.5.3 Universal quantification over parts and wholes

Though the Polish *whole*-adjective *cały* can combine with a wide range of expressions including singular count nouns, prototypical mass terms, granulars, object mass nouns, and plurals, in each case the interpretation of the whole NP depends on the type of referents denoted by the modified noun. The difference becomes apparent when we distinguish between subatomic quantification, i.e., quantification over material parts making up elements of the denotation of a certain noun, and quantification over wholes, i.e., quantification over entire elements. To illustrate the distinction, let us first consider the interpretative contrast between singular count nouns and liquid terms. For instance, the phrase in (3.62a) involves subatomic quantification, whereas (3.62b) does not. The *whole*-adjective *cały* in (3.62a) quantifies universally over material parts of an apple rather than over an apples

124

as such. On the other hand, in (3.62b) *cały* triggers universal quantification over entities in the extension of the modified mass noun. The translations of the sentences in (3.63) illustrate the difference.

(3.62) Polish

  a. całe   jabłko
     whole$_{\text{SG.N}}$ apple$_{\text{N}}$
     'whole apple'
     (i)   subatomic quantification
     (ii) #quantification over wholes
  b. cały   sok
     whole$_{\text{SG.M}}$ juice$_{\text{M}}$
     'all the juice'
     (i)   #subatomic quantification
     (ii) quantification over wholes

(3.63) Polish

  a. Marysia zjadła całe   jabłko.
     Marysia ate   whole$_{\text{ACC}}$ apple$_{\text{ACC}}$
     'Marysia ate the whole apple.'
  b. Marysia wypiła cały   sok.
     Marysia drank   whole$_{\text{ACC}}$ juice$_{\text{ACC}}$
     'Marysia drank all the juice.'

The distinction between subatomic quantification and quantification over wholes becomes more evident in the case of granular and object mass nouns since these expressions allow for both, see (3.64). In particular, the NP in (3.64a) can either refer to the total amount of rice or to rice involving whole grains. Similar, (3.64b) is either true of whole shoes or of the total number of shoes in a given context.

(3.64) Polish

  a. cały   ryż
     whole$_{\text{SG.M}}$ rice$_{\text{M}}$
     'all the rice'
     (i)   subatomic quantification
     (ii) quantification over wholes

  b. całe obuwie
    whole$_{\text{SG.N}}$ footwear$_{\text{N}}$
    'all the footwear'
    (i) subatomic quantification
    (ii) quantification over wholes

The contrast can be clearly demonstrated by considering non-trivially different truth conditions between (3.65a) and (3.66a) on the one hand and (3.65b) and (3.66b) on the other. The most prominent interpretation of (3.65a) is that there is no rice left. On the other hand, the sentence in (3.65b) does not infer that Marysia bought all the rice, but rather that she bought whole grain rice, i.e., rice that has not been milled. Similar, (3.66a) is true only if there was no shoe left in the shop last year, whereas the context rules out this interpretation in (3.66b). Instead, the available reading states, that the factory produces complete shoes.

(3.65) Polish

  a. Marysia zjadła cały ryż.
    Marysia ate whole$_{\text{ACC}}$ rice$_{\text{ACC}}$
    'Marysia ate all the rice.'
  b. Marysia kupiła cały ryż, natomist Jan mączkę
    Marysia bought whole$_{\text{ACC}}$ rice$_{\text{ACC}}$ whereas Jan flour$_{\text{ACC}}$
    ryżową.
    rice$_{\text{ADJ.ACC}}$
    'Marysia bought whole rice, whereas Jan bought rice flour.'

(3.66) Polish

  a. W tym roku nasz sklep wyprzedał całe
    in this$_{\text{LOC}}$ year$_{\text{LOC}}$ our shop sold-out whole$_{\text{ACC}}$
    obuwie.
    footwear$_{\text{ACC}}$
    'This year, our shop sold out all the footwear.'

  b. Nasza fabryka produkuje całe obuwie,
    our factory produces whole_ACC footwear_ACC
    natomiast konkurencja jedynie podeszwy.
    whereas competitors only soles_ACC
    'Our factory produces whole footwear, whereas our com-
    petitors produce only soles.'

Finally, as it has already been mentioned *cały* cannot scope over the plural. Consequently, a phrase such as (3.67a) denotes a plurality of whole objects and not a whole plurality of objects. This behavior is irrespective to whether the modified nominal is a regular plural or a plurale tantum noun, see (3.67b).

(3.67) Polish

  a. całe jabłka
    whole_PL apples
    'whole apples'
    (i) subatomic quantification
    (ii) #quantification over wholes
  b. całe nożyczki
    whole_PL scissors
    'whole scissors'
    (i) subatomic quantification
    (ii) #quantification over wholes

The quantificational behavior of *cały* modifying plurals is well illustrated in the sentence in (3.68). The following clause make it clear that in this case the *whole*-adjective indicates that no part of each relevant apple is exempt from being consumed.

(3.68) Polish
    Marysia zjadła całe jabłka, natomiast Jan zostawił
    Marysia ate whole_ACC apples_ACC whereas Jan left
    ogryzki.
    apple-cores_ACC
    'Marysia ate the whole apples, whereas Jan left the apple
    cores.'

The quantificational properties of the Polish *whole*-adjective *cały* 'whole' are summarized in Table 3.9. The crucial observation is that the domain of quantification depends in a non-trivial manner on the extension of the modified nominal. Specifically, *cały* quantifies over material parts of an entity denoted

	SINGULARS	PLURALS	OBJECT MASS	MESS MASS
subatomic quantification	✓	✓	✓	*
quantification over wholes	*	*	✓	✓

Table 3.9: Quantificational properties of Polish *cały* 'whole'

This concludes the investigations into the relevance of topological sensitivity with respect to part-whole structures in natural language.

3.6 Summary

In this section, I have provided additional evidence for the relevance of the notion of spatial integrity for subatomic quantification. In particular, novel evidence shows that the distinction between topology-neutral and topology-sensitive partitive words is lexicalized in Polish. Specifically, Polish has three morphologically distinct *half*-words, i.e., *połowa*, *pół*, and *połówka*. What they share is that when applied to an entity they return part constituting approximately 50% of a whole. They differ, however, regarding the type of entity they select for as well as the type of part they yield. Given different distributional and referential properties the data could be explained as follows. The half word *połowa* is topology-neutral, i.e., it measures halves of any type of entity, be it a solid individual, mass substance, aggregate, or an arbitrary plurality of individuals. The outcome of quantification is a portion of a whole which is topologically indeterminate. In other words, *połowa* can either designate an integrated entity within the part-whole structure of an individual or merely an arbitrary discontinuous sum of pieces making up the whole. Unlike *połowa*, *pół* is sensitive to the type of entity denoted by its complement. Specifically, it selects only for integrated individuals, i.e., individuated objects that come

in one piece.[19] Nonetheless, similar to *połowa* the resulting partitive phrase can refer either to a continuous or discontinuous half but in this case it has to be part of an integrated whole. Finally, the meaning of *połówka* is the most constrained. It shares selectional restrictions with *pół*, i.e., it excludes scattered entities such as substances and granular aggregates as denoted typically by mass terms as well as pluralities of individuals, but in addition it imposes topological constraints on the extension of the resulting partitive construction. To put it in a different way, *połówka* cuts out an integrated part constituting approximately 50% of the volume of a continuous individuated object.

Similar, the alternation between fractions and the *quarter*-words *ćwierć* and *ćwiartka* 'quarter' corresponds to the pattern observed in *half*-words. In particular, fractions are topology-neutral, whereas *ćwierć* patterns with *pół* and *ćwiartka* mirrors the behavior of *połówka* in that it requires a portion of an object to form a contiguous entity. Polish further distinguishes between distinct *piece*-words one of which, i.e., *kawałek*, yields an integrated object within a part-whole structure as long as the whole does not denote an arbitrary sum of individuals. Furthermore, the contrast between two types of Polish *whole*-adjectives, i.e., *cały* and *kompletny*, makes it easier to determine two distinct aspects of wholeness, namely maximality and integrity.

The cross-linguistic investigation revealed that the phenomenon observed in Polish partitives is not a peculiarity of one language but rather appears in at least several other languages. The distinction is not always expressed lexically and sometimes it manifests itself in different interpretations assigned to different syntactic structures. It turns out that Portuguese and Dutch distinguish between the topology-neutral *half*-words *metade* and *helft* on the one hand and the topology-sensitive adjectives *meio* and *half* semantically resembling Polish *połówka* on the other. In contrast, English does not differentiate between distinct lexical items but rather distinguishes two constructions, i.e., the topology-neutral *half* DP phrase as opposed to the *a half of* DP structure which again shows the semantic behavior similar to *połówka*. The same contrast is reported to hold between the *yí-bàn* and *bàn*-CL constructions

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19. This is a slight simplification since *pół* can also combine with collective nouns. However, as declared above I will ignore this issue here.

in Mandarin. Finally, the German data suggest a tripartite distinction similar to Polish involving the *die Hälfte*, *halb*, and *die eine Hälfte*.

The significance of the data discussed here lays primarily in revealing the relevance of topological relations holding between parts of individuals forcing us to recognize that natural language semantics is sensitive to whether parts come in one piece or constitute discontinuous entities. In the following section, I will introduce yet another set of novel evidence concerning subatomic quantification. In particular, I will show that similar to cardinal numerals which are dedicated to counting wholes there are numerical expressions in natural language dedicated to counting parts.

4 Multipliers

In the previous chapters, I have provided robust linguistic evidence for the relevance of subatomic quantification in natural language as well as for the significant role the topological notion of integrity plays with respect to that phenomenon. I have examined multiple types of partitive constructions including count explicit partitives and topology-sensitive proportional partitives. What these two types of expressions have in common is that they impose a certain constraint on how the denoted parts of objects are conceptualized. Specifically, they require the parts to constitute integrated continuous elements. On the other hand, we have seen that subatomic quantification is not restricted to partitive constructions and is also attested in the adjectival domain, e.g., German and Romance adjectival *half*-words as well as different types of *whole*-adjectives. At this point, one could expect that given the amount of evidence for the relevance of part-whole structures in different types of constructions there should be quantificational expressions in natural language that are specialized for counting parts of entities. It turns out that in fact in many languages there are such expressions.

In this chapter, I will present novel data concerning subatomic quantification in the adjectival domain. In particular, I will examine semantic properties of numerical expressions such as English *double* and *triple* which I will refer to as MULTIPLIERS (following Quirk et al. 1985, Huddleston and Pullum 2002). In particular, I will focus mostly on Slavic multipliers exemplified by Polish *podwójny*, Czech *dvojitý*, Russian *dvojnoj*, and Bosnian-Croatian-Serbian (BCS) *dvostruki* all 'double' which all seem to have identical properties. Unlike cardinal numerals, multipliers do not count entities, but rather their particular parts. I have chosen Slavic multipliers due to their morphological complexity which suggests a rich semantic structure that is not expressed formally in a language such as English. To foreshadow, I will argue that since multipliers in Slavic are derivationally complex, they are in fact compositional. Though the distribution of multipliers is relatively broad, I will mainly concentrate on a subset of environments in which they can

occur that allows us for novel insights with respect to the phenomena discussed in this study.[1]

4.1   Significance of multipliers

Since the early days of formal semantics (starting with Montague 1973) the meaning of quantificational expressions such as numerals has been constantly receiving a lot of attention. Over the last several decades, extensive and extremely important work has been done in this area leading to a number of influential theories of cardinals (e.g., Barwise and Cooper 1981, Scha 1981, Landman 2004, Ionin and Matushansky 2006). Nevertheless, it seems that there is still a vast territory left unchartered since natural languages exhibit multiple classes of numerical expressions that did not receive nearly as much recognition as cardinal numerals. In this study, I will contribute to the study of such somewhat neglected classes of quantifiers by providing novel evidence considering multipliers. Though such expressions are cross-linguistically common, see (4.1), as for now their semantic properties are surprisingly understudied and I am not aware of any formal attempt to account for their behavior and meaning.[2]

(4.1)   a.   English
            double
        b.   German
            doppelt
        c.   Italian
            doppio
        d.   Lithuanian
            dvigubas
        e.   Finnish
            kaksinkertainen

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1.   Most of the data discussed in this chapter come from linguistic corpora. However, I would like to sincerely thank Mojmír Dočekal, Pavel Caha, and Markéta Ziková, Tetiana Kamyshanova, Boban Arsenijević, Guy Tabachnick, and Viola Schmitt for confirming my intuitions concerning some of the Czech, Russian, BCS, English, and German phrases, respectively.

2.   An exception is Wągiel to appear a on which the whole chapter is loosely based.

> f. Hungarian
>   dupla
> g. Mandarin
>   shuāng

The fact that multipliers have been somewhat overlooked is even more striking given that they exhibit non-trivial quantificational behavior that differs significantly from the one observed in cardinal numerals. For instance, consider examples such as those in (4.2). The noun phrase in (4.2a) simply denotes a set of singular entities having the property of being a crown. Interestingly, although the phrase in (4.2b) is true of a plurality of two entities, (4.2c) can only be interpreted as referring to a single individual similar to (4.2a). There is, however, a crucial difference between the two expressions since (4.2c) seems to involve subatomic quantification. In particular, the multiplier appears to quantify over elements within the inner structure of a denoted entity. In other words, *double* in (4.2c) restricts the denotation of the noun to only those crowns that have a particular complex form.

(4.2)   English
>   a. crown
>   b. two crowns
>   c. double crown

In the following parts of this chapter, I will discuss distribution and semantic properties of Slavic multipliers with a special focus on Polish. I will argue that such expressions are compositional and involve quantification over objects that are conceptualized as integrated parts of integrated wholes. Furthermore, I will suggest a generalization concerning the meaning of a representative subset of data, i.e., multiplier phrases involving concrete singular nouns, and point out some non-trivial consequences as well as discuss other types of nominals multipliers combine with and how it relates to subatomic quantification.

4.1.1 Morphological complexity of Slavic multipliers

Since it is widely recognized that the derivational morphology in Slavic languages is particularly rich, in recent years increasing at-

tention has been drawn to the semantics of Slavic morphologically complex numerical expressions. The research has lead to a number of insightful investigations concerning such constructions in Czech (Dočekal 2012, 2013, Dočekal and Wągiel to appear), Polish (Wągiel 2014, 2015b, to appear a, to appear b), and Russian (Khrizman 2015). However, while previous research focused mainly on the impact of numeral morphology on the collective/distributive alternation with respect to different types of numerals, in this study I will examine the adjectival domain and present evidence of the significance of multipliers for subatomic quantification, i.e., quantification over cognitively salient parts of entities. In Section 3.1 and 3.2, we have seen how formal complexity correlates with topology-sensitive semantics in partitive words in Polish. In this section, I will focus on the morphology of Polish, Czech, Russian, and BCS multipliers which unlike their English counterparts exhibit a complex derivational structure. I believe this choice of data will be instructive since it will allow us to confront multipliers with cardinals in a straightforward manner that will reveal intriguing similarities as well as differences between the two classes.

The examined lexical items in question will be exemplified in the following sections by expressions derived from the numeral root $\sqrt{dw}/\sqrt{dv}$ corresponding to the number 2. Let us assume the morphological make-up of the basic cardinals and matching multipliers as in (4.3)–(4.6).[3] As can be witnessed in the morpheme orderings in (4.3b)–(4.6b), all the multipliers in question consist of numeral roots shared with corresponding cardinals and some additional morphology including the affixes *po*⟩...⟨*n-/-it-/-n-/-struk-* and the morphemes *-ój-/-oj-/-o-* which mark a non-cardinal stem as well as inflectional markers. Notice that the morphemes *-ój-/-oj-/-o-* appear in multipliers derived from numeral roots referring to numbers 2–3. In multipliers derived from roots referring to the number 4, the suffix *-ór-/-er-/-oro-*

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3. For convenience, I provide only unmarked forms for cardinals in (4.3a)–(4.6a). Similar, in (4.3b)–(4.6b) masculine forms represent whole declensional paradigms. For BCS in (4.6b), the derivationally complex adjective *dvostruki* 'double' has been chosen for sake of exposition despite the fact that there is also another BCS multiplier, namely *dupli* 'double'. Although it seems that there is a difference in the distribution of the two forms, I assume that there is no relevant semantic distinction between them with respect to the phenomena discussed here.

occurs instead, e.g., Polish *po⟩czw-ór⟨n-y*, Czech *čtv-er-n-ý*, Russian *četv-er-n-oj*, and BCS *četv-oro-stuk-i* all 'quadruple'.

(4.3) Polish

    a. dw-a
        numeral.root-inflectional.marker
    b. po⟩dw-ój⟨n-y
        circumfix⟩numeral.root-non.card.stem⟨circumfix-infl.marker

(4.4) Czech

    a. dv-a
        numeral.root-inflectional.marker
    b. dv-oj-it-ý
        numeral.root-non.cardinal.stem-deriv.suffix-infl.marker

(4.5) Russian

    a. dv-a
        numeral.root-inflectional.marker
    b. dv-oj-n-oj
        numeral.root-non.cardinal.stem-deriv.suffix-infl.marker

(4.6) BCS

    a. dv-a
        numeral.root-inflectional.marker
    b. dv-o-struk-i
        numeral.root-non.cardinal.stem-deriv.suffix-infl.marker

In general, Slavic multipliers exhibit adjectival morphology and agree with modified nouns in case, number, and gender, as in (4.7). I will refer to the derivational circumfix *po⟩...⟨n-* in Polish and suffixes *-it-*, *-n-*, and *-struk-* in Czech, Russian, and BCS, respectively, as multiplicative morphemes. Moreover, I will assume here that inflectional markers and morphemes constituting non-cardinal stems have no contribution to the semantics of multipliers. Therefore, in the analysis to come I will focus only on the interaction between numeral roots and multiplicative morphemes.

(4.7)   Polish
        Ta   podwójna   szyba   się stłukła.
        this_{NOM.SG.F} double_{NOM.SG.F} glass_{NOM.SG.F} REFL broke_{SG.F}
        'This double glazing broke.'

An approach neglecting the semantic role of some of the examined morphemes in composition might seem in fact a considerable simplification since affixes marking non-cardinal stems are attested not only in multipliers, but also in a variety of other numerical expressions. For instance, consider Polish denumeral group nouns such as *dwójka* '(group of) two', derived numerals presupposing that a plurality is heterogeneous with respect to the natural gender of referents such as *dwoje* 'two (one male and one female)', taxonomic numerals like *dwojaki* or *dwoisty* both 'twofold' as well as denumeral verbs such as *podwoić* 'to double'.[4] At first blush, it seems appealing to assume that the appearance of non-cardinal stem markers in all the above cases is non-coincidental from the semantic point of view. Therefore, an alternative approach could assign, e.g., Polish *-ój-/-oj-/-oi-* a meaning general enough to cover the whole variety of considered expressions, i.e., an underspecified operation on numbers that is different from the one employed in cardinals. Nevertheless, in this study I will remain agnostic with respect to that possibility. Instead, I will assume that the discussed markers are semantically vacuous and that their sole function is purely structural, specifically they simply form stems. Similar, for the sake of simplicity I will ignore all the intricacies related to the semantics of gender as well as its relationship with quantification (see, e.g., Arsenijević 2016 and Fassi Fehri 2016, 2017) and the role of inflectional markers in coding such an interaction.

4.1.2 Subatomic quantifiers

In the previous section, we have seen that in Slavic the morphological make-up of multipliers resembles to that of cardinals with the crucial difference that the first consistently involve more structure. Let us now consider the meaning of phrases in which multipliers modify a noun. I will start with a set of examples in (4.8)–(4.11) which are

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4. See Wągiel (2014, 2015b) for a discussion and possible analysis of expressions such as *dwójka* and *dwoje*. Notice also that *-ój-*, *-oj-*, and *-oi-* are allomorphs.

semantically analogous and equivalent to the English phrase in (4.2c). These examples will be considered as illustrative since they are all attested in representative text corpora such as the National Corpus of Polish (NCP), Czech National Corpus (CNC), Russian National Corpus (RNC), and Croatian National Corpus (CrNC). Furthermore, the Polish noun *korona* 'crown' ranks at the 13th place on the list of collocation candidates within two-words collocations for the form *podwójna* 'double' in the balanced sample of the National Corpus of Polish whereas the Czech noun *koruna* 'crown' ranks at the 7th place for the form *dvojitá* 'double' in a corresponding Czech sample in the Czech National Corpus. Moreover, the example is of historical relevance since it resembles the famous case of the complex papal crown discussed in the philosophical literature (Wiggins 1980) which I will also briefly address.

(4.8) Polish (NCP)
podwójna korona
double$_{\text{SG.F}}$ crown$_{\text{F}}$
'double crown'

(4.9) Czech (CNC)
dvojitá koruna
double$_{\text{SG.F}}$ crown$_{\text{F}}$
'double crown'

(4.10) Russian (RNC)
dvojnaja korona
double$_{\text{SG.F}}$ crown$_{\text{F}}$
'double crown'

(4.11) BCS (CrNC)
dvostruka kruna
double$_{\text{SG.F}}$ crown$_{\text{F}}$
'double crown'

As already mentioned, noun phrases such as those in (4.8)–(4.11) denote sets of singular (and not plural) entities despite the fact that in the morphological make-up of multipliers modifying a noun there are numeral roots which involve reference to the number 2. Given the classical perspective on the meaning of numerals, at first sight this fact

might appear somewhat puzzling. Nevertheless, it seems clear that in all the multiplier examples discussed in this chapter quantification is involved albeit it is not quantification over wholes. To illustrate this claim, let us consider the historical example in (4.12).[5] The sentence in (4.12a) indicates a complex inner structure of the Pschent, and thus entails the sentence in (4.12b). If it is true of an object that it is a double crown, it has to be a crown and it has to consist of two elements.

(4.12) Polish

 a. Pszent to podwójna korona.
   Pschent this double crown
   'The Pschent is a double crown.'

 b. $\models$ Pszent składa się z dwóch części.
   Pschent consists REFL from two$_{GEN}$ parts$_{GEN}$
   'The Pschent consists of two parts.'

However, it is not sufficient that the object referred to by the noun *Pszent* consists of any two elements since it is not the case that every crown consisting of two parts is a double crown. Rather, the two relevant elements need to have a particular property, i.e., a property comparable to the property of the whole. In other words, a double crown consists of two parts which can be considered crowns themselves. For instance, the reader familiar with ancient history will recognize the Pschent as a crown worn by rulers of Ancient Egypt. Historically, as demonstrated in Figure 4.1 the Pschent combined two parts, i.e., the Red Deshret Crown of Lower Egypt, see Figure 4.2, and the White Hedjet Crown of Upper Egypt, see Figure 4.3. Its design represented the pharaoh's power over all of the unified kingdom. Due to the discussed features of the relevant parts of the Pschent, one could say it is a two-in-one crown.

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5. In the following text, the phenomena will be discussed mainly on the basis of Polish examples. However, the generalizations are intended to hold for every language in question.
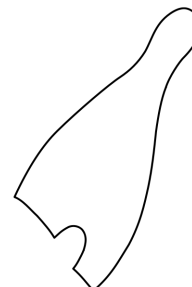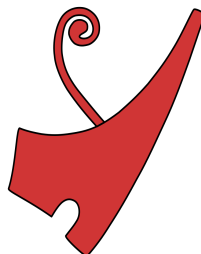
Figure 4.1: Pschent   Figure 4.2: Deshret   Figure 4.3: Hedjet

Note that there is only one cut that divides the Pschent into the Deshret and Hedjet and this characteristic can be generalized to any hypothetical double crown. In other words, not every part of a double crown is a crown itself. On the contrary, in this and similar cases within the set of all the parts of a whole there are exactly two parts of which a property comparable to the property of the whole holds. Therefore, the semantics of Slavic predicates such as those in (4.8)–(4.11) and others corresponding to the English phrase *double crown* does not involve the property of divisive reference which is sometimes assumed to hold for mass nouns (e.g., Cheng 1973).[6] Notice also that the referents of multiplier phrases differ from things designated by nouns like *twig*, *rock*, and *fence*. Though such entities also involve parts that have the property of a whole, e.g., parts of a twig are also twigs and parts of a rock are also rocks (e.g., Zucchi and White 2001, Rothstein 2010), the crucial distinction lies in the arbitrariness of subdivisions. While objects in the extensions of *twig*-like expressions can be partitioned in an arbitrary manner and the resulting parts will still have the property of a whole (assuming some level of granularity), complex individuals denoted by multiplier phrases typically require a specific subdivision in order to identify the salient parts. This property makes them on a par with the already mentioned example of the papal tiara which is a complex object constituting a triple crown. What I believe to be the signature property of such individuals is that they are simultaneously

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6. I do not share this view. For an overview of the problems related to treating divisive reference as the hallmark of mass terms see, e.g., Grimm (2012b, pp. 111–119).

perceived as one integrated entity and as a configuration of parts that could be considered independent objects in their own rights.

At this point, one might object that the double crown example is somewhat fancy and does not really reveal the meaning of multipliers. However, the characteristic described above is also valid for many other examples including nouns ranked relatively high as collocation candidates in the NCP. For instance, consider (4.13a). The noun *drzwi* 'door' ranks at the 23rd place on the NCP collocation list for the non-virile/neuter form *podwójne*.[7] An object referred to by the phrase is a door made of two independently moving leafs, i.e., entities that could be perceived as self-sufficient doors. Similar, a double garage denoted by the expression in (4.13b) is a building with two entrances leading to separate parking spaces. Again, it appears that the structure of the entity in question involves parts that can be considered as having a property similar to the property of the whole.

(4.13)   Polish (NCP)

         a.   podwójne drzwi
               double$_{\text{PL}}$ doors
               'double door'

         b.   podwójny garaż
               double$_{\text{SG.M}}$ garage$_{\text{M}}$
               'double garage'

The same way of thinking about multiplier phrases also applies to examples that at first blush appear to be less obvious than those discussed above. For instance, Polish multipliers frequently occur in constructions such as those in (4.14). This is corroborated by the fact that, e.g., the noun *warstwa* 'layer' ranks at the 7th place on the NCP collocation list for the feminine form *podwójna*. At first sight, it seems that expressions such as (4.14a) and (4.14b) refer to homogeneous entities and as such do not designate objects involving a complex structure. However, under closer inspection it turns out that even in these cases division into parts is not arbitrary. Specifically, a double layer is normally taken to be a layer consisting of two layers merged together. Since layers are flat entities spread over some surface, only a cut alongside a certain plane can divide the two parts constituting

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7.   Notice that in Polish the noun *drzwi* 'door' is a plurale tantum.

an entity denoted by the phrase in (4.14a). Similar, the expression in (4.14b) designates an entity whose structure can be partitioned only in a particular manner in order to separate the salient parts. Intuitively, a double coating consists of two parts that themselves are coverings applied to the surface of some object.

(4.14)   Polish (NCP)

    a.   podwójna warstwa
        double$_{\text{SG.F}}$ layer$_{\text{F}}$
        'double layer'

    b.   podwójna powłoka
        double$_{\text{SG.F}}$ coating$_{\text{F}}$
        'double coating'

Certainly, examples such as those in (4.8)–(4.11) and (4.13)–(4.14) do not exhaust the combinatorial potential of Slavic multipliers and some more problematic cases will be discussed subsequently in Section 4.2. Nevertheless, the data discussed above indicate that at least a subset of constructions involving multipliers indicates clear cases of subatomic quantification. Noun phrases such as those in (4.8)–(4.11) and (4.13)–(4.14) denote sets of things conceptualized as singular objects with a complex inner structure. The role of multipliers in the discussed constructions is to quantify over particular parts of such structures, i.e., parts that have a property comparable to the property of the whole. I will refer to such elements as self-sufficient parts. However, before we turn our attention to some less obvious examples and see how far the generalization proposed above can get us, let us dedicate some space to examine to what extent the described semantic properties are representative for multiplier phrases in general.

4.1.3   Cross-linguistic similarities

In this section, I will discuss corpus-based evidence suggesting that the semantic behavior of multipliers discussed in the previous section is not a Slavic idiosyncrasy but it is actually widespread in languages such as English and German. In particular, I will inspect a number of phrases mostly involving some of the most frequent collocates of *double* as provided by the Corpus of Contemporary American English

141

(COCA) and supplement them with German equivalents found in the German Reference Corpus (GRC). This brief investigation will both provide a slightly broader cross-linguistic perspective and prepare the ground for a discussion of more problematic cases in Section 4.2.

Let us start with a somewhat metalinguistic example. Specifically, the symbol $[\![\ldots]\!]$ used to indicate the semantic evaluation function every formal semanticist knows so well, is referred to by the pluralized phrase in (4.15a), i.e., *double brackets*. The structure of the entity is such that each of the two parts of the, say, opening bracket can be considered a bracket itself. However, since they are spatially arranged in such a way that they form a perceptibly one integrated object, under ordinary circumstances it would be rather awkward to refer to a double bracket as *two brackets*. An analogous effect can be observed with respect to the meaning of the phrase in (4.15b) which refers to a basin designed in such a way that it includes two parts each of which is a sink itself. Similar, (4.15c) is true of a cooking device that consists of two merged ovens, i.e., two parts that have a property comparable with the property of the whole. The expression (4.15d) refers to a knot wound twice which typically results in an object consisting of two recognizable parts that are knots themselves.

(4.15)   English (COCA)
      a.   double bracket
      b.   double sink
      c.   double oven
      d.   double knot

Other examples of the discussed sort involve multiplier phrases such as those in (4.16). Each of these expressions refers to an entity perceived as a complex object involving two salient parts with a property comparable to the one of the whole. Specifically, (4.16a) and (4.16b) denote a staircase consisting of two connected staircases and a tomb built in such a way that it comprises two tombs in one structure, respectively. Similar, the phrases in (4.16c)–(4.16e) designate entities conceptualized as one object consisting of two merged individuals in their own right. Finally, the frequent expression in (4.16f) refers to a body part that due to the excessive amount of fat or skin looks as if it were a two-layered chin.

(4.16)   English (COCA)
      a.   double staircase
      b.   double tomb
      c.   double canoe
      d.   double flute
      e.   double needle
      f.   double chin

Similar to the phrases such as (4.14), the COCA provides examples of homogeneous entities involving self-sufficient parts, see (4.17). The same observations I have discussed with respect to the Polish data also apply to the English expressions.

(4.17)   English (COCA)
      a.   double layer
      b.   double glazing
      c.   double stitching

Sometimes, parts of an individual denoted by a multiplier phrase are not actually connected but remain in such proximity that they are perceived as forming one complex entity. For instance, the phrases in (4.18a) and (4.18b) refer to close-set pairs of meteorological and astronomical objects, respectively. In such cases, the perceived distance between individual rainbows and stars is considered so small that a pair is conceptualized as a complex entity consisting of two parts each of which has a property comparable to the one of the whole. Similar examples include, e.g., *double nebula* and *double cluster* which also refer to astronomical objects perceived as complex dual entities.

(4.18)   English (COCA)
      a.   double rainbow
      b.   double star

The results of several quick searches in the German Reference Corpus (GRC) consulted with native speakers demonstrate that the reported effects are also attested in German. The phrases in (4.19) exemplify different types of referents of multiplier phrases including homogeneous

143

entities, see (4.19b), and phenomena perceived as wholes consisting of similar objects remaining in relative proximity, see (4.19c).[8]

(4.19)   German (GRC)
    a.   Doppelkrone
         double-crown
         'double crown'
    b.   Doppelverglasung
         double-glazing
         'double glazing'
    c.   Doppelregenbogen
         double-rainbow
         'double rainbow'

The quick overview of some of the frequent contexts multipliers appear in shows that the double crown example discussed in Section 4.1.2 is not some extravagant case. Rather, it represents common semantic behavior of multipliers. I conclude that the English and German evidence based on the collocation list from COCA and and searches in GRC suggests that the pattern is robust.

4.1.4   Multipliers as degree modifiers

Briefly, I will return to the data both from Slavic and English to examine somewhat more problematic examples. Nonetheless, before I move on to the discussion of cases where at least at the first sight intuitions concerning the subatomic structure of entities denoted by multiplier phrases are not that clear, let us discuss yet another use of the expressions in question. In English, multipliers can be used as predeterminer modifiers, i.e., external modifiers of the whole DP occurring before the central determiner such as the definite article (see, e.g., Quirk et al. 1985, pp. 257–258, Huddleston and Pullum 2002, pp. 433–436).[9] Examples of such uses are given in (4.20). The nominals

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8.   In fact, German distinguishes between two distinct though morphologically related multiplier expressions, e.g., adjectival *doppelt* and suffixal *Doppel-*. Though it seems that their distribution differs, for the sake of brevity I will not examine the alternation here and leave such an investigation for future research.

9.   Actually, the term predeterminer modifier is used only by Huddleston and Pullum (2002). Quirk et al. (1985) use the term predeterminer instead.

provided in (4.20) appear to be degree-related (cf. Morzycki 2009). For instance, the meaning of the noun *size* in (4.20b) could be thought of as corresponding to some degree of volume. Similar, the noun *fun* is clearly gradable since it allows for degree modification as witnessed by the well-formedness of the comparative phrase *more fun*. Consequently, I conclude that multipliers used as predeterminer modifiers involve degree modification and as such are expressions of a different type.

(4.20)   English (Quirk et al. 1985, p. 257, Huddleston and Pullum 2002, p. 434; adapted)
   a.   double the amount
   b.   triple the size
   c.   quadruple the fun

At some abstract level it might seem appealing to try to associate the use of multipliers as degree modifiers with the use related to subatomic quantification we are interested here. However, I will not pursue such an attempt here for two reasons. First, the phrases in (4.20) have a different structure than, e.g., those in (4.15). In the first, multipliers combine with NPs whereas in the latter they modify whole DPs. I argue that this fact indicates that there are good reasons to believe that in each case they introduce a semantic operation of a different type. The second reason has to do with cross-linguistic distribution of multipliers. For instance, Czech distinguishes lexically between subatomic multipliers such as *dvojitý* 'double, consisting of two parts' and *dvojnásobný* 'double, twice as large' (see Dočekal and Wągiel to appear for a related discussion). This fact further suggests that the semantics of English multipliers used as degree modifiers differs from what is the main interest of this study.

4.2   Less obvious cases

In the previous section, I have discussed model examples of multiplier phrases denoting complex entities with self-sufficient parts. I have signaled that the distribution of multipliers in the examined Slavic languages as well as in English and German is richer. In this section, I will discuss other types of nominals that can be modified by numerical

expressions such as *double* and show how the perspective posited above allows us to capture the meaning of less obvious cases.

4.2.1 Essential parts

So far, I have discussed examples of multiplier phrases denoting objects involving self-sufficient parts, i.e., elements that have a property comparable to the property of the whole. However, one does not need to search for a long time to come across phrases in which the multiplier does quantify over parts of an individual, but crucially those parts are not self-sufficient. Examples of such expressions are frequent in the fast food culture and involve, e.g., Slavic phrases such as those in (4.21)–(4.24). In each of these examples, it is not the case that its referent consists of two parts that are hamburgers themselves, but rather that it consists of two patties in a bun. This seems to differ significantly from the cases discussed so far. Notice, however, the fact that parts multipliers quantify over are not self-sufficient does not mean that they are arbitrary. In fact, they appear to be the most salient parts of the whole thing since in the fast food context it is commonly assumed that the essential element of a hamburger is a patty whereas other parts of a sandwich are considered to be merely a garnish.

(4.21)   Polish
         podwójny hamburger
         double$_{\text{SG.M}}$ hamburger$_{\text{M}}$
         'double hamburger'

(4.22)   Czech
         dvojitý   hamburger
         double$_{\text{SG.M}}$ hamburger$_{\text{M}}$
         'double hamburger'

(4.23)   Russian
         dvojnoj   gamburger
         double$_{\text{SG.M}}$ hamburger$_{\text{M}}$
         'double hamburger'

(4.24) BCS
        dupli   hamburger
        double~SG.M~ hamburger~SG.M~
        'double hamburger'

The fact that a certain part of an entity is perceived as more salient than others corresponds to what is sometimes called structured parthood (Champollion and Krifka 2016). As mentioned in Chapter 1, the term has been postulated to account for the fact that some parts appear to have a more significant status than others or, in other words, are perceptibly distinguished within a whole.[10] The difference between structured and unstructured elements is that the former are cognitively salient parts of a whole whereas the latter are not. Table 4.1 gives some examples of structured part relations.

WHOLE	PART
a (certain) man	his head
a (certain) tree	its trunk
a house	its roof
a mountain	its summit
a battle	its opening shot
an insect's life	its larval stage
a novel	its first chapter

Table 4.1: Examples of structured parthood (Simons 1987, p. 10)

For some time, it has been recognized that structured parthood plays an important role in organizing lexicons of natural languages, specifically via semantic relations such as meronymy and hyponymy (see, e.g., Cruse 1986). However, in contemporary research on natural language, the impact of structured part relations on compositional semantics seems to be somewhat neglected (e.g., Champollion 2012b and Champollion and Krifka 2016). However, it appears that multipliers provide evidence that cognitively salient parts are relevant from the quantificational perspective. In particular, I would like to suggest that counting self-sufficient parts is a particular instance of a more

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10. For an overview and discussion of related philosophical issues see, e.g., Simons (1987) and Varzi (2016).

general subatomic quantification over particular elements within a make-up of an entity. Consequently, I posit that the inner structure of objects such as hamburgers is in fact linguistically encoded in some way and that multipliers are sensitive to its most cognitively salient parts. In particular, they select for such essential elements within the part-whole structure of a certain whole and quantify over them. Within such a view, self-sufficient parts constitute a particular case within a class of structured parts, specifically a subclass of cognitively most salient elements, i.e., essential parts.

However, what counts as an essential part is somewhat vague and can be subject to different conceptualizations under different circumstances. To illustrate this claim, let us consider another sample from the COCA collocate list for the English multiplier *double*. For instance, consider the term in (4.25a) one can stumble across in the context of typography. Though in the Unicode standard the expression refers to the symbol $\Rightarrow$, it is commonly used to designate also the symbols $\leftrightarrow$ and $\gg$. In the Unicode terminology, what appears to be considered the most salient part, and thus multiplied is the horizontal line. However, in the more common and lax use of the expression, it seems that it is the arrowhead that is taken to be the essential part. Similar, in typography the phrase in (4.25b) refers to the symbol ‡. Its design indicates that the parts worth quantification over are the handles. However, in a military, martial arts, or gaming context the very same expression or a similar phrase such as (4.25c) is much more likely to denote thrusting weapons with two blades. On the other hand, there are some examples that display a stable meaning. For instance, the phrase in (4.25d) refers to a gun with two joint barrels. From a ballistic point of view, the barrel ensures functionality of a gun, whereas in terms of size it also constitutes its most significant portion. Consequently, it is perceived as the essential part of a weapon and when the noun *shotgun* combines with a multiplier, the resulting phrase designates objects consisting of two merged barrels.

(4.25)   English (COCA)
   a.   double arrow
   b.   double dagger
   c.   double sword
   d.   double shotgun

148

At this point, it is clear that the generalization concerning multipliers as quantifiers over self-sufficient parts requires an adjustment. In fact, multipliers count essential, i.e., cognitively most salient, elements within the subatomic part-whole structure encoded by the modified noun. Self-sufficient parts constitute only a subset of such entities. In the next section, I will discuss the case of mass nouns where at first blush talking about essential parts does not make any sense.

4.2.2 Portions

In the previous sections, I have discussed examples involving count nouns. Nonetheless, similar to equivalent expressions in other languages Slavic multipliers co-occur also with mass terms. At first sight, it might appear to be problematic for the proposed perspective since it is somewhat odd to think about referents of prototypical mass nouns in terms of consisting of essential parts. For instance, consider the phrases in (4.26)–(4.29). Prima facie, it may seem that there is no straightforward correlation between the semantic behavior of multipliers we have observed so far and the examples introduced above since none of the expressions refers to some cognitively salient parts of coffee, e.g., coffee grounds. However, a more careful examination reveals some non-trivial facts about such examples. More specifically, the contrast appears to fade when we realize that multiplier phrases involving mass terms in fact are not mass. Notice that the expressions in (4.26)–(4.29) neither denote a scattered entity, i.e., a substance in general, nor do they refer to some arbitrary portion thereof. Instead, they designate a standard amount, i.e., a big cup of coffee, which consists of two standard amounts of coffee or, in other words, it is equivalent to two regular cups of coffee.

(4.26) Polish
        podwójna kawa
        double$_{\text{SG.F}}$ coffee$_{\text{F}}$
        'double coffee'

(4.27) Czech
        dvojitá   káva
        double$_{\text{SG.F}}$ coffee$_{\text{F}}$
        'double coffee'

(4.28)   Russian
dvojnoj   kofe
double$_{\text{SG.M}}$ coffee$_{\text{M}}$
'double coffee'

(4.29)   BCS
dupla   kahva/kava/kafa
double$_{\text{SG.F}}$ coffee$_{\text{F}}$
'double coffee'

Let us consider the relationship between bear mass nouns and mass nouns modified by multipliers more closely on the basis of Polish data. As already mentioned in Section 3.3, Polish mass terms are generally incompatible with cardinal numerals unless they are coerced to a portion reading.[11] Consequently, a phrase such as (4.30a) lacks a substance interpretation and can only refer to a standard or contextually determined portion of coffee. As I have already stated, the same is true of a multiplier phrase such as that in (4.26). With this respect multipliers pattern with cardinals in that they coerce the meaning of the modified mass noun. Furthermore, as can be witnessed by the felicity of (4.30b) multiplier phrases involving mass terms are countable. When combined with a cardinal, they denote a plurality of portions that are some number larger than standard.

(4.30)   Polish

    a.   trzy   kawy
       three coffees
       'three coffees'
       (i)   portion interpretation
       (ii)   #substance interpretation
    b.   trzy   podwójne kawy
       three double$_{\text{PL}}$ coffees
       'three double coffees'

Given the portion interpretation of examples such as (4.26)–(4.29), describing their meaning in terms of an interaction between quantifi-

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11. Another possible coercion involves a taxonomic interpretation where the cardinal quantifies over subkinds of a particular kind. For the sake of brevity, I ignore this phenomenon here.

cation over self-sufficient parts and some sort of a mass-count shift appears as a plausible possibility. For this purpose, one can adopt the standard Universal Packager operation (see, e.g., Bach 1986, Jackendoff 1991, and Landman 1991) which yields quantized portions or packages of substance for a given mass entity.[12] Assuming that the mass noun *kawa* 'coffee' is shifted by the Universal Packager operation before it is modified by the multiplier,[13] the resulting count noun refers to a standard or contextually determined portion of coffee, specifically a cup of coffee. The multiplier then asserts that within the package there are two standard or contextually determined portions, i.e., two cups of coffee. As a result, we obtain a double coffee, i.e., a standard portion consisting of two smaller parts that have a property of being a standard portion of coffee as well. This is actually analogous to the cases provided in (4.8)–(4.11). Furthermore, note that in the discussed coffee scenario the portion of beverage is defined pragmatically and at least to some extend fixed. In other words, there are exactly two parts of the amount of coffee constituting double coffee that are standard portions. Smaller or bigger amounts are simply not standard with respect to coffee. Yet of course the standard can differ with respect to other beverages or food-stuffs. For instance, multiplier phrases such as those in (4.31) refer to different amounts of substance.

(4.31)   English (COCA)
      a.   double vodka
      b.   double martini
      c.   double ice cream

Interestingly, it seems that there is a deep relationship between multipliers and packaging. One could easily imagine that multipliers would also interact with another mass-count shift, specifically with Universal Sorter (see, e.g., Bunt 1985) which converses the mass noun into a count denotation referring to quantized subkinds. However, it seems that at least in Slavic there is no such interaction since phrases such

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12. An alternative approach to be taken is to enrich the structure with a classifier. As suggested by Selkirk (1977) and subsequent work, it is plausible to assume that possibly every natural language uses classifiers which are often null.
13. There are good reasons to presume that it is in fact the case since only after the shift mass nouns can combine with number words.

as those in (4.26)–(4.29) lack the relevant interpretation. In particular, (4.26) is not true of a substance consisting of two kinds of coffee, say a drink brewed from a mixture of *Coffea arabica* and *Coffea canephora* beans. This fact seems to suggest that multipliers operate only on mereologies involving object-level entities, i.e., tokens, and cannot count parts of kind-level individuals, i.e., types.[14]

Up to this point, I have focused on multiplier phrases involving concrete nouns since they are the subject of this study. However, as can be witnessed by the collocation lists in the examined corpora many of such constructions comprise also nominals of different sorts. For the sake of completeness, in the next two sections I will briefly discuss how the observations collected so far can inform us about the meaning of two other types of multipliers phrases.

4.2.3 Events

I believe that the discussion of the novel data discussed so far gives some intriguing insights into issues concerning subatomic quantification in the adjectival domain. The proposed generalization seems to cover a significant subset of constructions in which multipliers appear including examples such as those in (4.8)–(4.11) and many others provided in this chapter. Nonetheless, as attested in the examined corpora multipliers can co-occur also with nouns that at first sight might appear to be problematic since they do not refer to concrete entities such as crowns, hamburgers, or portions of coffee. For instance, as indicated by examples such as those in (4.32) Polish multipliers can also combine with deverbal nouns which do not involve reference to entities, but rather to some abstract objects. Neither murder nor victory are spatial objects as opposed to all previously discussed cases.

(4.32) Polish (NCP)

    a. podwójne morderstwo
       double$_{\text{SG.N}}$ murder$_{\text{N}}$
       'double murder'

    b. podwójne zwycięstwo
       double$_{\text{SG.N}}$ victory$_{\text{N}}$
       'double victory'

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14. I leave aside the question what a part of a kind could be.

It is usually assumed that deverbal nominalizations with the exception of participant and result nominals denote properties of events (e.g., Grimshaw 2011), i.e., spatiotemporal particulars with a location and time (Davidson 1967). A standard test reveals that a noun such as Polish *morderstwo* 'murder' refers to an event since it can be felicitously modified by event-modifying predicates such as *mieć miejsce* 'take place' and *być świadkiem* 'witness' (e.g., Grimshaw 1990), as indicated in (4.33a) and (4.33b), respectively.

(4.33)   Polish

    a.   Morderstwo miało miejsce w piątek.
        murder   had   place$_{ACC}$ in Friday$_{ACC}$
        'The murder took place on Friday.'

    b.   Jan był świadkiem morderstwa.
        Jan was witness$_{INS}$ murder$_{GEN}$
        'Jan witnessed the murder.'

If multipliers can count essential parts of not only entities but also events, the proposed view naturally extends to examples such as those in (4.32). An intuitive way to understand subatomic quantification within the structure of an event involves temporal characteristics. Within this view, a self-sufficient part of an event is a subevent of which the same property as the one of the whole event holds. For instance, in a murder scenario in which a maniac stabbed to death two victims with a knife one after another there would be two relevant parts of the whole murdering event, i.e., stabbing victim 1 and stabbing victim 2, and each of those subevents can be considered a murder as well, see Figure 4.4. In such a case the multiplier in, e.g., (4.32a) quantifies over self-sufficient parts of the murdering event, i.e., subevents that are murders themselves. Since there is no murder without a victim, the phrase infers that there were two individuals murdered during the complex murdering event which seems to be the expected result.

Nonetheless, one could object that such an approach cannot account for situations such as a bomb scenario in which an intended explosion affected two victims leading to a double homicide. In such a case, quantification over temporally subsequent subevents is not possible since the explosion had an immediate effect and there was virtually no running time of the event. However, I believe there is no
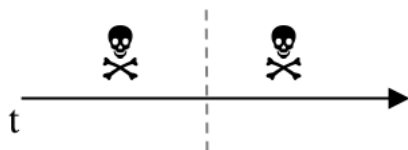
Figure 4.4: Parts of a double murder event in a knife-stabbing scenario

reason to assume that temporal chunks are the only possible parts that can constitute an event. Since events are complex peculiarities involving not only times, but also locations, in a bomb scenario described above it is still possible to differentiate between parts of an event in spatial terms. Consider the situation depicted in Figure 4.5. Even with no temporally defined subevents it is quite straightforward that one could specify two areas within the space in which the explosion took place, as indicated by the dashed line. As a result, we obtain two parts of the murdering event each of which turns out to have the property of the whole and since individuals are in general assumed to occupy space, it should be possible to draw divisions like that with respect to any event.
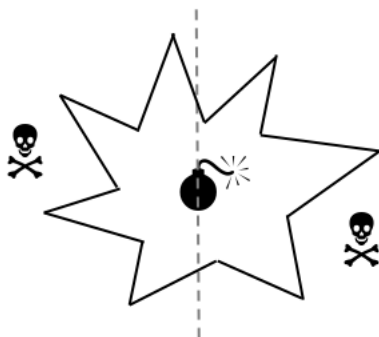
Figure 4.5: Parts of a double murder event in a bomb scenario

All things considered, it seems that the way of thinking about the meaning multipliers I argue for can be easily extended to phrases involving reference to abstract peculiarities such as eventualities. Adopting a typal distinction between entities and events as typically assumed in standard Neo-Davidsonian frameworks (e.g., Carlson 1984, Dowty

1989, Parsons 1990) along with the proposal that pluralities of events are obtained similar to pluralities of entities (Bach 1986) would simply require to ensure that the semantics of multipliers is general enough to cover both types of objects. In the next section, I will inspect yet another type of expression that is often modified by multipliers and does not necessarily denote concrete things.

4.2.4   Social roles

The final piece of data that appears to be problematic for the claim that multipliers are in fact subatomic quantifiers is constituted by phrases with nouns referring to social roles in examples such as those in (4.34). Under ordinary circumstances, the phrase in (4.34a) does not refer to an individual who consists of two parts, e.g., Siamese twins working as spies, but rather they seem to involve a relationship between an individual and two other entities. To be an agent means to be an agent for some intelligence agency, i.e., one cannot be an agent without an employer for whom they spy. Similar, (4.34b) refers to one person who is a champion in two disciplines. In both cases, the multiplier seems to quantify over entities denoted by the argument of the noun which at first blush is not expected given the previously discussed semantic behavior.

(4.34)   Polish (NCP)

      a.   podwójny agent
           double$_\text{SG.M}$ agent$_\text{M}$
           'double agent'

      b.   podwójny mistrz
           double$_\text{SG.M}$ master$_\text{M}$
           'double champion'

At this stage, it is not entirely clear how to integrate the ideas developed in this paper with the treatment of examples such as those in (4.34). However, a possible extension of the proposed view is to assume that nouns such as *agent* involve some kind of an abstract association to entities related to denoted individuals, e.g., an association between agents and intelligence agencies, and that such an association is subject to part-whole relations. A promising perspective to pursue

155

involves adopting the notion of roles into semantic theory (see Sowa 1984, Steimann 2000, Zobel 2017). Intuitively, roles are functions or capacities of individuals, i.e., social constructs independent of the individuals that bear them. Crucially for our purposes, for an individual to bear a particular role, it must stand in a certain relationship to other individuals. Following Zobel (2017), one could distinguish between class nouns such as *man*, i.e., properties of individuals of type $\langle e, t \rangle$, and role nouns such as *agent* by adding a new type r referring to roles and a shifting mechanism relating roles and individuals. Role nouns would then essentially denote properties of roles at type $\langle r, t \rangle$, but they could be shifted to refer to properties of entities as well. Within this view, multipliers would quantify over essential parts of roles rather than individuals and the whole phrase could then be shifted to refer to entities if required. This would account for the fact that expressions such as those in (4.34) denote individuals performing a complex role, i.e., individuals involved in activities and bearing responsibilities related to two distinct intelligence agencies. However, exploring the interaction between subatomic quantification and roles is rather challenging and goes beyond the scope of this study. Instead, in the next section I will return to the relationship between multipliers and cardinals.

4.3 Multipliers vs. cardinals

As we have seen in Section 4.1.1, Slavic multipliers display morphological complexity which suggests semantic compositionality. For instance, the Polish multiplier *podwójny* 'double' consists of the numeral root $\sqrt{dw}$ corresponding to the number 2 present also in other types of numerical expressions, e.g., the cardinal *dwa* 'two', as well as additional morphemes including a special multiplicative circumfix $po\rangle \ldots \langle n$-. This fact indicates that what multipliers and cardinals have in common is some sort of reference to integers. Though they both indicate that the number of quantified things equals 2, they differ in that they are devised to count entities of a distinct type. In particular, cardinals are semantically foreordained to count wholes. It is true that in Chapter 2 and 3 we have seen that they can be used in count explicit partitives in order to quantify over parts of objects, but notice that it

is only possible when they modify a partitive word. In such a case, entities denoted by the whole partitive are treated as objects in their own right. Hence, the source of subatomic quantification is in fact the partitive and the cardinal simply counts provided entities in their relative entirety. Bare cardinals can never access elements within a part-whole structure of a given entity. Rather, they always count entire objects in the denotation of a modified noun. On the other hand, multipliers are essentially subatomic quantifiers. They always count parts of objects referred to by a modified nominal.

To better illustrate the claim introduced above, let us closely consider the contrasts between truth conditions of the sentences involving numerical expressions in (4.35). Under normal circumstances, there is no way the statement in (4.35a) could be judged true if there were only two parts of a crown or crowns, e.g., an orb and a jewel, in the vault. The numeral phrase simply cannot be understood as referring to two parts instead of two entire objects.[15] In other words, cardinals are semantically devised to counting wholes. This is not in contradiction with the fact that in a sentence including a count explicit partitive construction as in (4.35b) the cardinal does count parts of a whole object. Consequently, (4.35b) would be true if there were only, say, an orb and a cross in the vault. Notice, however, that it is the partitive word that employs subatomic quantification. The cardinal here simply counts provided parts as if they were considered objects in their own right. In other words, it does not operate on the part-whole structure of a given part. This contrasts with the meaning of (4.35c) which would be true in a scenario where there were just one object in the vault. Crucially, however, that object would have to have two essential parts or, to put it differently, two parts having a property of being sort of a crown. Finally, the felicity of the sentence in (4.35d) shows that both types of quantification can co-occur within one phrase. Here, the multiplier triggers subatomic quantification constrained as discussed above, whereas the cardinal quantifies over wholes having the inner structure imposed by the multiplier. Consequently, the sentence would be true in a situation where in the vault there were three crowns such

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15. Of course, what portion of an entity counts as an entire object is rather vague and might differ with respect to a particular context. However, this is not at issue here.

that each of them consisted of two self-sufficient parts. Notice that the relative order of the cardinal and multiplier as well as adjectival properties of the latter further suggest that subatomic quantification takes place first and only after the resulting wholes are counted.

(4.35)   English (Guy Tabachnick, p.c.)
      a.   There are two crowns in the vault.
      b.   There are two parts of the crown in the vault.
      c.   There is a double crown in the vault.
      d.   There are three double crowns in the vault.

The discussed contrast between cardinals and multipliers becomes even more salient if we consider what happens when the latter modify partitive words. I have already dedicated a lot of attention to count explicit partitives such as (4.36a), but let us now contemplate the meaning of an equivalent multiplier phrase such as (4.36b). Crucially, the multiplier quantifies over parts of a part of the adapter. The whole expression would be true of an entity that is an adapter part and consists of two comparable parts. Yet again, the multiplier employs quantification within the part-whole structure of a thing denoted by the nominal no matter what that thing is. Though such a use is for sure very rare, the phrase is definitely interpretable.[16] A natural example of a sentence with such a construction from the NCP is provided in (4.37).

(4.36)   Polish
      a.   dwie części adaptera
          two   parts   adapter$_\text{GEN}$
          'two parts of the adapter'
      b.   podwójna część adaptera
          double   part   adapter$_\text{GEN}$
          'double part of the adapter'

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16. The example was actually found on a website of a producer of adapters with a picture of a described part.

(4.37) Polish (NCP)
Skrzynkę horyzontalną otwiera się   podobnie jak
box$_{\text{ACC}}$ horizontal$_{\text{ACC}}$ opens REFL similar   how
podwójną część   przedniej ścianki…
double$_{\text{ACC}}$ part$_{\text{ACC}}$ front$_{\text{GEN}}$ wall$_{\text{GEN}}$
'The horizontal box can be opened similar to the double part
of the front wall…'

I conclude that the purpose of cardinals is essentially to count whole entities in the extension of a modified nominal. If they modify a regular noun, they simply quantify over objects in its denotation. Similar, if they appear in a count explicit partitive, they count provided parts in their entirety as if they were wholes. At the same time, natural language developed a special type of expression dedicated to numerical subatomic quantification, namely multipliers. Multipliers differ from cardinals in that they always count essential parts of objects denoted by a modified nominal be it a whole or a part. In other words, they are equipped to count parts within a whole. The quantificational properties of the two numerical expressions in question are summarized in Table 4.2.

	CARDINALS	MULTIPLIERS
subatomic quantification	*	✓
quantification over wholes	✓	*

Table 4.2: Properties of cardinals and multipliers

This concludes the examination of some non-trivial properties of the neglected class of multipliers with respect to the issues concerning subatomic quantification.

4.4   Summary

In this chapter, I have focused on a key topic concerning subatomic quantification. Specifically, I have presented novel evidence demonstrating that in natural language there are numerical expressions devised for the purpose of quantification over parts, namely multipliers such as English *double*. The fact that such lexical items are

cross-linguistically widespread indicates the relevance of part-whole structures of singular objects for the interaction between parthood and quantification in natural language semantics. The morphological evidence from Slavic demonstrates that Polish, Czech, Russian, and Bosnian-Croatian-Serbian multipliers are formally complex expressions derived from numeral roots of corresponding cardinals. I have argued that the morphological complexity of Slavic multipliers suggests semantic compositionality.

Multipliers are sensitive to the internal part-whole structure of objects denoted by nouns they modify and involve quantification over cognitively most salient elements of those objects. Often, such elements are self-sufficient parts, i.e., entities that have a property comparable with the property of the whole. Sometimes, they are not self-sufficient but rather they are considered the most significant parts within the inner structure of an object. I have generalized that multipliers count essential parts, i.e., parts perceived as crucial for an entity to be considered as having a particular property, with self-sufficient parts constituting a subset thereof. Moreover, I have examined multiplier phrases involving mass terms. Interestingly, such expressions receive a portion interpretation and as such are countable. This fact indicates that multipliers either perform or require a mass-count shift in terms of packaging. Furthermore, I have proposed how the view argued for here could be extended to cover cases of multiplier phrases involving other types of NPs, namely event nominals and role nouns. Incorporating abstract objects such as eventualities and roles into the ontology would allow us to capture the meaning of virtually all multiplier phrases in terms of one unified mechanism. Finally, I have discussed how multipliers differ from cardinals. The crucial distinction between the two boils down to what type of entity they count in terms of part-whole structure. In particular, while cardinals always quantify over wholes, multipliers always count parts of a singular object which, similar to complex entities or events they constitute, have themselves a property considered essential for an entity to fall in the denotation of a modified noun.

This chapter concludes the empirical part of this thesis. The most important insights could be summarized as follows. First, singulars and plurals share a unified part-whole structure. Second, natural language is sensitive to topological relations holding between parts of

entities in the denotations of nominal expressions and allows for quantification at the subatomic level. Third, only entities that constitute integrated wholes are countable. Finally, though different types of numerical expressions specialize in quantification either over wholes or over parts, the underlying mechanism is the same. In the next chapter, I will introduce a conceptual background that will serve as an informal notional basis for a formal account for subatomic quantification.

5 Conceptual background

In the previous three chapters, I have discussed novel data concerning the interaction between partitivity and countability. Several novel observations have been made. In particular, I have presented linguistic evidence suggesting that singulars and plurals do not differ with respect to the part-whole relation they employ but rather that parts of singularities remain in a particular topological configuration ensuring that the whole constitutes an integrated object, whereas parts of pluralities do not, and thus form a scattered entity. This claim is motivated by two linguistic facts. First, cross-linguistically the same partitive words can appear both in entity and set partitives. Second, the contrast between regular plurals and Italian irregular plurals reveals the crucial topological distinction between the two. Furthermore, I have shown that natural language does not allow for counting arbitrary sums of topologically unrelated parts. Only entities that are conceptualized as constituting continuous objects can be subject to counting. This restriction appears to hold both on the level of wholes and on the subatomic level, as the evidence concerning topology-sensitive partitive words as well as the semantic behavior of multipliers indicate. In other words, quantification in natural language is sensitive to topological relations between parts of entities it operates on.

In this chapter, I will discuss the conceptual background concerning countability and subatomic quantification I assume here. In particular I will present three claims regarding the relevance of topological notions in nominal semantics, general counting principles, and their significance for subatomic quantification. This informal conceptual framework will motivate the formal account for the core phenomena explored in this study that I will develop in Chapter 6 and 7. However, before I present in detail the notional core of this study, let us consider a more general cognitive context. Though the linguistic evidence presented so far is compelling and entirely motivates the claims, I believe it is useful to confront it with what we know from psychological research in order to provide supplementary support.

5.1 Psychological evidence

In this section, I will review psychological evidence that demonstrates several factors concerning human cognition that correlate with the linguistic evidence introduced in Chapter 2, 3, and 4. First, I will discuss the relevance of how spatial categories such as solidity, shape, and most importantly integrity of entities are conceptualized with respect to the mass/count distinction in grammar. Furthermore, I will consider the significance of the part/whole distinction for language acquisition and review the evidence that humans have simultaneous perception of an object as a whole and as a collection of parts. Finally, I will discuss what cognitive studies tell us on number sense in humans, especially on the relation between integrity and counting in young children. The literature on each topic is abundant. For the sake of brevity,in each case I will only refer to a few representative studies.

5.1.1 Object/substance distinction

I will start with a brief overview of the research in cognitive psychology relating individuation with the difference in perception of solid objects and amorphous substances that suggests that this distinction is relevant to the phenomenon of countability in natural language. In the past 30 years, convincing evidence has been presented that indicates that the distinction between objects and substances is not merely an alternation based on grammar. Contrary to Quine (1960)'s influential claim that it is language what provides the means for individuation of objects (see also Pelletier and Schubert 1989), it appears that count and mass syntax reflect on how we see entities in the world rather than the other way round. In other words, the research to be discussed here shows that the distinction between objects and substances is not something conventional, formal, or arbitrary imposed by the way a particular language is devised, i.e., a way of speaking so to say. To the contrary, it appears to be a deeply embedded component of human cognition manifesting itself in non-verbal infants at a few months of age and shared with non-human species.

Multiple experiments demonstrate that children associate count nouns with solid discrete objects at an early age (see, e.g., Landau et al. 1988, Soja et al. 1991, Imai and Gentner 1997). The evidence come

from the results of the so-called word extension task which concerns learning new words by subjects. The procedure is as follows. First, a child is presented with a novel entity and instructed on what it is called. That entity can be either a solid object or a shapeless portion of a substance. Next, two other entities resembling the initial one with respect to different perceptual features are introduced. One would match the already introduced entity in shape, whereas the other would be of the same material. Interestingly, the extensive research shows that children extend the name of a novel item to items of the same shape only if the initial entity was a solid discrete object. On the other hand, if they were first presented an amorphous portion of a novel substance, they do not extend its name to items of a similar shape, but rather to objects made out of the same material.

Based on such evidence, Soja et al. (1991) argue that certain onto-logical commitments are prior to the linguistic mass/count distinction, contrary to Quine's claim. Additional support for such a conclusion comes from the fact that the results were replicated in populations using languages with syntax significantly different from English. For instance, Imai and Gentner (1997) demonstrate parallel evidence based on the same experimental paradigm using the word extension task in the Japanese speaking environment. Though Japanese lacks a straight-forward morpho-syntactic equivalent of the distinction between mass and count nouns present in languages such as English, Japanese speak-ing children distinguish between objects and substances just as well as their English speaking peers.

A number of studies provide varied and carefully controlled evi-dence that preverbal children are endowed with certain assumptions concerning the nature of objects (e.g., Carey 1985, Spelke 1990, Soja et al. 1991, Carey and Spelke 1996). For instance, they expect objects to be bounded and cohesive. The first assumption reflects on an en-tity having natural boundaries, whereas the latter translates into an anticipation that objects have parts that stick together. Furthermore, objects are expected to move across space as wholes along continuous paths and to retain identity upon collisions with other entities. On the other hand, substances lack any of those properties. An exemplary experimental paradigm is as follows. An infant is introduced either to a solid object, e.g., a teddy bear, or a portion of substance, e.g., clay, displayed in front of them. Next, the entity is covered by a screen and

the child sees that a second item is placed behind the screen. Finally, depending on an experimental condition after the screen was removed either there were two items revealed or due to the manipulation by an experimenter only one item could be seen. Interestingly, if the initially introduced entity was a discrete solid object, children reacted with great surprise if after removing the screen they could find only one item. On the other hand, no such reaction was recorded if they were introduced to a portion of a substance at the beginning. The conclusion is that preverbal children appear to know the different between objects, pluralities thereof, and substances. They expect that if one adds a teddy bear to a teddy bear, the result should be two teddy bears, whereas if one combines clay with clay, one does not get a plurality of clay. Since similar results were also found in non-human primates (see, e.g., Hauser and Carey 2003, Hauser and Spaulding 2006 on rhesus monkeys), it seems plausible to assume that this property of human cognition has some evolutionary history. I will come back to the issue of number in Section 5.1.4.

However, though the difference between discrete solid objects and amorphous substances has a significant status in cognitive and linguistic development of children, it turns out that the correlation between this distinction and mass/count syntax is imperfect. A corpus study on toddler vocabulary pursued by Samuelson and Smith (1999) shows a significant asymmetry between particular syntactic categories. Specifically, while the large majority of nouns that English speaking children learn early are count nouns (74% of the vocabulary), only a small portion involves mass nouns (10% of the vocabulary). There is also a third class of expressions that considered ambiguous between count and mass (16% of the vocabulary). The results of the examination in order to find out whether the object/substance distinction correlates with the mass/count distinction are somewhat surprising. In general, a correspondence between solid objects and count nouns on the one hand and non-solid entities on the other was in fact observed. However, it was imperfect and and interesting mismatches were recognized. Surprisingly, solidity and shape turn out to be good predictors of

166

count nouns, whereas mass nouns appear to be less correlated with non-solidity and shapelessness.[1]

Yet another strand of research provides support for the findings discussed above. In particular, Prasada et al. (2002) demonstrate that categorization of a noun as countable or uncountable does not reduce to establishing an immutable ontological distinction between an object and a substance. Rather, it seems that speakers of languages such as English categorize nouns in terms of countability based on whether they construe entities they refer to as objects or as substances. A series of experiments utilizing novel vocabulary tasks show that a novel item that has an irregular shape is less probable to be referred to by a novel count noun when contrasted with entities of a regular shape. However, the same item is significantly more probable to be considered countable when displayed among other similar entities having such an irregular shape. This means that it is not the case that some objective properties of things influence the way grammar is directly. Instead, the mass/count distinction appears to reflect how humans conceptualize entities in the world.

To conclude, the psychological research reviewed in this section indicates that contrary to Quine's claim, human beings apparently do have certain ontological commitments prior to acquiring syntax of a particular language. Specifically, the way how children are biased towards perceiving objects as opposed to substances correlates with the mass/count distinction in grammar. However, this correspondence is not of a one-to-one type and there are well-known cases of mismatches. Yet, it is crucial that the distinction in grammar is not due to different ontological categories in some objective sense, but rather it reflects on how entities in the world are conceptualized. In the next section, I will discuss the relevance of part-whole structures in the process of language acquisition. In particular, I will briefly review how a certain bias concerning what it means to be a whole guides children in how they learn lexical meanings of nouns.

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1. Notice that probably the main reason is the existence of the category of object mass nouns such as *furniture*.

5.1.2 Whole object assumption

Since at least early 1990s, language acquisition studies have shown that vocabulary development is constrained by certain mechanisms that govern it in order to ensure its effectiveness. In particular, children are guided by a number of word learning biases i.e., endowed assumptions that allow us to eliminate unlikely alternatives in order to efficiently process and learn new lexical meanings (see, e.g., Markman 1990, Hansen and Markman 2009, Hollich et al. 2007). Such assumptions begin to manifest around the age of 18 months when rapid expansion of a child's vocabulary starts. They are crucial in deciding what the reference of a new noun is, e.g., what aspect of an object it designates, as well as solving the problem of indeterminacy. One of the word learning biases that will interest us here is the so-called whole object assumption (Markman 1990).

The whole object assumption allows children to constraint the meaning of novel words by guaranteeing that a child relates a new noun with an object in its entirety rather than with some arbitrary part or property of that object (Markman 1990). For instance, if an adult points to an item and labels it, e.g., a doll, a child assumes that the noun *doll* is meant to refer to the whole object and not to its part or characteristic. Though in principle the adult's verbal behavior might have been interpreted as referring to the doll's head, leg, dress, color, or size, children intuitively rule out such possibilities and associate the label with the whole item.[2] This bias holds even in situations where object's color or a certain dynamic activity are made salient to a child (Hansen and Markman 2009).

Though the original research regarded 18-month-year old children and older, later studies have demonstrated that in fact infants can associate nouns with whole objects already at the age of 12 months (Hollich et al. 2007). Crucially, the effect was attested despite the fact that test items could be viewed as two separate objects and even when a certain part of an item was made salient. Interestingly, similar results were obtained in experiments where participants were adult. Hence, the mechanism appears to be deeply embedded. Its relevance relies in that it enables children to determine which of numerous logically possible meanings a word could have is actually the correct one.

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2.   See Quine (1960) for a related problem of the inscrutability of reference.

In this section, we have seen that children distinguish intuitively between parts and wholes. In language acquisition, when exposed to a novel noun, they do not attribute its meaning to a part of an object it refers to but rather immediately assume it is true of a whole thing. The whole object assumption has often been related to the findings concerning infants' perception of objects discussed in the previous section. It has been hypothesized that the constraint in question reflects the non-linguistic status of objects (Hollich et al. 2007). This might suggest that from the perspective of a child the fact that wholes involve parts is somewhat discriminated. However, the evidence is much more complex. In the next section, I will provide a brief overview of the study of part-whole perception in children.

5.1.3 Part-whole perception

It is a cognitive fact that we often conceive entities as being made up of smaller entities related to each other in a particular manner. Experimental evidence shows that humans have two simultaneous perceptions concerning part-whole structures of entities (e.g., Witkin 1950, Meili-Dworetzki 1956). On the one hand, we possess the ability to discriminate parts from a whole, i.e., decompose an entire object into distinct elements making it up. On the other hand, we are able to integrate the total sum of the parts into a complete whole. This phenomenon is usually referred to as part-whole perception and it is standardly attributed to an ability of an individual to decenter, i.e., to shift attention from parts to the whole or vice versa (see Piaget and Morf 1958).

Though adults in general show part-whole perception, it has been subject to controversy in the psychological literature at what age it emerges. In the early Piagetian studies, the experimental results suggested that this ability is absent in young children (Elkind et al. 1964). In a typical experiment, a child was presented a drawing of a whole consisting of parts that were independent objects in their own right, e.g., a person made out of fruit as in Figure 5.1, and was instructed verbally to report what they see. According to Elkind et al., children report that they perceive simultaneously a whole and parts attributed to the same form, e.g. both a man and fruit, only at the age of 8. Before that age, subjects failed in complete decentration with 5-to-6-year-

169

olds showing complete centration, i.e., they reported that they saw only parts, e.g., fruit but not a person, and older children gradually improved.

Figure 5.1: Part-whole perception (Elkind et al. 1964)

However, later studies suggest that part-whole perception develops much earlier. For instance, Kimchi (1993) provides evidence that 5-years-old children show sensitivity to parts and to part-whole relations and that this sensitivity improves with age. Even more interestingly, Boisvert et al. (1999) show that 3-year-olds demonstrate good performance in part-whole perception tasks using multiple-choice tests instead of Piagetian verbal tasks used in earlier studies. Finally, Quinn et al. (1993)'s results indicate that 3-month-old infants can group elements of visual pattern information into larger perceptual units based on lightness similarity in a way that suggests at least some component of part-whole perception.

I interpret the experimental results presented above as suggesting that human cognition is devised to be sensitive to part-whole structures since even very young individuals can decompose a whole into pieces by discriminating its parts as well as represent individual elements as making up an integrated collection that is a whole. Since these two perceptions are simultaneous, one could expect that we intuitively categorize entities as certain configurations of smaller things that despite their often complex inner structure are singular objects. At the same time, parts of such objects remain cognitively salient and if required, can be easily accessed. If the experiments reported above are on the right track, this property of human mind appears to be either innate or at least emerge at an early stage of cognitive development.

170

5.1.4 Number sense

The final piece of cognitive evidence to be discussed here concerns human number sense, i.e., an intuitive understanding of what numbers mean and how they can be affected by various operations (see, e.g., Dehaene 1997 for an overview). This mental mechanism allows for a special type of sensory perception through which the cardinality of a given set of entities can be perceived with similar ease as their size, color, shape, or position and as such provides the basis for various forms of calculation. In humans, number sense is based on two different cognitive systems, specifically the approximate number system and the object tracking system (see, e.g., Hyde 2011). The first is a cognitive system supporting the estimation of the magnitude of a collection of objects without relying on symbolic representations (see, e.g., Feigenson et al. 2004, Nieder and Dehaene 2009 for an overview). It manifests already in infants and gets more developed with age (Cantlon et al. 2006). On the other, the parallel object tracking system is the mental ability to immediately enumerate small sets without counting by means of individuation (see, e.g., Carey 1998, 2009 for an overview). It allows us to rapidly recognize values from 0 to 4, whereas the approximate number system is credited with the non-symbolic representation of all greater numbers (Piazza 2010).

It appears that there is evidence indicating that humans are at least to some extent predisposed to develop certain numerical abilities such as the concept of exact number and simple arithmetic based on their number sense. For instance, Gelman and Gallistel (1978) argue that children are endowed with innate principles of counting and have intuitive understanding of the cardinality of a set of entities and its conservation under changes that do not affect quantity. It seems that a lot of knowledge concerning how to put objects in one-to-one correspondence with numbers is intuitively understood by children though it is never taught or formulated in an explicit way. In particular, when learning to count children are not taught that each entity must be counted once and only once or that one number cannot be associated with more than one entity, but this principle is taken for granted. Similar, children know intuitively that number words are supposed to be recited in fixed order and that the last numeral represents the cardinality of the whole enumerated set. According to this view, innate

171

knowledge regarding counting principles precedes and facilitates the acquisition of that part of lexicon that includes number words and guides its application in a particular situation of counting.[3]

The above introduced hypothesis seems to be corroborated by further experimental evidence that suggests that at the age of 2,5 years children understand that counting is an abstract procedure that can be applied to different kinds of objects including concrete entities as well as events (Wynn 1990). 3,5-year-olds know that the order in which they recite numerals is crucial, whereas the order of pointing at counted items is irrelevant as long as each item is counted and none item is counted twice. Furthermore, children are able to indicate and correct subtle errors that result from violations of basic counting principles such as reciting numerals out of order, counting the same object twice, or omitting an item while counting (Gelman and Meck 1983, Gelman et al. 1986). By the age of 4 years, children show that they have already mastered the basics of counting and they can generalize the procedure to novel situations. Older children often spontaneously reinvent arithmetic developing new strategies for calculation best fit for a particular problem (Dehaene 1997, p. 119).

However, probably the most intriguing finding from the perspective of the interest of this study is a tight link between spatial integrity and numerical information observed in young children. Specifically, Shipley and Shepperson (1990) provide a particularly striking demonstration of the relevance of objects understood as integrated discrete physical entities for cognition of 3-to-4-year-olds. In the experiment, children were presented sets of items and instructed specifically what they were supposed to count. For instance, an array of forks with one item broken in two pieces as illustrated in Figure 5.2 was displayed in front of a child who was then asked to count the forks. Unlike adults and older children, children between 3 and 4 years of age did not answer that there are five forks, but rather that there six of them, i.e., they counted each detached part of a fork as a separate entity. Similar, when instructed to count kinds, e.g., different kinds of animals, or

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3. However, see Fuson (1988) for an opposing view claiming that children learn counting by imitation. For instance, Fuson argues that children recite strings such as *onetwothreefourfive*…as uninterrupted sequences and only later on they segment them in order to delimitate numerals, learn to extend them to larger values and apply to particular situations.

172

properties, e.g. different colors, in a setup where each kind or property was exemplified by a number of distinct discrete items, children included each separate object in their count.
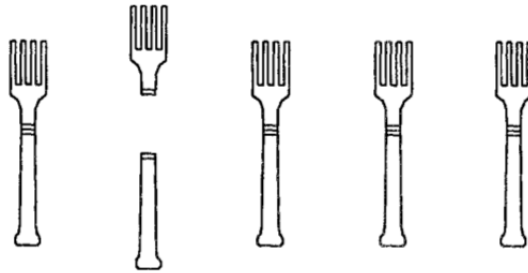
Figure 5.2: Relevance of integrity in counting (Dehaene 1997, p. 60; adapted from Shipley and Shepperson 1990)

The results demonstrate that the canonical countable entity for a young child is a discrete physical object that comes in a single piece. The role of spatial integrity is crucial in determining what counts as one and this bias precedes the process of learning how to count. In other words, at an early stage of development of arithmetic abilities young children simply cannot avoid counting each discrete integral object as one unit. It is very likely that the assumption that number is a property of sets consisting of discrete spatially contiguous objects is a deeply embedded principle that facilitates mastery of counting.

The findings of Shipley and Shepperson were further confirmed for other forms of linguistic quantification in experiments based on quantity judgment tasks involving comparative constructions and pluralization (Melgoza et al. 2008). In particular, when presented an object divided into three parts, e.g., a broken fork, on the one hand and two whole objects, e.g., two integral forks, on the other, 4-year-old children judged the first to be more objects than the latter. Furthermore, in an elicitation task children often labeled broken objects using plural morphology. For instance, a phrase *some forks* was used to refer to three separated pieces of a broken fork. The results presented by Melgoza et al. not only confirm the findings of Shipley and Shepperson but also indicate the existence of a spatial bias for many forms of quantification in natural language. In all tested cases, the spatial criterion of integrity

was preferred over other factors in order to establish what counts as one. In other words, when children quantify sets, they seem to define individual set members in terms of spatial contiguity in the first place. I take these results to be evidence suggesting that countability as a property of sets of discrete separate entities is a deeply embedded principle. Though as we know from every-day life experience adults can develop a mechanism to retrieve the original part-whole structure of counted entities, e.g., reconstruct a whole fork by combining separate pieces in thought, I hypothesize that the underlying quantificational procedure is based on a principle relating numbers with things that come in one piece. However, before I conclude, I will briefly discuss how human quantification differs from number sense in other animals.

Massive evidence indicates that number sense is not exclusive to human mind. Well-established research in etiology shows that apprehension, comparison, and even approximate addition of quantities is also present in animals (see, e.g., Davis and Pérusse 1988, Gallistel 1989, and Dehaene 1997, pp. 13–40 for overview). The most well-known evidence concerns the ability to represent and discriminate quantities of relative sizes in primates (e.g., Woodruff and Premack 1981, Matsuzawa 1985, Rumbaugh et al. 1987, Washburn and Rumbaugh 1991, Boysen et al. 1996), but it is also found in other mammals such as dolphins (Mitchell et al. 1985), cats (Thompson et al. 1970), and rats (Capaldi and Miller 1988) as well as birds (Pepperberg 1987) and even fish (Agrillo et al. 2012). Furthermore, recent studies in botanics suggest that there are reasons to also assume plant arithmetic (Böhm et al. 2016). The evidence indicates either that number sense is an evolutionary ancient part of cognition or that there were multiple convergent evolution events. In any case, it is a widespread phenomenon shared by a wide range of species.

However, despite the large body of evidence indicating that many non-human animals have the approximate number system, their number sense seems to differ significantly from that of humans. Though there are well-documented cases of approximate quantification in animals, no case of symbolic addition is known in any species other than the chimpanzee and only after long training dedicated to learning a small set of digits and with frequent errors in computation (Dehaene 1997, p. 39). On the other hand, young children spontaneously count

on their fingers, often up to 10 before the age of three and acquire syntax of complex numerals with ease. Thus, the more lax concept of numerosity is often used with respect to animal quantification in order to distinguish it from the symbolic and verbal representation of number in humans. To conclude, though some forms of quantification appear to be frequent in the animal kingdom, they seem to differ significantly from what humans do when they count. While non-human animals estimate quantities on a regular basis, it seems that the ability to establish one-to-one correspondence between an object and a discrete symbol which allows for computation of the exact number of entities in a given set is an extremely rare property among species.

I conclude that counting as performed by humans is a quite unique ability which associates discrete entities with abstract representations of number. I hypothesize that the core mechanism underlying all counting manifests in early childhood and concerns establishing one-to-one correspondence between integral physical objects that come in one piece and integers. In other words, I assume that the topological notion of spatial integrity is the basis for what counts as one and only later humans develop ways of abstracting away from this principle, e.g., by extending it to quantification over abstract entities such as kinds and properties or being able to retrieve the original connection between detached parts. In the following sections, I will attempt to relate the linguistic evidence presented in Chapter 2, 3, and 4 with the conclusions of cognitive psychology in order to develop the three main claims of this study that will serve as a conceptual background for the formal account for subatomic quantification in natural language.

5.2   The three claims

I have already presented a broad range of data demonstrating both the relevance of subatomic structures in natural language and the important role the way entities including parts of a whole are spatially related with each other plays with respect to quantification. Furthermore, in the previous section I have reviewed additional psychological evidence suggesting that these issues are deeply embedded in human cognition. Now is the time to discuss in detail what I believe to be the

conceptual core of this study, namely the three claims I make with respect to the interaction between parthood, topology, and quantification in natural language. I will start with postulating that natural language semantics is sensitive to topological relations holding between elements within part-whole structures designated by nominal expressions. Next, I will posit a set of general counting principles including constraints concerning non-overlap, maximality, and integrity of counted things. Finally, I will postulate that the very same universal mechanism applies irrespective whether one quantifies over entire entities or over parts of a whole.

5.2.1 Relevance of topological notions with respect to part-whole structures

It is well-known that topological notions play an important role in natural language. The existence of locative expressions involving prepositions such as *inside*, *near*, *under*, *between*, and *far* shows that relations concerning how we conceptualize position and space are deeply rooted in grammar. Arguably, they constitute universal components of human cognition and natural language semantics. Though the research on locative PPs is well-established and contributed to our understanding what means language uses to encode information regarding location of objects with respect to each other (e.g., Clark 1973, Herskovits 1985, Zwarts and Winter 1997, and Kracht 2002), the question concerning spatial constitution of entities remained somewhat elusive in the study of meaning. One line of argumentation justifying why those kind of issues should remain unaddressed is that they simply stem from every-day world knowledge, and thus as extra-linguistic factors are not supposed to be incorporated into the semantic theory. For instance, Schwarzschild (1996) famously argues that the fact that (5.1a) entails (5.1b) has nothing to do with referential properties of the subject DPs in both sentences. Rather, the inference simply results from facts concerning what we know about where brains are located. This might seem plausible for examples such as (5.1). However, in previous chapters of this study we have seen that there are a number of natural language expressions that are sensitive to topological properties of part-whole structures corresponding to their referents. My view is that if we want to account for the meaning of nouns in

176

its entire complexity, topology-related phenomena deserve serious consideration.

(5.1)   English (Schwarzschild 1996, p. 187)
    a.   Bill is in Texas.
    b.   Bill's brain is in Texas.

It has been acknowledged for a long time that nominal semantics differentiates between count singulars referring to integrated objects on the one hand and mass terms and plurals denoting scattered entities and arbitrary sums of individuals, respectively, on the other. As we have seen in Section 5.1.1, there are good reasons to assume that this contrast correlates with some fundamental properties of human cognition. Soon, we will see that the distinction can be captured in terms of how parts constituting a whole are spatially arranged. However, before I even start considering how to develop a proper account, it is crucial to realize that the extent to which topology plays a role in part-whole structures associated with referents of nominals is highly underestimated, if not neglected, in the contemporary mainstream research on pluralities and countability. One of the important empirical contributions of this study is about compensating this deficit.

The data presented in Chapter 2 and 3 show that certain classes of nouns encode certain types of spatial configurations of entities making up a whole. In particular, at least some Italian irregular plurals discussed in Section 2.3 denote integrated pluralities, i.e., sums of individuals that form cohesive wholes. On the other hand, different types of topology-sensitive partitive words explored in Section 3.1 and 3.2 involve reference to continuous parts whereas extensions of their topology-neutral counterparts comprise also from discontinuous portions of a whole. Though the evidence explored in this study emphasizes specifically the role of integrity in subatomic quantification, recent research on different types of collective nouns and mass terms points to a similar conclusion regarding the relevance of topology in nominal semantics on independent grounds. In particular, Grimm (2012b) proposes that a subset of mass nouns that denote aggregates of granular objects, hence granular mass terms like *rice* and *gravel*, involve reference to clustered individuals, i.e., bundled entities transitively connected to each other. Similar, Henderson (2017) argues that

swarm nouns such as *grove*, *horde*, and *swarm* denote large pluralities whose constituents remain in proximity within a certain spatial configuration. Moreover, Grimm and Dočekal (to appear) discuss a particular class of Slavic derived mass nouns such as Czech *list* 'leaf' ~ *listí* 'foliage' that refer to collections of singular entities that are easily distinguishable yet conceptualized as aggregates.[4] Finally, as proposed by Grimm (2012b) referents of concrete count nouns can be viewed as entities whose part-whole structure is constrained in such a way that it forms an integrated object in its own right.

The psychological evidence discussed in Section 5.1 emphasizes the role of spatial integrity of objects on the one hand and our ability to simultaneously perceive individual parts of such cohesive wholes as entities in their own right on the other. Given the amount of linguistic evidence supplemented with findings concerning human cognition, I argue that semantic theory needs to accommodate to those facts. In other words, my first claim is as follows. There is more to the meaning of common nouns than usually assumed and without a proper approach incorporating the insights concerning the role of topological notions in part-whole structures many phenomena will be left unexplained or even unnoticed. I posit that a good starting point is to try to capture the difference between count singulars, regular plurals, and mass nouns in terms of different topological relations encoded within corresponding part-whole structures. Specifically, count singulars refer to entities that constitute integrated wholes whose parts stick together rather than mere collections of spatially unrelated portions of matter. On the other hand, plural nouns denote arbitrary sums of such individuals, i.e., they require their parts to be constrained by spatial integrity but impose no topological restrictions on configurations in which they appear. Finally, a proper treatment of mass terms should account for the fact that their prototypical referents are scattered unbounded entities. An additional advantage of such a novel perspective on is that it suggests that different part-whole structures do not emerge due to distinct parthood relations. Instead, it seems that different part-whole structures arise as a result if the interaction between one unified parthood relation with distinct topological no-

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4. For experimental investigation into the meaning of such expressions in Czech and Polish see Dočekal and Wągiel (2018).

tions. In the next section, I will posit that the postulated view on what it means to be a whole can shed new light on countability.

5.2.2 General counting principles

In general, I follow a well-established linguistic tradition assuming that the mass/count distinction corresponds to the contrast concerning how different types of entities are conceptualized (e.g., Wierzbicka 1988, Chierchia 1998a,b, 2010, 2015, Grimm 2012b). As we have seen in Section 5.1, a substantive body of research in cognitive psychology provides evidence that human beings categorize stable bounded things that preserve shape differently than amorphous substances. The first are considered delimited objects whose spatial identity can be tracked easily throughout time, whereas the latter are merely unindividuated portions of matter. On the other hand, there appears to be rich intuitive knowledge concerning what counting is that seems to be part of human cognitive endowment. Though the exact nature of how cognitive facts relate to natural language semantics and grammar, i.e., the structure and meaning of certain syntactic constructions, might not be straightforward, I propose a most probably simplified view that will nevertheless reveal what I argue is an advantageous perspective on what it means to be countable.

Usually in the study of countability, at some point when it comes to defining what counting is something like 'counting is quantification over what counts as one' is stated. Often, 'what counts as one' is understood in terms of atomicity. In particular, an atom in a technical sense is an object that has no proper parts. Though at first blush this notion seems rather counterintuitive (see Champollion 2010, 2017 for discussion),[5] it has been very influential in the research on countability. To the extent that counting is often implicitly assumed to be simply quantification over atoms. In this section, I will attempt to elaborate a bit more on what counting is and how it differs from other forms of quantification. For this purpose, I will consider three conditions entities need to satisfy in order to be able to be put in one-to-one correspondence with numbers based on what we know what humans do when they count. In particular, I will propose three general counting

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5. I will come back to this issue in Section 7.1.

principles that will allow us to better understand countability in a more general manner. As we will soon see, this novel perspective will prove of great significance with respect to subatomic quantification.

Counting is usually understood as establishing one-to-one correspondence between what is being counted and numbers. This kind of association is necessary to determine the cardinality of a particular set of entities. Though there are a number of aspects of counting as well as many morphologically distinct numerical expressions dedicated to particular quantificational purposes, including cardinal numerals, ordinals, fractions, percentage expressions, nominals such as *dozen*, approximators like *hundreds*, and numerical adverbials, e.g., *twice*, it is usually assumed that at the end their interpretation is based on number (see Rothstein 2017).[6] In particular, different types of numerical expressions appear to represent enumerations of different types of sets, e.g., sets of entities as opposed to sets of events.

However, though the definition introduced above involves a significant component of counting, it is incomplete, and thus fails to capture what counting really is and how it differs from other quantificational operations such as measuring. As we will soon see, there are a number of situations where despite successfully establishing one-to-one correspondence between entities and numbers we would intuitively reject a particular enumeration as counting. Therefore, a more specific characterization is required. In particular, I take counting to be a quantificational operation that is governed by three general principles restricting what kind of object can be assigned a number. I will refer to those constraints as the principle of non-overlap, maximality, and integrity. It is not unlikely that their knowledge is inherent, and thus when talking about counting, their existence is taken for granted. That is precisely why I believe it might be useful to formulate them explicitly in order to get better understanding of the phenomenon of countability.

The principle of non-overlap ensures that things that count as one need not overlap, i.e., do not share a part (cf. Landman 2011, 2016). Guaranteeing disjointness of units of counting is necessary to avoid

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6. Notice also that both complex numerical expressions are necessary if a language is to be able to enumerate the infinite series of numbers since in order to do so it requires a recursive system (Rothstein 2017, pp. 12–13).

a possibility of an entity being counted twice. For instance, assume portions of matter a, b, c, and d arranged in such a way that c overlaps with both a and b, specifically $c = a \sqcup b$. Now, one could imagine an operation that would assign numbers to all a, b, c, and d. Summing them up would yield number 4 but this result is incorrect if one wanted to count how many portions of matter there are. The reason is that c is not disjoint from a and b, and thus should not be associated with a number. To put it differently, when counting it is disallowed to count a thing two times.

The second principle concerns maximality understood in terms of mereological exhaustivity. It states that counting requires that what is associated with a number needs to be a maximal entity of which a certain property holds. In other words, objects need to be counted in their entirety, i.e., all parts of a thing need to be put in correspondence with a particular number and it is disallowed to leave some of them out. To illustrate this, let us consider a situation where there are three distinct entities a, b, and c such that c consists of two parts d and e. Now, assume a quantificational operation that satisfies the non-overlap constraint but is not restricted by the principle of maximality. When applied to the set of entities in question, it might very well yield 4 since a, b, d and e are disjoint, whereas c shares a part with both d and e. However, such an operation would not be of great help if one wanted to know how many entities are there since it fails to differentiate between wholes and their parts. Counting, on the other hand, is about recognizing that though what counts as one might be constituted of smaller elements, for the purpose of quantification the maximal sum of those elements is considered a whole unit. Again, counting ensures that an entity is not assigned two numbers. Notice also that this constraint accounts for how we count homogeneous entities such as twigs and rocks. Given a particular counting situation, what counts as one is always the maximal entity irrespective how its part-whole structure is construed in that situation.

Finally, the principle of integrity requires what counts as one to be an integrated whole. It is crucial that for an individual to be assigned a number it is not enough to be disjoint from other entities and mereologically maximal. In addition, its parts need to be in a particular spatial configuration, i.e., they need to be connected. This means that scattered entities such as substances and arbitrary sums

of individuals normally are not assumed to count as one. In the next chapter, I will introduce a theory of wholes spelling out those concepts in a formal way. Notice, however, that the use of the words 'integrated' and 'connected' is somewhat liberal here and should be rather understood as 'conceptualized as integrated/connected'. If the connection between parts is easily retrievable, e.g., parts are physically dissected but remain in proximity to the original place of attachment, then with great probability the principle of integrity would be satisfied. Thus, individuation in terms of integrity is about how human beings categorize objects in the world rather than about objective properties of mind-external entities. As we will see in the next section, this property is crucial for distinguishing counting from another quantificational operation, namely measuring. However, before I discuss the difference between the two in detail, let us consider examples of proper counting as opposed to what I call illegal counting.[7]

Given the counting principles of non-overlap, maximality, and integrity discussed above, we expect that counting works as illustrated schematically in Figure 5.3. Each of the apples can be associated with integers since they all satisfy the requirements in question. First, the apples are disjoint from each other, i.e., they do not share parts. Second, in each case what is assigned a number is a whole apple, i.e., a maximal sum of parts making up an object. This means that counting is complete. In other words, after the procedure is finished, there are no entities left that have not been associated with an integer. Finally, individual apples constitute objects that are conceptualized as integrated wholes, i.e., configurations of parts that connected to each other. This fact guarantees that each apple can be tracked in space and time, and thus is easily recognizable as an object distinct from other entities. The interplay of the three factors in question results in that the procedure illustrated in Figure 5.3 satisfies conditions on counting. Given the depicted set of apples, after performing it we would get an integer corresponding to the total number of apples.

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7. Notice that in accordance with the focus of this study the proposed principles are intended to account for quantification over concrete physical objects. In fact, they might be more abstract in order to be extendable to quantification over abstract entities such *two ideas* and *three proposals*. However, I will not pursue this possibility here.
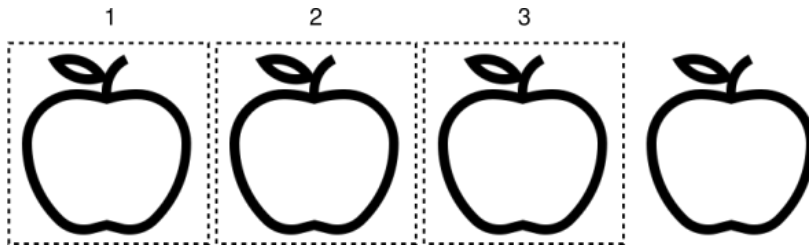
Figure 5.3: Counting

According to the general counting principles, counting is devised to be what we see in Figure 5.3. However, one could easily imagine an operation where, e.g., assigning a number to less than a whole entity or summing up complementary parts of corresponding entities to make up what counts as one is not prohibited. For instance, consider a quantificational operation that satisfies the general counting principle of non-overlap, but violates the principle of integrity. To illustrate this, let us assume illegal counting as depicted in Figure 5.4. In contrast to Figure 5.3, in this case numbers are associated with arbitrary sums of parts of distinct entities. Notice that those sums can be guaranteed to cover the total volume of all the relevant apples. If this were somehow ensured, in the discussed scenario we could obtain the same number as in Figure 5.3. However, intuitively this operation is very different from counting. It is simply not what we do when we count.
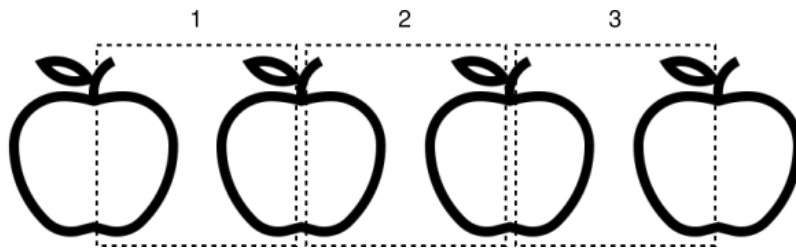
Figure 5.4: Illegal counting

Though the procedure illustrated in Figure 5.4 is not counting, it does not mean that it does not represent another quantificational operation human beings utilize for different purposes. In the next paragraphs, I will discuss what I believe to be the crucial difference between counting and measuring.

Intuitively, the difference between counting and measuring relies essentially on that the first is about specifying how many objects of a certain kind are there in a particular context, whereas the latter concerns determining the quantity of a particular substance in relevant measure units. In principle, there are three possible views on the relationship between the two operations in question. The first one reduces measuring to a particular type of counting (e.g., Gil 2013). The intuition behind such an approach is that measure words individuate in terms of quantity (Lyons 1977). Consequently, instead of quantification over objects measuring is about counting units determined be measure words. An opposing approach is to view counting as a form of measuring. In particular, counting can be understood as measuring a quantity of an object in terms of natural units (Krifka 1989, 1995). Finally, the third option is to postulate that counting and measuring are two independent operations (Rothstein 2017). In this study, I adopt the third perspective and present novel evidence in its favor. In particular, I propose that the distinction between counting and measuring can be reformulated in terms of quantificational principles introduced in the previous section. Though both counting and measuring obey the principle of non-overlap and maximality, only the first satisfies the principle of integrity. Measured quantities of a substance cannot overlap in order to ensure that things are not measured twice. Furthermore, a unit of measurement needs to correspond to the maximal quantity of matter to guarantee that measuring is exhaustive. However, only counting is sensitive to the topological make-up of entities it applies to.

To better understand the essence of the distinction, let us consider the following two scenarios. In the first scenario, illustrated in Figure 5.5, someone has spilled some liquid on the table in such a way that there are two separate blobs a and b whose volume is one and a half milliliters each. In the second scenario, see Figure 5.6, someone has simply put two cubes c and d on the table. Let us also assume that apart from the liquid and the cubes there is nothing else on the table in each of the cases, respectively.
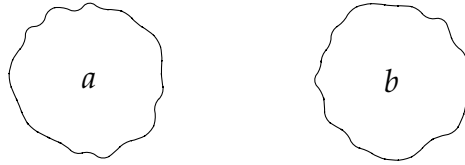
Figure 5.5: Measuring and integrity

Figure 5.6: Counting and integrity

Now, let us examine the sentences in (5.2a) and (5.2b) describing the situations depicted in Figure 5.5 and 5.6, respectively. The statement in (5.2a) is true despite the fact that one of the three milliliters must be split between a portion of *a* and a portion of *b* since each of the blobs consists of one and a half milliliters of liquid. On the other, hand (5.2b) is simply false. This contrast shows that units of measurement such as milliliters are not objects. In other words, unlike counting measuring does not care about individuation in terms of integrity and it indeed appears to be a distinct operation.

(5.2)   English (Guy Tabachnick, p.c.)
   a.   There are three milliliters of liquid on the table.
   b.   #There are three objects on the table.

I argue that this is because counting is topology-sensitive whereas measuring is not. In other words, in measuring units of measurement are not required to be assigned only to contiguous entities. There are multiple ways how to assign particular milliliters u_1, u_2, and u_3 to particular portions of *a* and *b* and Figure 5.7 illustrates one possible distribution with u_2 corresponding to the volume of liquid of a sum of one-third of *a* and one-third of *b*. As long as the total volume of *a* and *b* equals three milliliters (5.2a) is true with respect to Figure 5.5.

185
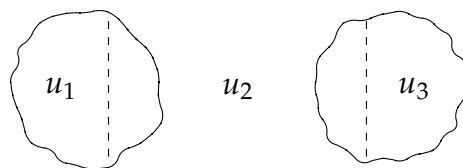
Figure 5.7: Measuring in units

Unlike measuring, counting is sensitive to what kind of topological structure entities in its domain have. In parallel to 5.7, one could imagine that two-thirds of c correspond to an object o_1, one-third of c plus one-third of d make up o_2, and the remaining two-thirds of d correspond to o_3, as depicted in Figure 5.8. However, this is not how counting works. The statement in (5.2b) is false with respect to Figure 5.6 because counting assigns numbers to integrated individuated entities rather than arbitrary portions thereof.
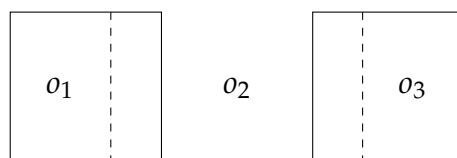
Figure 5.8: Counting objects

I argue that what the contrast between truth-conditions of (5.2a) and (5.2b) discussed above shows is that counting and measuring are in fact two distinct semantic operations, as proposed by Rothstein (2017). The core difference between the two boils down to the fact that counting is topology-sensitive, whereas measuring is not. In other words, it is misleading to think of measuring in terms of counting measure units (pace Gil 2013). If it were the case, similar to 5.6 we would expect (5.2a) to be false with respect to Figure 5.5, contrary to fact.

The distinction gets even more salient if we realize that many numeral phrases can get a measure interpretation.[8] For instance consider the contrast between the scenarios described in (5.3) and (5.4). In

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8. See Rothstein (2017, p. 3) for an extensive discussion of ambiguities another sort, namely the alternation between individuating and content readings of container words such as *glass of water*.

186

(5.3a), the numeral phrase *three apples* denotes a plurality of integrated wholes, i.e., distinct individuated objects, and thus it can be felicitously continued by (5.3b). In other words, since the referents of the phrase satisfy conditions specified by the general counting principles, they can be put in one-to-one correspondence with numbers. On the other hand, the same phrase in the context of (5.4) gets a measure reading. Given that the slices in the bowl are placed in such a way that it is impossible to retrieve the original part-whole structures of individual apples, *three apples* does not refer to distinct objects but rather to the volume corresponding to three apples. Consequently, it is distinctively odd to continue the sentence in (5.4a) with (5.4b). All things considered, (5.3a) presumes an operation depicted in Figure 5.3, whereas (5.4a) does not.

(5.3) English (Guy Tabachnick, p.c.)
Context: John is cooking with his child. They put three whole apples on a table. John says:
a. There are three apples on the table…
b. Let's count them together: one, two, three.

(5.4) English (Guy Tabachnick, p.c.)
Context: John is cooking with his child. They sliced three apples and put the slices into a bowl. John says:
a. There are three apples in the bowl…
b. #Let's count them together: one, two, three.

The contrasts discussed above prove that counting indicates integrity or at least easily retrievable traces of integrity of entities it applies to whereas measuring ignores this factor. Though monotonic systems of measurement track part-whole relations (Schwarzschild 2002), they do not seem to be sensitive to topological notions. On the other hand, counting does care about the spatial arrangement of parts making up wholes. This strongly suggests that the two operations in question are distinct from each other. Perhaps, one could argue that despite syntactic as well as semantic differences between numeral phrases and measure phrases (see Rothstein 2017) counting is a very special case of measuring (see Krifka 1989, 1995). In particular, measuring could be thought of as a more general quantificational procedure. In

principle, I remain agnostic on this issue and am open to an argument. However, for the sake of what this study is about I will assume that measuring and counting are two distinct semantic operations. In the next section, I will propose how general counting principles extend to subatomic quantification.

5.2.3 Subatomic quantification

In Chapter 2, 3, and 4, we have seen that there is substantial evidence showing that natural language semantics is sensitive to the fact that entities we refer to by means of nominal constructions consist of parts and that there are linguistic means to quantify over such parts. On the other hand, in the previous section I have postulated general counting principles of non-overlap, maximality, and integrity. So far, we have seen that there are good reasons to believe that these quantificational constraints capture what humans do when they count objects. In this section, I argue that the proposed set of rules constitutes a universal mechanism that can be applied not only to whole individuals but also when counting parts of objects. In other words, I posit that subatomic quantification is subject to the very same constraints as quantification over wholes.

Similar to what we have seen in Figure 5.3, counting at the subatomic level presumes mereological maximality and topological integrity of entities subject to quantification. Typically, counting is sensitive to the fact that some parts are cognitively more salient within the part-whole structure of an object than others. This seems to correspond to what Champollion and Krifka (2016) call structured parthood as well as to the distinction between specific and arbitrary subdivisions of a whole into parts introduced in philosophical considerations, as already mentioned in Chapter 1 (e.g., Krecz 1986, Markosian 1998, Jennings 2010). Notice that what counts as a cognitively salient part can vary with respect to the context. For instance, assume John's daughter has a teddy bear called Fuzzy Wuzzy. In a situation where she has covered Fuzzy Wuzzy's entire left half with red paint and its entire right half with black paint, John might want to say (5.5a). In such a case, the total volume of matter making up the teddy bear is partitioned in such a way that the cognitively salient parts are its left

half and its right half.[9] On the other hand, in a scenario where John's daughter painted Fuzzy Wuzzy's left paw red and right paw black, the count explicit partitive in (5.5b) refers to the teddy bear's body parts. Nevertheless, what cognitively salient parts have in common is that given a particular context they are disjoint. This brings us to the relevance of the principle of non-overlap in subatomic quantification.

(5.5)   English (Guy Tabachnick, p.c.)
    a.   Both parts of the teddy bear are painted.
    b.   Two parts of the teddy bear are painted.

Since there are numerous ways how to divide an object into parts, there are multiple portions of matter of which the property of being part of that object holds. For instance, Figure 5.9 illustrates a number of entities the phrase *part of the apple* could refer to. Notice that none of them is disjoint from the others, i.e., all of them share a part with at least one other part. However, the principle of non-overlap states that such entities cannot be counted. Similar to arbitrary portions of mass, e.g. juice, arbitrary parts of the apple abide individuation since they are not well-defined bounded integrated objects and it is virtually impossible to distinguish them from other parts. This means that not all parts are equal with respect to countability. Some of them are countable, whereas others are not. Crucially only those divisions of a whole into parts can be enumerated that involve only non-overlapping parts.
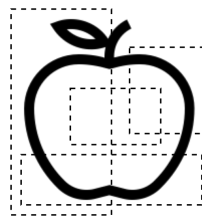

Figure 5.9: Overlapping parts

Another issue concerns the principle of maximality. What counts as part of an object can also consist of smaller parts. Notice that such parts of parts also satisfy the property of being part of that object. However, the general counting principles require that only entities in their mereological entirety can can be associated with a number. This means that once a particular division of an individual into parts has been executed in a given counting context, those parts are immutable and treated as objects in their own right. Consequently, the principle of maximality applies as it would in a situation when one counts whole individuals. In other words, given a partition non-overlapping parts are assumed to be maximal with respect to how the whole has been divided.

Finally, as we have seen in the previous section countability is also governed by the principle of integrity. However, as we have seen in Chapter 3 extensions of expressions referring to parts of objects do not necessarily involve topological commitments, i.e., parts need not be continuous. For instance, let us consider explicit entity partitives. There is definitely a sense in which two or more separated portions of matter within an object are part of that object, and thus a topology-neutral explicit entity partitive construction would be true of such a configuration. Nevertheless, similar to any arbitrary sum such entity is not countable since associating it with a number would clearly violate the principle of integrity. Therefore, only sets including parts that are mereologically maximal integrated entities that do not overlap can be enumerated. In other words, only a subset of possible divisions of an object is fit for counting.

Having this in mind, let us see how the general counting principles apply in a context of subatomic quantification. For this purpose, let us assume John's daughter wants to count parts of her teddy bear Fuzzy Wuzzy. Figure 5.10 represents an exemplary situation that intuitively fits what we expect from counting. Specifically, the illustrated operation assigns numbers to cognitively salient non-overlapping parts of Fuzzy Wuzzy, namely its ear, leg, and paw. Those parts constitute mereologically maximal and integrated entities, i.e., though they are elements of a larger whole, when counted they are treated as objects in their own rights. This means that, e.g., number 1 is associated with Fuzzy Wuzzy's whole ear and there is no part of that ear that is left out. Also, an ear is a continuous part that comes in one piece. To sum

up, since the depicted division of the teddy bear consists of elements that are disjoint, maximal, and integrated, all the counting principles are satisfied and the set of parts can be enumerated.
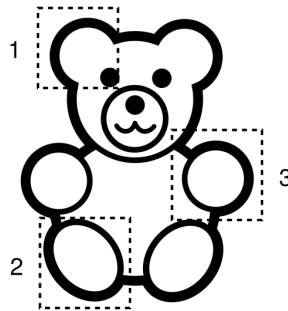
Figure 5.10: Counting of parts

On the other hand, one could imagine another operation, e.g., one such as that illustrated in Figure 5.11. Though it is definitely logically possible, it seems weird and would intuitively be rejected as counting. There are two reasons why it is so. The first reason is that the indicated ear and leg do not constitute a continuous integrated part that would be cognitively salient. Therefore, assigning them number 1 violates the principle of integrity. In other words, there is a sense in which an ear and a leg are part of the teddy bear, but there are not a part of it and only entities that can be considered as such can satisfy the countability condition. The second reason is that the marked paw is not disjoint with the right half of the teddy-bear which of course violates the principle of non-overlap. Given the division in Figure 5.11, the paw in question would be counted twice and since counting requires that an entity can be associated with a number once and once only, the depicted operation fails to satisfy the general counting principles and represents what I call illegal counting at the subatomic level.

All things considered, counting at the subatomic level presumes assigning numbers to salient parts of a whole constituting disjoint integrated entities that are maximal with respect to a given partition. In other words, once it is decided what counts as a part, it is treated as an individuated object in its own right and all discussed constraints concerning counting apply. The key conclusion is that there is one
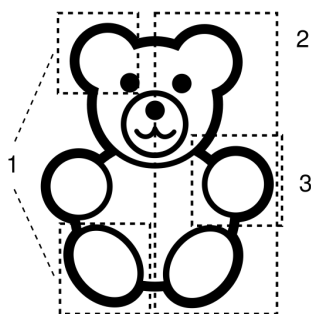
Figure 5.11: Illegal counting of parts

universal mechanism governing quantification over wholes as well as subatomic quantification and exploring the latter can reveal some fundamental properties of countability in general that otherwise might be difficult to recognize.

5.3 Summary

In this chapter, I have provided a general conceptual framework that will serve as a basis for developing a formal account for subatomic quantification in natural language. I have suggested how the linguistic evidence presented in Chapter 2, 3, and 4 correlates with findings in cognitive psychology. In particular, in the first part of the chapter I have reviewed several representative studies in the research indicating the following. First, there is compelling evidence that humans possess an innate ability to perceptually distinguish between objects, i.e., bounded integrated entities, and substances, i.e., shapeless scattered portions of matter, and that this contrast correlates with the mass/count distinction in grammar though the correspondence is imperfect. Specifically, integrated solid things are predictably referred to by count nouns, but there is a class of mass terms that also refer to objects. Importantly, the object/mass distinction is not based on ontological properties of entities in some objective sense but rather it is construed by human cognition. Second, the ability to simultaneously perceive parts as elements of a whole and a whole as a collection of parts manifests itself in early childhood. The capacity to intuitively dis-

tinguish between parts and wholes guides vocabulary acquisition. Finally, human number sense appears to be sensitive to whether counted items are conceptualized as integrated objects. Experimental evidence shows that young children always count each separate physical entity as one. Even if there are instructed to do otherwise or given clues that two elements might be considered one broken thing, they ignore them and simply cannot avoid counting contiguous entities. This suggests that further development of quantification in older humans rests on a mechanism individuating in terms of spatial integrity.

In the second part of this chapter, I have presented the three claims that constitute the conceptual core of this study. I have postulated that natural language is sensitive to how the spatial relationship holding between parts of a particular entity is conceptualized. The relevance of the topological notion of integrity manifests itself primarily in how nominal lexicon is classified into different grammatical categories. In particular, count singular nouns prototypically designate integrated wholes, i.e., encode information concerning a particular spatial configuration of parts their referents consist on, whereas plurals denote arbitrary sums thereof, i.e., presuppose integrated wholes as parts but impose no topological constraints on them.

The second claim regards what I call the general counting principles. I posit that counting is a special kind of quantificational operation that presumes certain properties of entities that are put in one-to-one correspondence with numbers. Specifically, the principle of non-overlap guarantees that enumerated entities are disjoint, and thus no thing is counted twice. On the other hand, the principle of maximality requires that numbers are associated with entities in their mereological entirety, i.e., no part is left out. Finally, the principle of integrity ensures that what can be counted needs to be conceptualized as something that comes in one piece. This constraint excludes arbitrary sums of individuals as well as scattered entities such as substances. Altogether, the general counting principles guarantee that sets that can be enumerated consist only of elements that are discrete object.

The final claim extends the general counting principles to subatomic quantification. In other words, I postulate the quantificational mechanism described above is a universal mechanism that can apply both at the level of wholes and at the level of parts. In the next chapter, I will introduce a theory of wholes called mereotopology in which

the formal account for subatomic quantification will be grounded. In particular, it will enable us to capture various subtle topological distinctions including the intuitive notion of an integrated whole as opposed to an arbitrary sum of parts. This will provide means to model not only the fact that some entity is part of something else but also how individual parts are spatially arranged within a whole configuration.

6 Theory of parts and wholes

Given the conceptual background described in the previous chapter, an account for subatomic quantification is supposed to capture the intuitive notions of parthood and integrity. In this chapter, I will introduce a theory of parts and wholes that provides a formal apparatus devised to model both concepts. This theory is usually referred to as mereotopology and as suggested by the name it involves two interrelated components. In the following sections, I will first introduce standard mereology, its axioms, and advantages. Next, I will turn to its limitations and discuss how it can be extended with topological notions such as connectedness. As a result, the sophisticated mereotopological framework to be presented will allow us to model integrated wholes as opposed to other types of entities. The mereotopological distinctions to be developed will play a crucial role in the formal account for subatomic quantification in natural language I will propose in the next chapter.

6.1   Mereology

Mereology (from the Greek $\mu\epsilon\rho o\zeta$ 'part') is the study of parthood, i.e., relations between part and whole as well as between parts within a whole. It has been proposed by Leśniewski (1916) and further developed by Leonard and Goodman (1940) and Goodman (1951) as an alternative for set theory. In particular, since set theory is founded on two distinct part-whole notions of set membership $\in$, i.e., the relation between an element and a set, and subset relation $\subseteq$, i.e., the relation between sets, it distinguishes between a singleton set and its member, i.e. $\{a\} \neq a$, and postulates the empty set $\varnothing$. In order to avoid such assumptions mereology does not postulate abstract entities such as sets, and thus does not distinguish between $\in$ and $\subseteq$. Instead, only one primitive relation of parthood is postulated. Therefore, there is no need to draw the distinction between a singleton set and its member nor to assume the notion of the empty set.

The part-whole relation can hold between portions of masses, parts of individuals, members of groups, locations, events, times, and even abstract entities (Simons 1987, Winston et al. 1987). It is a prominent no-

tion in how the world appears to be structured, how human beings perceive things and talk about things. Therefore, mereology plays an important role in ontology, cognitive psychology, and natural-language semantics. Since Quine (1960)'s discussion of mass nouns as scattered objects (a clearly mereological concept), considering countability in mereological terms became standard in philosophy (see especially Sharvy 1980 for a treatment of mass definite descriptions in this spirit). However, it was Link (1983) who introduced formal mereology to linguistics in his lattice-theoretic approach to pluralities.

There are different ways how to spell out mereological intuitions and axiomatize the parthood relation (see, e.g., Simons 1987, Casati and Varzi 1999, and Varzi 2016 for thorough surveys of the field). However, in this chapter I will discuss only a system that gives rise to algebraic structures, i.e., sets with binary operations defined on them, that is often referred to as classical extensional mereology. For brevity, however, I will simply call it standard mereology since it is considered to be a standard in mereology (though see, e.g., Rescher 1955 for an alternative proposal) and is most commonly used in formal semantics for natural language (e.g., Link 1983, Krifka 1989, Landman 2000, and Champollion 2017, to name just a few prominent approaches). The axiomatization of standard mereology presented here largely follows the extensive discussions provided by Simons (1987) and Casati and Varzi (1999) as well as the encyclopedic entry on linguistic applications of mereology by Champollion and Krifka (2016). To avoid terminological confusion I will follow Grimm (2012b) in distinguishing between INDI-VIDUALS, i.e., a pre-theoretic notion regarding well-defined physical entities, and M-INDIVIDUALS, i.e., objects in the mereological sense.

The most received view on the axiomatization of mereology takes the concept of PART to be the basic notion. The part-of relationship $\sqsubseteq$[1] is the primitive relation which is reflexive, transitive, and antisymmetric. The axioms given in (6.1)–(6.3) constrain $\sqsubseteq$ to be a partial order.

(6.1)   Reflexivity
$\forall x [x \sqsubseteq x]$
(Every thing is part of itself.)

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1.   I use the symbol $\sqsubseteq$ following Landman (1989a,b, 2000). Other authors use $\leq$ instead. The main motivation behind this notational decision is to avoid potential confusion with the 'less than or equal to' relation between numbers.

(6.2)   Transitivity
$\forall x \forall y \forall z[(x \sqsubseteq y \wedge y \sqsubseteq z) \rightarrow x \sqsubseteq z]$
(Any part of any part of a thing is itself part of that thing.)

(6.3)   Antisymmetry
$\forall x \forall y[(x \sqsubseteq y \wedge y \sqsubseteq x) \rightarrow x = y]$
(Two distinct things cannot both be part of each other.)

Given that the primitive $\sqsubseteq$ relation states that an m-individual is part of itself, it might be useful to introduce an auxiliary notion of parthood which is not reflexive. The formula in (6.4) provides the definition of PROPER PART.

(6.4)   Proper part
$x \sqsubset y \overset{\text{def}}{=} x \sqsubseteq y \wedge \neg(y \sqsubseteq x)$
(A proper part of a thing is a part of a thing that is distinct from it.)

Furthermore, one might also want to account for the fact that entities can share parts. For this reason, an ancillary relation OVERLAP can be defined, as in (6.5).[2] From the mereological point of view, identity can be understood as a special case of overlap.

(6.5)   Overlap
$x \circ y \overset{\text{def}}{=} \exists z[z \sqsubseteq x \wedge z \sqsubseteq y]$
(Two things overlap if and only if they share a part.)

The parthood relation together with the derived notions of proper part and overlap constitute the core of standard mereology. However, without any further constraints the system developed so far gives rise to structures which have been traditionally dismissed as objects that a theory of part-whole relations should represent. For instance, Figure 6.1 illustrates a mereological relation between the m-individuals a and b such that b is a proper part of a and there is no other proper part of a.[3] Similar, Figure 6.2 depicts a model in which two distinct m-individuals

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2.   I use the $x \circ y$ notation following Simons (1987), Link (1983), and Champollion and Krifka (2016). Other authors use $\mathrm{o}(x, y)$ instead.
3.   The line represents $\sqsubseteq$ with the m-individual corresponding to the lower node being part of the m-individual corresponding to the higher node.

a and *c* both share all parts. Though such models are allowed within the system, intuitively they are not what a theory of parts and wholes should account for. To remedy this, further constraints have to be introduced.

Figure 6.1: Object with a solitary proper part
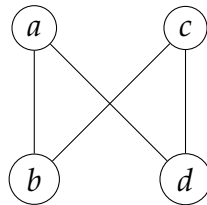
Figure 6.2: Objects sharing all parts

Standard mereology can be devised in order to restrict how an m-individual can be decomposed into parts. In particular, an additional axiom can be added that constrains mereological objects in such a way that every proper part must be supplemented by another disjoint part. In other words, there is always a mereological difference between a whole and its proper parts. An extension known as REMAINDER PRIN-CIPLE or SUPPLEMENTATION, see (6.6), guarantees that m-individuals cannot consist of a single proper part, and thus models such as that in Figure 6.1 are ruled out.

(6.6)   Supplementation
$x \sqsubset y \rightarrow \exists z[z \sqsubseteq y \wedge \neg(z \circ x)]$
(Whenever a thing has a proper part, it has more than one.)

However, supplementation is insufficient to exclude structures like the one in Figure 6.2, and thus another extension needs to be implemented in order to ban them. Given $\sqsubseteq$, the notion of SUM can be defined. Sums

are devised to capture a pretheoretical concept of collections, i.e., the result of grouping several entities together. In natural language, conjoined terms and definite descriptions like *the water* have been analyzed in terms of sums. In particular, in his influential paper Link (1983) treats expressions such as *John and Mary* as referring to the sum of two individuals, namely John and Mary. Similar, Sharvy (1980) proposes that a definite expression such as *the water* denotes the sum of all water. I will follow here the classical definition of sum due to Tarski (1929) as presented in (6.7) (for alternative definitions see Simons 1987 and Casati and Varzi 1999).

(6.7) Sum
$$\text{SUM}(x, P) \overset{\text{def}}{=} \forall y[P(y) \to y \sqsubseteq x] \land$$
$$\forall z[z \sqsubseteq x \to \exists z'[P(z') \land z \circ z']]$$

In prose, a sum of (the things in) a set P is a thing that consists of everything in P and whose parts each overlap with something in P. Since the part structures in standard mereology are closed under sum formation, for any two individuals there is also a sum of those two individuals. This is ensured by UNIQUENESS OF SUMS which requires two things composed of the same parts to be identical, see (6.8). This principle excludes structures such as that in Figure 6.2. Since b and d form two different sums, namely a and c, uniqueness of sums is violated and the model is ruled out, as desired. Notice also that introducing uniqueness of sums makes the axioms of reflexivity and antisymmetry redundant since any transitive relation that satisfies uniqueness of sums is provably reflexive and antisymmetric (see Hovda 2009 and Champollion 2017). The axioms have been discussed for completeness though.

(6.8) Uniqueness of sums
$$\forall P[P \neq \varnothing \to \exists! z \; \text{SUM}(z, P)]$$
(Every non-empty set has a unique sum.)

Finally, the additional notions BINARY SUM and GENERALIZED SUM can be derived, as given in (6.9) and (6.10), respectively.[4] The first operation

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4. Again, I follow Landman (1989a,b, 2000) in the use of the symbols $\sqcup$ and $\bigsqcup$, respectively, which correspond to the symbols $\oplus$ and $\bigoplus$ one can often find in the literature.

allows us to refer explicitly to the sum of two m-individuals, whereas the latter to the sum of an arbitrary set.

(6.9)   Binary sum
$$x \sqcup y \stackrel{\text{def}}{=} \iota z \,\textsc{sum}(z, \{x, y\})$$

(6.10)   Generalized sum
$$\bigsqcup X \stackrel{\text{def}}{=} \iota z \,\textsc{sum}(z, X), \text{ where } X \text{ is any non-empty set}$$

Consequently, the meaning of the coordinate phrase *John and Mary* can be represented as $j \sqcup m$, whereas the semantics of the definite description *the water* as $\sqcup$WATER.

Alongside philosophical arguments for standard mereology, a significant advantage of this framework is that it involves a well-understood algebraic structure. In particular, as demonstrated by Tarski (1935) models delivered by mereology are essentially isomorphic to Boolean algebras with their bottom element removed, or equivalently complete semi-lattices without the null element (see also Pontow and Schubert 2006).[5] Figure 6.3 gives an exemplary mereological structure licensed by standard mereology. Note that it is isomorphic to the powerset of the set $\{a, b, c\}$ with the bottom element removed. I will follow the well-established tradition in the semantic literature and refer to such models as complete semi-lattices, however, as pointed out by Champollion (2017) due to the lack of the null element the use of the term *complete* is rather inadequate since it deviates from standard mathematical practice (see also Landman 1989a). The definition of such a structure is given in (6.11).

(6.11)   Let S be a set and $\sqsubseteq$ be a relation from S to S. A pair $\langle S, \sqsubseteq \rangle$ is called a complete semi-lattice iff $\sqsubseteq$ satisfies the axioms of transitivity and uniqueness of sums.

Before we move on to the discussion of the limits of standard mereology that seem to have posed some serious problems in the study of natural language semantics adopting lattice-theoretic approaches,

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5.   The reason is that standard mereology does not allow for an object that is part of everything and since the empty set is a subset of every other set, it must be eliminated.
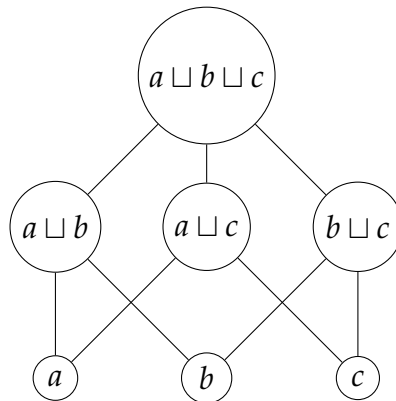
Figure 6.3: Semi-lattice

let us briefly contemplate some correspondences between mereology and set theory.

6.1.1 Mereology vs. set theory

It can be easily noted that the notion of parthood in standard mereology has basically the same properties as the subset relation in standard set theory. Therefore, since $\sqsubseteq$ and $\subseteq$ are very much alike, there a number of deep correspondences between the two axiom systems, some of which are illustrated in Table 6.1. Consequently, in many contexts it might be convenient to regard sums as sets and a question arises on why mereology is preferable over set theory in modeling pluralities in natural language semantics.

Though early approaches to the semantics of plural expressions were grounded in set theory (Bennet 1974, Hausser 1974), a substantive body of contemporary research embraces systems based on the notion of parthood. The original motivation behind preferring mereology over set theory in the logic of plurality developed by Link (1983, 1998) was formulated on philosophical grounds. In particular, Link argues that the traditional set-up of semantics taking the domain of individuals to be simply a non-empty set fails to represent various relations between objects. In other words, it is devised to capture only flat domains and cannot account for structured domains, as in modeling natural language. In terms of ontology, Link describes his position

201

PROPERTY	MEREOLOGY	SET THEORY
reflexivity	$x \sqsubseteq x$	$x \subseteq x$
transitivity	$x \sqsubseteq y \wedge y \sqsubseteq z \to x \sqsubseteq z$	$x \subseteq y \wedge y \subseteq z \to x \subseteq z$
antisymmetry	$x \sqsubseteq y \wedge y \sqsubseteq x \to x = y$	$x \subseteq y \wedge y \subseteq x \to x = y$
interdefinability	$x \sqsubseteq y \Leftrightarrow x \sqcup y = y$	$x \subseteq y \Leftrightarrow x \cup y = y$
unique sum/union	$P \neq \varnothing \to \exists! z[\text{SUM}(z, P)]$	$\exists! z[z = \cup P]$
associativity	$x \sqcup (y \sqcup z) = (x \sqcup y) \sqcup z$	$x \cup (y \cup z) = (x \cup y) \cup z$
commutativity	$x \sqcup y = y \sqcup x$	$x \cup y = y \cup x$
idempotence	$x \sqcup x = x$	$x \cup x = x$
unique separation	$x \sqsubset y \to \exists! z[x \sqcup z = y \wedge \neg(x \circ z)]$	$x \subset y \to \exists! z[z = y - x]$

Table 6.1: Correspondences between mereology and set theory (Champollion and Krifka 2016; adapted)

as "relative nominalism" and argues that a set-theoretic approach is not desirable because by treating pluralities as sets, it commits us to assuming additional abstract objects. On the other hand, since in a mereological framework pluralities are treated on par with singularities as individuals, such an approach bears no additional ontological commitments, and thus is to be preferred.[6]

Another arguable advantage of using mereology rather than set theory in plural semantics is that it conveniently allows us to distinguish type-theoretically between the denotations of common nouns, i.e., set-denoting expressions of type $\langle e, t \rangle$, and plural individuals, i.e., individual-denoting expressions of type e (see also Vaillette 2001, Champollion 2017). Due to the lack of sums set-theoretic approaches to pluralities conflate the meanings of these two types of expressions which can be considered an undesired result.

A prominent linguistic application of standard mereology is the theory developed by Champollion (2017) who adopts the order-theoretic perspective and treats $\sqsubseteq$ as primitive. On the other hand, Krifka (1998) derives $\sqsubseteq$ from $\sqcup$. Other influential theories that were intended to characterize standard mereology were developed by Link (1983, 1998) and Landman (1989a, 1991, 2000).[7]

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6. But see Landman (1989a) for discussion challenging such a view.
7. But see a scrupulous review by Hovda (2009) who demonstrates in detail that these systems contain certain flaws which result in that they fail to describe standard mereology, as intended.

6.1.2 Limits of mereology

As it has been pointed out by many authors, there is independent motivation for extending standard mereology with auxiliary notions. One factor concerns the criticism mereology faced with respect to its inadequacy in modeling objects in the real world. In particular, mereology is committed to unrestricted sum formation and as such it is insufficient to capture what it means to be a whole. This results in diametrical discrepancies between intuitions regarding the nature of entities in the world and objects mereology actually delivers. To use Casati and Varzi (1999)'s example, imagine a cup and broken glass. Intuitively, the first constitutes an object, i.e., an individuated whole, something that counts as one, whereas the other is just a collection of shards. However, this distinction cannot be captured by describing entities purely in terms of parthood. This is because in mereology for every whole there is a set of parts and to every arbitrary collection of parts there is their sum, i.e., a complete whole. As a result, a cup and broken glass have the very same mereological status. In other words, the allowance of scattered entities makes it impossible to differentiate between them and individuals constituting integrated wholes. Consequently, mereology has faced criticism that in principle it fails as a theory of individuals.

As suggested by Grimm (2012b), it is very likely that at least some of the shortcomings of mereological approaches to pluralities and countability in natural language may be due to the general flaws of standard mereology in attempting to capture what it means to be a whole. Arguably, extending standard mereological frameworks with additional notions in order to develop a better theory of objects, as proposed, e.g., by Casati and Varzi (1999), may provide superior tools for semantic treatments of quantification in natural language. As already signaled, a challenging problem concerns the requirement of unrestricted sum formation which states that for every two elements in the domain, there is a plural individual corresponding to the sum of those two elements. Specifically, the ontological status of such a plural entity has often been disputed. As wittily put by Landman (1989a)[8], if three children messed up the living room, the correct answer to the question "How many individuals were involved in

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8. See also Cresswell (1985).

messing up the living room?" is "three", whereas "seven" clearly seems incorrect. However, if pluralities were conceptualized as having the same ontological status as singular individuals, we would expect this answer to be adequate since that number includes all distinct sums of the children in question. In other words, if the children were Anne, Betty, and Carl, i.e., a, b, and c respectively, then 7 is the number of the elements of the set consisting of all entities that are part of the total sum of the children, i.e., $\{a, b, c, a \sqcup b, a \sqcup c, b \sqcup c, a \sqcup b \sqcup c\}$. The fact that natural language does not allow for quantification over arbitrary sums suggests that the way individual objects are conceptualized differs from how we see pluralities of objects. However, standard mereology lacks notions fine-grained enough to distinguish between objects that come in one piece and entities that do not.

The next section will introduce an extension of standard mereology within which more fine-grained notions can be defined. These notions will enhance mereology in that in addition to parthood they will allow us to capture also the topological configuration of entities making up a particular sum, i.e., their spatial arrangement. In other words, the topological extension will make it possible to discriminate between scattered entities and wholes that come in one piece.

6.2 Mereotopology

Topology (from the Greek $\tau o \pi o \zeta$ 'place') is the study of those properties of space that are unaffected by continuous deformations of shape or size of figures such as stretching or bending as opposed to tearing or gluing. Important contributions to the early development of the field were made by Fréchet (1906), Hausdorff (1914), and Kuratowski (1922) and two of the crucial notions studied in topology are connectedness and compactness.

Theories which extend standard mereology with topological relations are known as mereotopology. Though early attempts to formalize such a system trace back to Whitehead (1920, 1929), for many decades there was no continuation in the systematic study of mereotopological issues until relatively recent work in artificial intelligence began to investigate the interaction between mereological and topological notions in order to develop formal representations of spatial relations

such as NEAR or INSIDE. Consequently, new developments in philosophy and ontological modeling (e.g., Clarke 1981, Smith 1996, Roeper 1997) urged to extend standard mereology with topological relations in order to advance an improved theory of objects.

Though topological notions have been widely used within the study of locatives (e.g., Clark 1973, Herskovits 1985, Zwarts and Winter 1997, and Kracht 2002), it was not until Grimm (2012b,a) that mereotopology was introduced to natural language semantics.[9] In this chapter, I will define basic topological concepts and review how they interact with mereological notions as discuss by Casati and Varzi (1999) and adopted for the semantic treatment of countability by Grimm (2012b,a).

There are two main approaches with respect to relating mereology and topology: i) choosing mereology as a basic level and augmenting it with topological notions or ii) choosing topology as a basic level and adding mereological relations. I will follow Grimm (2012b) who employs a technique simply extending mereology with topological notions. In particular, he adopts the approach developed in Casati and Varzi (1999) which I will describe in the following paragraphs.

The crucial topological notion for the purpose of this study is CONNECTEDNESS (C). This relation is introduced in such a way that it interacts with other definitions and axioms of standard mereology. Connectedness is reflexive and symmetric, as defined in the axioms (6.12) and (6.13).

(6.12) Reflexivity
$\forall x[\mathrm{C}(x, x)]$
(Every thing is connected to itself.)

(6.13) Symmetry
$\forall x \forall y[\mathrm{C}(x, y) \leftrightarrow \mathrm{C}(y, x)]$
(If x is connected to y, then y is also connected to x.)

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9. Mereotopology has been further applied to particular topics concerning countability in Lima (2014) and Grimm and Dočekal (to appear). The theory has also inspired approaches that do not develop full-fledged mereotopological accounts but employ some spatial relations in order to capture certain issues relating to individuation such as Scontras (2014, 2017), Sutton and Filip (2017a,b), and Henderson (2017)

Notice, however, that unlike the parthood relation $\sqsubseteq$, as defined in standard mereology, the connectedness relation c does not need to be transitive. For instance, consider the configuration depicted in Figure 6.4. Although *a* and *b* are connected to each other and *b* and *c* are connected to each other, it is not the case that *a* and *c* are connected to each other.
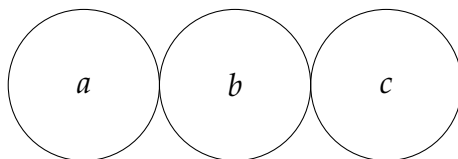
Figure 6.4: Connectedness and transitivity

    As discussed by Casati and Varzi (1999), there are a number of intuitive interactions between the topological notion c and mereological relations $\sqsubseteq$ and $\circ$. These intuitive interactions can be formalized as the so-called bridging principles which interrelate the mereological and the topological component of the theory (Varzi 2007). The main aim of the bridging principles is to secure that irrespective of their full characterization $\sqsubseteq$ and c are related in such a manner that a whole and its parts are firmly connected. In particular, the principle of INTEGRITY ensures that connectedness is implied by parthood, see (6.14), and consequently mereological overlap is in fact a form of connection as guaranteed by the principle of UNITY, see (6.15). Finally, the principle defined in (6.16) secures MONOTONICITY.[10]

(6.14)   Integrity
        $\forall x \forall y [x \sqsubseteq y \rightarrow \mathrm{c}(x,y)]$
        (If an m-individual is part of another m-individual, then they are connected.)

(6.15)   Unity
        $\forall x \forall y [x \circ y \rightarrow \mathrm{c}(x,y)]$
        (If two m-individuals overlap, then they are connected.)

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10. In fact, (6.16) implies both (6.14) and (6.15) (Varzi 2007). However, I provide all the bridging principles for the sake of completeness.

(6.16)   Monotonicity
$$\forall x \forall y \left[x \sqsubseteq y \rightarrow \forall z [\text{c}(z, x) \rightarrow \text{c}(z, y)] \right]$$
(If an m-individual is part of another m-individual, then whatever is connected to the first, is also connected to the latter.)

Given these extensions, a number of mereotopological properties like INTERNAL PART (IP), INTERNAL OVERLAP (IO), and TANGENTIAL OVERLAP (TO) can be defined. Such notions enable us to draw subtle distinctions to account for different spatial configurations that entities may be in. For instance, the individual b in Figure 6.5 is an internal part of the individual a since every entity that is connected to b overlaps with a or, in other words, a includes b. On the other hand, though part of b is an internal part of a in Figure 6.6, there is also part of b which is not included in a. Finally, Figure 6.7 illustrates a configuration where there is overlap between a and b but there is no internal overlap, i.e., the individuals only share their edges. The relevant definitions are given in (6.17)–(6.19).
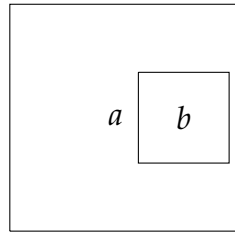
Figure 6.5: Internal part
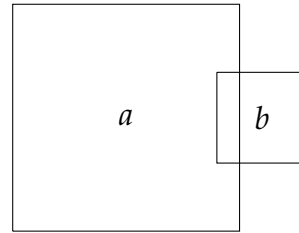
Figure 6.6: Internal overlap
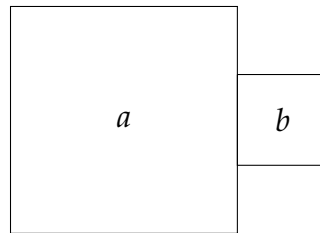
Figure 6.7: Tangential overlap

207

(6.17) Internal part
$$\text{IP}(x,y) \overset{\text{def}}{=} x \sqsubseteq y \land \forall z[\text{C}(z,x) \to \text{O}(z,y)]$$

(6.18) Internal overlap
$$\text{IO}(x,y) \overset{\text{def}}{=} \exists z[\text{IP}(z,x) \land \text{IP}(z,y)]$$

(6.19) Tangential overlap
$$\text{TO}(x,y) \overset{\text{def}}{=} \text{O}(x,y) \land \neg\text{IO}(x,y)$$

In addition, within the mereotopological framework standard topological notions can be defined. (6.20)–(6.23) give definitions for INTERIOR, EXTERIOR, CLOSURE, and BOUNDARY, respectively, whereas Figures 6.8–6.10 illustrate the intended configurations. In particular, the shaded area in the Figure 6.8 represents the interior of an object which is taken to be the sum of m-individual's internal parts. The dashed line is the boundary of the object which is not comprised in its interior. On the other hand, Figure 6.9 represents the exterior of an object which again does not include the boundary, whereas the solid line in Figure 6.10 indicates that both the interior and boundary make up the closure of an entity.

(6.20) Interior
$$ix \overset{\text{def}}{=} \sqcup X \text{ where } X = \{y \ : \ \text{IP}(y,x) = \text{TRUE}\}$$

(6.21) Exterior
$$ex \overset{\text{def}}{=} i(-x)$$

(6.22) Closure
$$cx \overset{\text{def}}{=} -(ex)$$

(6.23) Boundary
$$bx \overset{\text{def}}{=} -(ix \sqcup ex)$$
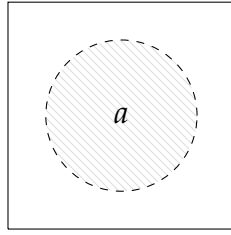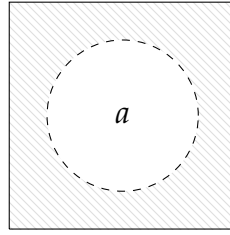
Figure 6.8: Interior   Figure 6.9: Exterior   Figure 6.10: Closure
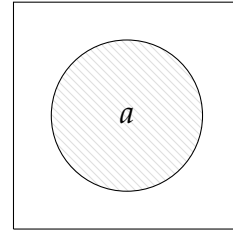
The framework described so far extends standard mereology with the primitive topological relation of connectedness and several derived notions which allow us to talk about different configurations that entities can be in. Such a system provides means to define individuals as integrated wholes, i.e., objects characterized in terms of different degrees of connectedness. The following section will discuss a mereotopological approach to such individuals.

6.2.1 Integrated wholes

A great advantage of the mereotopological approach to natural language semantics is that by means of the topological relations defined so far it is possible to capture what it means to be an individual understood as an integrated whole. In particular, a distinction can be drawn between entities which come in one piece as opposed to pluralities, i.e., scattered entities which bear no topological commitments. To distinguish the first from the latter, it is essential to introduce the property of SELF-CONNECTED (SC). Unlike arbitrary sums, SC entities cannot be divided into separated parts. The definition of SC is provided in (6.24).

(6.24)   Self-connected

$sc(x) \stackrel{\text{def}}{=} \forall y \forall z [\forall w (o(w,x) \leftrightarrow (o(w,y) \lor o(w,z))) \rightarrow c(y,z)]$
(An entity is self-connected if and only if any two parts that form the whole of that entity are connected to each other.)

Given the definition in (6.24), it is possible to differentiate inseparable individuals from arbitrary sums of entities, i.e., disconnected configurations of objects. To illustrate this let us assume a model containing four entities, specifically suppose *a* and *b* are two halves of a cube, whereas *c* and *d* are a pyramid and sphere, respectively, see Figure

6.11. Intuitively, there is a difference between the sum $s_1 = a \sqcup b$ and the sum $s_2 = c \sqcup d$. While s_1 forms an integrated whole, the plurality s_2 is just an arbitrary collection of disconnected objects, and thus it does not make up a solid whole. In other words, $\mathrm{sc}(s_1)$ is true, whereas $\mathrm{sc}(s_2)$ is false.
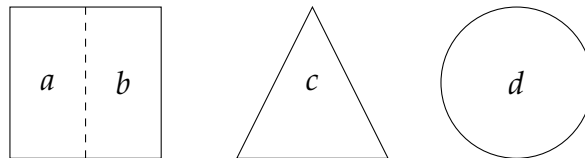
Figure 6.11: Wholes vs. sums

Self-connectedness is a great improvement in the theory of wholeness. However, this notion is still insufficient to capture what it means to be a one-piece object since it allows for configurations involving only an external connection holding between individuals. In other words, cases when the closure of one object overlaps the other or vice versa are not ruled out. For instance, Figure 6.12 depicts two spheres a and b which only touch each other, i.e., their boundaries are connected at a single point. Given the definition of sc in (6.24), a and b are self-connected but intuitively such a spatial configuration is not sufficient to count as an integrated whole.
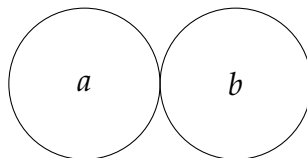
Figure 6.12: External connection

In order to rule out cases such as the one illustrated in Figure 6.12, a stronger relation is required. Ostensibly, what is necessary is that not only boundaries of parts of a whole are connected to each other, but also that their internal parts are shared. The basic io relation as defined in (6.18) allows for modeling strong connection by means of internal overlap but it is too weak to capture the nature of what it means to be an integrated whole since it does not exclude certain undesirable configurations. For instance, consider two partially-merged spheres a

and b in Figure 6.13. Though their interiors are connected, there are parts of a that are not shared with b and vice versa. Consequently, in spite of the fact that the sum $s = a \sqcup b$ satisfies io, it is not what would intuitively count as a one-piece individual.
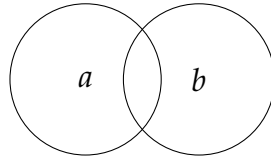
Figure 6.13: Strong connection

To solve this problem one needs to define an additional restriction on the concept of interior given in (6.20). Accordingly, Casati and Varzi (1999) postulate the property of being STRONGLY SELF-CONNECTED (SSC), as defined in (6.25). This refinement will get us much closer to the type of entities we would like to single out, i.e., integrated wholes.

(6.25)   Strongly self-connected
$$\text{ssc}(x) \stackrel{\text{def}}{=} \text{sc}(x) \wedge \text{sc}(ix)$$
(An m-individual is strongly self-connected if it is self-connected and its interior is self-connected.)

The restriction introduced in (6.25) guarantees that not only boundaries of parts are shared but also that the interior of an entity is self-connected. Therefore, the property of SSC rules out objects that merely touch each other, e.g., the spheres in Figure 6.12, as well as objects that are partially merged, e.g., Figure 6.13. However, the fact that an entity is strongly self-connected still does not ensure that such an entity is an integrated whole in the sense a theory of wholeness is supposed to capture. For instance, consider the entity a, i.e., the left half of the cuboid represented in Figure 6.14. Although a qualifies as a strongly self-connected object since any two parts that form its interior are connected, intuitively it cannot be characterized as a whole. Obviously, the reason is that there is yet another half of the cuboid and only the two together make up the entire individual. Consequently, SSC is insufficient for our purposes and an even stronger notion is required to capture what is perceived as an integral whole.
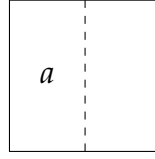
Figure 6.14: Strongly self-connected part

Describing wholes is about capturing an intuitive notion of unity. This means that a proper definition should accommodate both topological integrity and mereological exhaustivity. For this purpose, Casati and Varzi (1999) introduce the property of being MAXIMALLY STRONGLY SELF-CONNECTED (MSSC), as defined in (6.26).

(6.26) Maximally-strongly-self-connected
$\text{mssc}(x) \stackrel{\text{def}}{=} \text{ssc}(x) \wedge \forall y[\text{ssc}(y) \wedge \circ(y,x) \leftrightarrow y \sqsubseteq x]$
(An m-individual is maximally strongly self-connected if (i) every (interior) part of the individual is connected to (overlaps) the whole (strongly self-connected) and (ii) anything else which overlaps it and is strongly self-connected is once again part of it (maximality).)

More generally one could single out entities that are maximally strongly self-connected relative to a particular property. The relativized MSSC property can be characterized as in (6.27) which provides the final mereotopological definition of what a whole is.

(6.27) Maximally strongly self-connected relative to a property
$\text{mssc}(P)(x) \stackrel{\text{def}}{=} P(x) \wedge \text{ssc}(x) \wedge$
$\forall y[P(y) \wedge \text{ssc}(y) \wedge \circ(y,x) \leftrightarrow y \sqsubseteq x]$
(An m-individual is maximally strongly self-connected relative to a property if (i) every (interior) part of the individual is connected to (overlaps) the whole (strongly self-connected) and (ii) anything else which has the same property, is strongly self-connected, and overlaps it is once again part of it (maximality).)

If an entity satisfies MSSC, then it is the largest entity satisfying that property which is self-connected. For instance, if P is the property of

being a cuboid, then the definition in (6.27) will select the largest such entities among those that come in one piece. In particular, it will single out the whole cuboid in Figure 6.14 as opposed to, e.g., its left half *a* though *a* itself also satisfies the property of being a cuboid. That is because *a* is part of the cuboid whereas the whole cuboid is not part of any other strongly self-connected object.

Though the structure of the mereotopological framework of Casati and Varzi (1999) discussed above is relatively simple, the distinctions developed within it are fine-grained enough for the representation of spatial objects. Similar to standard mereology, mereotopology gives rise to algebraic structures isomorphic to Boolean algebras with their bottom element removed, however such lattice representations are further endowed with regions indicating which elements are associated with the connectedness relation c. For instance, consider Figure 6.15. Given a model with 3 entities, namely two halves of a cube *a* and *b* and a pyramid *c*, the semi-lattice represents all the possible sums generated by the set of corresponding m-individuals. The vertical axis illustrates the parthood relation with individual lines between particular nodes symbolizing $\sqsubseteq$, whereas the horizontal axis represents connectedness. The shaded area then covers those entities in the structure which are connected. In other words, the mereological component of the theory defines part-whole relations between *a*, *b*, and *c* and all the possible sums thereof, whereas the topological component singles out those parts that form self-connected wholes. In particular, while *a* in the model is part of $a \sqcup b$ (as well as of itself due to the axiom of reflexivity), it is not part of *b*. However, it is connected to *b* (as well as to $a \sqcup b$ due the axiom of integrity and to itself – again reflexivity).

Moreover, as discussed by Grimm (2012b) the subtle niceties between different types of connectedness, provide means to capture differences between distinct types of nominals such as object, substance, and aggregate nouns as well as eliminate old problems concerning cumulative singular count nouns such as *fence* or *twig* (see, e.g., Zucchi and White 2001, Rothstein 2010). Specifically, among multiple parts that have a property of being, say, a rock only the maximal (improper) part counts as a whole object. Hence, I conclude that compared to standard mereology mereotopology proves to be a superior theory of parts and wholes and its application in natural language semantics

213
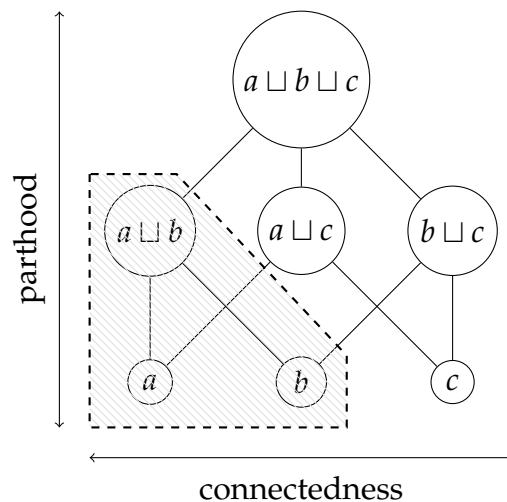
Figure 6.15: Parthood and connectedness (based on Grimm 2012b, p. 136)

should be considered advantageous. In the next section I will discuss several other kinds of connection.

6.2.2 Other types of connection

In the previous section, we have seen that given the primitive relation c it is possible to derive a notion that can capture what it means to be an integrated whole as opposed to an arbitrary sum, namely the mssc property. This, however, does not deplete the potential of mereotopology in modeling different types of objects and spatial configurations. Based on c, many other types of topological relations representing distinct varieties of connectedness may be defined. In this section, I will discuss some of them.

The first auxiliary mode of connection to be discussed is the property of FIRMLY CONNECTED (FC), as defined in (6.28).[11] This property holds between two entities when they overlap in a substantive sense. In other words, two m-individuals are firmly connected if their sum is

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11. I use the term following Varzi (2007), whereas Casati and Varzi (1999) and Grimm (2012a,b) talk about strong connection. My motivation is mainly to avoid potential confusion with the ssc property which plays a crucial role in defining FC but is definitely distinct from it.

214

strongly self-connected. Such a definition excludes cases of tangential overlap, i.e., configurations where entities only touch each other, since the ssc property implies that not only the boundaries but also the interiors of m-individuals are connected. With an additional restriction regarding the local scope of connectedness, the notion of FC seems to be suitable to capture what it means to be a substance as opposed to an integrated individual that we count as one (Grimm 2012a,b).

(6.28)   Firmly connected

$\text{FC}(x, y) \stackrel{\text{def}}{=} \exists w \exists z [w \sqsubseteq x \wedge z \sqsubseteq y \wedge \text{ssc}(w \sqcup z)]$
(Two m-individuals are firmly connected if a sum of their parts is strongly self-connected.)

In contrast to FC, the property EXTERNALLY CONNECTED (EC), as defined in (6.29), concerns entities that are merely tangentially connected, i.e., it is only their boundaries that are connected, whereas their interiors do not overlap. This notion enables us to model configurations of entities that are not merged but simply touch each other.

(6.29)   Externally connected

$\text{EC}(x, y) \stackrel{\text{def}}{=} \text{C}(x, y) \wedge \neg\text{C}(ix, iy)$
(Two m-individuals are externally connected if they are connected but it is not the case that their interiors are connected.)

Another topological notion it might be useful to derive is a one describing a configuration of objects touching each other in such a way that they form, say, a row. This variety of connectedness can be captured by the property BY-CONNECTED (BC), as provided by Varzi (2007). It holds between entities that are not connected to themselves but are associated by virtue of being connected to another mediating entity. The definition in (6.30) describes BC as a three-place relation establishing indirect connectedness between the outermost entities within a configuration.

(6.30)   By-connected

$\text{BC}(x, y, z) \stackrel{\text{def}}{=} \text{C}(x, z) \wedge \text{C}(z, y)$
(Three m-individuals x, y, and z are by-connected if x is connected to z and z is connected to y.)

215

A related notion is called MEDIATELY CONNECTED (MC). This is a binary relation that holds between entities that are not necessarily connected to each other, but are part of a by-connected configuration, e.g, a row of individuals. In other words, there is an object that mediates the connection between two such entities. The formal definition of BC is given in (6.31).

(6.31)   Mediately connected
          $\text{MC}(x, y) \overset{\text{def}}{=} \exists z[\text{BC}(x, y, z)]$
          (Two m-individuals are mediately connected if they are by-connected through a third m-individual.)

To illustrate the BC and MC relations, consider the configuration of two spheres a and c and a cube b, as depicted in Figure 6.16. The three entities are by-connected since a is externally connected to b and b is externally connected to c. At the same time, the spheres are mediately connected since both a and c are connected to the cube b, i.e., b in a way mediates the connection between a and c.
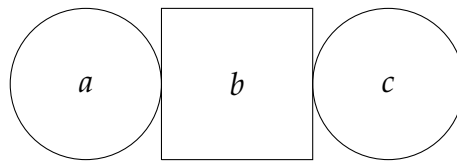
Figure 6.16: By-connected and mediately connected

As pointed out by Grimm (2012b), the notion of MC may prove useful in modeling natural language expressions denoting entities such as eyes or fingers. Such objects seem to be conceptualized in a particular way which is further reflected in grammar. In particular, in some languages they belong to a category of inherently plural or dual expressions.

Furthermore, the BC relation can be generalized to hold for any number of entities. This gives rise to the property TRANSITIVELY CON-NECTED (TC). As defined in (6.32) (cf. Grimm 2012a,b), it determines whether two individuals are connected through a series of mediating entities. For instance, it would hold of two opposite entities in a scenario similar to the one illustrated in Figure 6.16 with the exception

that between the spheres a and c there is not only the cube b, but also a pyramid d such that it touches both b an c.

(6.32) Transitively connected

$\text{TC}(x, y, P, C, Z) \stackrel{\text{def}}{=} \forall z \in Z[P(z) \land (x = z_1 \land y = z_n) \land Cz_1 z_2 \land Cz_2 z_3 \cdots \land Cz_{n-1} z_n]$
where $Z = \{z_1, z_2, \ldots z_n\}$
(Entities x and y are transitively connected relative to a property P, a connection relation C, and a set of entities Z, when all members of Z satisfy P and x and y are connected through the sequence of z_is in Z.)

Finally, the weakest variety of connectedness can be captured by the notion of PROXIMATELY CONNECTED (PC).[12] This relation concerns entities that are neither contiguous, nor do they touch each other, but rather they are 'very close'. The formula in (6.33) defines PC in terms of the distance function d which yields the distance between two entities it is applied to. For instance, if a sphere a is 1 meter away from a cube b, the distance function d would yield 1 meter. The value n is determined with respect to the predicate in question so that entities that satisfy it are conceptualized as being sufficiently near each other relative to the relevant property.

(6.33) Proximately connected (Grimm 2012b, p. 135; adapted)

$\text{PC}(x, y, P) \stackrel{\text{def}}{=} d(x, y) \leq n(P)$
(Two m-individuals are proximately connected if the distance between them is lesser than or equal to the value determined for a given predicate.)

The overview provided here does not by any means exhaust possible configurations of objects and different types of connectedness. Nonetheless, for our purposes the notions introduced in this section will be absolutely sufficient. The introduced distinctions allow us to distinguish between several types of entities. First, due to the MSSC relation it is possible to differentiate between arbitrary sums and inte-

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12. In order to refer to a similar concept, Casati and Varzi (1999) use the term quasi-connected. However, I will follow Grimm (2012a,b)'s terminology and formalism here.

grated wholes. On the other hand, the fc property might be useful in capturing the nature of substances, i.e., entities that come in multiple instances which overlap with each other. Furthermore, the tc relation enables us to single out entities consisting of connected objects arranged in configurations such as rows and piles, whereas pc provides means to describe entities involving multiple parts remaining in a proximate distance from each other such as clusters and swarms (cf. Henderson 2017).

6.3 Summary

In this chapter, I have presented a theory of parts and wholes called mereotopology. The described system involves a mereological core devised to capture the intuitive notion of parthood. This allows us to relate elements that are part of certain larger entities with those entities. However, what standard mereology cannot account for are certain relations between parts that result in different spatial configurations. This is where the topological component introduces the notion of connectedness. Based on this primitive concept more sophisticated properties can be derived. From the perspective of this study, the most prominent is the property of being a maximally strongly self-connected individual. This notion enables us to capture integrated wholes, i.e., objects that come in one piece. In the next chapter, I will demonstrate how the mereotopological framework introduced here can be used to model individuals in term of integrated wholes as well as continuous fragments of such wholes as integrated parts. Such treatment will allow us for a better understanding of countability and developing a novel account for subatomic quantification.

7 Mereotopological account for subatomic quantification

Given the mereotopological framework introduced in the previous chapter, let us now attempt to account for some of the relevant expressions employing quantification over parts such as partitives and multiplier phrases as discussed in Chapter 2, 3, and 4. The conceptual basis of the general mechanism of subatomic quantification outlined in Chapter 5 can now be spelled out formally with the notions of parthood and connectedness playing the central role in modeling the meaning of expressions that provide objects to be counted as well as of those that do the counting.

In this chapter, I will propose an analysis of different types of partitives and multiplier phrases. I will start with the general discussion of the implausibility of atomicity based approaches to subatomic quantification. Next, I will turn to my own proposal spelled out in in terms of the mereotopology supplemented with additional components such as measure functions, partitions, and the individuation operation. First, I will address the mass/count distinction by distinguishing between different types of nominals as well as give the meaning of the plural in the spirit of Grimm (2012b). Subsequently, I will turn to defining other semantic objects I assume in the structure of different types of partitives and multiplier phrases including the meanings of different types of partitive words as well as numerical expressions which I treat as complex constituents comprising a classifier component. Finally, I will propose a compositional analysis of the expressions in question by showing how the pieces fit together.

7.1   Doing without atoms

In standard theories of pluralities and countability the mass/count distinction is often formulated in terms of atomicity (e.g., Link 1983, Landman 1991, 2000, Chierchia 1998a, 2010). Though particular theories differ significantly with respect to the exact character of the alternation, the contrast between count and mass nouns usually boils down to the existence or lack of minimal building blocks in their de-

notation or, alternatively, to a distinct nature of those building blocks. The approach developed here rejects the view that what counts as one is best represented as an atomic entity. Instead, I build on the mereotopological notion of MSSC introduced in Section 6.2.1 to capture what can be an object of counting. There are three main motivations behind this decision. The first one is the linguistic relevance of topological relations holding between parts of a whole object as empirically attested in multiple constructions in various languages discussed in Chapter 2 and 3. The second stems from the general counting principles described in Section 5.2.2 which I believe constitute a deeply rooted part of human cognition and as such interact with that part of the language faculty that generates grammar of countability. Finally, the last reason for abandoning atomicity is simply that it does not seem helpful with respect to subatomic quantification. Before we move on to the proper proposal, let us briefly consider why that is.

Things that count as one are typically defined as atomic individuals, i.e., entities that have no proper parts, see (7.1) for the definition of ATOM and (7.2) for an optional mereological axiom of ATOMICITY that delivers atomic domains, i.e., domains consisting of atoms.

(7.1)   Atom
        ATOM$(x) \leftrightarrow \neg \exists y[y \sqsubset x]$
        (An atom is an entity which has no proper parts.)

(7.2)   Atomicity
        $\forall x \exists y[y \sqsubseteq x \wedge \neg \exists z[z \sqsubset y]]$
        (For any element, there is a part for which there does not exist a proper part.)

At first blush, such an approach to what it means to count as one seems counterintuitive since we know from our everyday life experience that things we count as one very often do have multiple parts. However, one should not confuse intuitive concepts with formal notions devised to represent the semantics of natural language expressions, e.g., the meaning of the English partitive word *part* with $\sqsubseteq$. Thus, my view is that there is nothing wrong in dissociating intuitive entities from their intuitive parts in terms of mereological modeling as long as it serves a certain purpose. For instance, Champollion (2010, 2017) assumes that the relation between, say, a teddy bear named Fuzzy Wuzzy and its

paw is not mereological parthood. In other words, despite the fact that Fuzzy Wuzzy consists of a number of material parts it is considered a mereological atom, i.e., an inseparable unit in the domain of entities. This might seem plausible since it corresponds to yet another intuition, namely that though we know that things have parts, we seem to ignore this fact when we count those things. However, a significant problem for such an account arises when we decide to quantify not over whole things but over parts of those things. Thus, I will argue for two claims. First, having a notion of atomicity is not enough for a full analysis of quantification of parts. Second, atomicity is actually not needed to analyze quantification over parts since it can be reduced to topological notions which are required independently.

To illustrate this, let us consider the following case. Assume we wanted to count parts of Fuzzy Wuzzy, e.g., its paws. Of course, natural language allows for that since we can say (7.3a) and it would be true if, say, Fuzzy Wuzzy's paws were brown whereas all of its other parts were of a different color. However, if referents of proper names are mereological atoms, there are no parts to be counted since the entity designated by the expression *Fuzzy Wuzzy* cannot be split. Without saying anything else, this is untenable if we want to account for (7.3a). Consequently, one could follow Link (1983) and propose to distinguish between a domain of entities and a domain of portions of matter over which a different parthood relation would be defined, i.e., the material parthood relation $\sqsubseteq_m$ as opposed to the individual parthood relation $\sqsubseteq_i$. Assuming the domains are related by a function from entities to portions of matter, one could argue that subatomic quantification is about triggering this mapping and operating on portions of matter corresponding to the mereological atom in the domain of entities. However, such a proposal raises questions. If the domain of portions of matter is atomless, which at first blush might seem intuitive, then how it is possible to count entities of such a domain? Since atomicity is a necessary condition for countability,[1] we need to ensure that at least some portions of matter are atomic. However, this is problematic.

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1. Notice that it is not a sufficient condition due to the existence of object mass nouns which are also assumed to have atomic reference yet are uncountable (see, e.g., Chierchia 1998a, Barner and Snedeker 2005).

221

(7.3)   English
       a.   Exactly two parts of Fuzzy Wuzzy are brown.
       b.   Exactly two parts of one part of Fuzzy Wuzzy are brown.

Let us imagine that for some reason we wanted to count parts of a part of an individual. Again, natural language does allow for that. The sentence in (7.3b) is not a common thing to say but it is definitely interpretable and it would be true if, say, exactly two digits of Fuzzy Wuzzy's left paw were brown and the remaining part of the paw were not. Notice that truth-conditionally (7.3b) is not equivalent to (7.3a). Consider, for instance, a scenario where the brown parts of Fuzzy Wuzzy included only its head, right leg, and two digits of its left paw. In such a scenario, (7.3b) would be true but (7.3a) would not. Now, if countable portions of matter are atomic, then the same problem as discussed with respect to atoms in the domain of entities arises. Specifically, if a portion of matter corresponding to Fuzzy Wuzzy's left paw is modeled as having no proper parts, then how can we quantify over its parts? To account for that we would be forced to postulate another domain and relate it with the domain in which the material part corresponding to Fuzzy Wuzzy's paw is an atom. However, this seems weird and even if we did it, the same problem would arise again if we wanted to count parts of a digit of Fuzzy Wuzzy's paw. Consequently, distinguishing between just two distinct parthood relations is not enough. In fact, since there are multiple possible divisions of matter, we might end up establishing numerous or even potentially infinitely many domains in order to be able to define parts as atomic objects in a particular domain. This of course is far from desirable and it seems to me that the discussed example strongly suggests that sorting domains does not offer a tenable solution to the issues concerning subatomic quantification and atomicity.

Since sorted domains proved unsatisfactory for our purpose, one might think of another idea. As discussed in Section 3.3, one of very few attempts to propose a solution to the problem of countability of portions of matter (as opposed to uncountability of matter) preserving atomicity was developed in Chierchia (2010).[2] In this system, an

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2.   Another account worth mentioning was proposed by Landman (2016) but this theory also abandons atomicity.

expression such as English *part* is modeled as a variable over partitions of an entity, i.e., an expression of type $\langle e, \langle e, t \rangle \rangle$ that selects an entity and returns a set of relative atoms that are 'spatio-temporally included' in that entity. Since in principle such partitioning can be applied recursively, we can obtain parts of parts of entities etc. This seems to be a significant improvement compared to an attempt to account for subatomic quantification in terms of sorted domains and distinct parthood relations. However, there are still several issues with this approach. First, as already discussed an analysis along the lines of Chierchia's proposal fails to account for the fact that parts can be either continuous or discontinuous and, crucially, natural language turns out to be sensitive to this distinction. Second, the notion 'spatio-temporally included' introduced above is very loose. Compared to the fine-grained concepts discussed in the previous chapter, it appears to be very unsatisfactory. In general, I agree with Chierchia's 'mereotopological' intuition, however its formulation leaves a lot to be desired. For instance, imagine that Fuzzy Wuzzy was left in a car. Does it mean that for linguistic purposes it is now part of that car? Or if an elephant accidentally swallowed Fuzzy Wuzzy in a zoo only to throw it up after some time, would the poor teddy bear be considered part of that elephant in the period between swallowing and throwing up? Intuitively, it does not seem so. Finally, from my point of view postulating partitions of entities is an attempt to compensate deficits of atomicity rather than a genuine development. On the one hand, it seems like circumventing the ban on having proper parts by building blocks of denotations of count nouns. On the other, though it seems to point in a desirable direction, it somewhat stops halfway. As we will see, given the potential of a mereotopological account the vague concept of a partition of entities does not make a tenable alternative. In particular, if maintaining one unified parthood relation proved successful, then partitions of entities could be reformulated in terms of such unified $\sqsubseteq$, as we will see in Section 7.5.5. Though I will return to partitioning as a useful tool in modeling partitive words denoting continuous parts, I do not find Chierchia's proposal an overall satisfactory analysis.

I believe that at this point it is plausible to conclude that in order to account for subatomic quantification it is desirable to find a substitute for atomicity. One might think that a plausible alternative would be to adopt frameworks that model model building blocks of denotations

in terms of natural or object units as proposed by Krifka (1989, 1995). At first blush, this might seem attractive because what counts as one, i.e., an entity to which we can assign the number 1, is not defined in terms of not having proper parts. Thus, in principle accounting for quantification over parts of such an object would not have to face the issues described above. Nonetheless, such an approach would run into a nagging conceptual problem. Intuitively, it is quite straightforward what a natural unit of, say, entities that have the property of being an apple is or, alternatively, what kind of concept they realize. But what is a natural unit of a part? Or what kind of concept a part instantiates? Given the systems mentioned above, it those notions would have to refer in some way to the denotations of partitive words. But it seems to me that this is a very strange way of thinking about parts. However, even if one accepts the existence of natural units or concepts of this sort, other questions arise. Since there are numerous ways how to divide portions of matter making up an entity, does it mean there are numerous natural units or concepts corresponding to such portions, e.g., a natural unit/concept of a tiny part, half, great part, and not-that-small-but-not-that-big part? Can both continuous and discontinuous parts be considered as natural units? Are they realizations of the same concept or two different concepts? Is there a difference between natural units or concepts corresponding to parts of a singularity and those corresponding to parts of a plurality? If yes, then how do quantificational operations know that one is to be selected over the other? If no, then how to distinguish between the semantic subtleties the meaning of partitive words are associated with? It feels like all of those are quite odd questions but they would need to be addressed seriously in order to account for the data presented in Chapter 3. Therefore, it appears to me that trading atomicity for natural or object units does not make a satisfactory alternative with respect to subatomic quantification.

Given the problems considered above, I conclude that an approach based on mereotopology proves more advantageous for accounting for subatomic quantification. Not only it provides means to better capture what it means to be a whole but also as we will see it turns out to be extremely useful in modeling those entities that can be counted. In the following sections, I will introduce a minimal set of tools necessary to model two different types of constructions concerning quantifi-

cation over parts, namely multiplier phrases and partitives. I treat this repertoire of means alongside the standard principle of function application as the first attempt to develop a compositional analysis of the syntactic structures in question that could shed new light on the somewhat neglected phenomenon of subatomic quantification as well as on countability in general. At the same time I believe the real journey begins afterwards.

7.2 Common nouns

I will start my proposal with a brief discussion of the semantics for different types of concrete nouns, i.e., expressions denoting sets of individuals such as apples and teddy bears as well as scattered entities such as juice and rice. Though everything I have to say about the semantics of nouns extends naturally to NPs, for the sake of brevity I will focus only on nouns. In general, I assume that all common nouns of the sort discussed in this study as well as nominals derived from them, e.g., by adjectival modification, are standard $\langle e, t \rangle$ predicates, i.e., functions from entities to truth values. Among such expressions there some that are countable and some that are not. Though countability is, of course, a grammatical category, I assume that the mass/count distinction correlates with cognitive factors determining how human beings conceptualize things in the world, as discussed in Chapter 5. In particular, I take the counting principles of non-overlap, integrity, and maximality as the benchmark for deciding what is countable and what is not. In other words, only nouns whose referents satisfy all three criteria are countable. In the next section, I will propose how the meaning of such predicates can be captured.

7.2.1 Count nouns

Building on Grimm (2012b), I model the distinction between count and mass nouns in terms of mereotopological distinctions developed in the previous chapter. In particular, I propose that defining countable predicates is about ensuring that their extensions include only entities conceptualized as objects, i.e., discrete integrated wholes that are disjoint from each other. Intuitively, the core distinction between

225

count nouns on the one hand and mass nouns including substance terms, granulars, and object mass nouns on the other can be reduced to the fact that the first denote only individuated objects, whereas the latter either do not have integrated wholes in their extension at all, or they do but in addition they also refer to other types of entities. For instance, the noun *apple* simply denotes a set of individuals, i.e., distinct apples. On the other hand, the referents of *juice* do not have properties integrated wholes have, i.e., they are not constrained as bounded non-overlapping discrete objects. The case of granular terms such as *rice* is somewhat more complicated. Arguable, those predicates do denote integrated wholes since one can point at a single grain of rice and truthfully call it *rice*. Intuitively, grains of rice have similar properties as apples, i.e., they are disjoint, integrated, and mereologically maximal, however *rice* does not refer exclusively to individual grains of rice. In addition, it can also denote arbitrary sums as well as clusters of rice which lack those properties. Similar, object mass nouns such as *footwear* have both integrated wholes, i.e., individual shoes, as well as groups of such individuals, i.e., pluralities of shoes, in their extensions. Thus, though there are important differences between particular types of mass nouns, they all contrast with count nouns with respect to what kinds of entities fall into a denoted set.

Since mereotopology provides powerful means to distinguish formally between objects conceptualized as integrated wholes and other types of entities, I will use the notion of mssc defined in (6.26) in order to capture the main intuition behind the contrast discussed above. Specifically, I will employ the variant of mssc relativized to a property as provided in (6.27) and repeated here as (7.4). As discussed in Section 6.2.1, this allows us to distinguish between integrated objects such as individual apples and scattered entities like juice on the one hand and arbitrary sums such as pluralities of apples on the other.

(7.4)   Maximally strongly self-connected relative to a property

$$\text{mssc}(P)(x) \stackrel{\text{def}}{=} P(x) \wedge \text{ssc}(x) \wedge \forall y[P(y) \wedge \text{ssc}(y) \wedge \circ(y, x) \leftrightarrow y \sqsubseteq x]$$

(An m-individual is maximally strongly self-connected relative to a property if (i) every (interior) part of the individual is connected to (overlaps) the whole (strongly self-connected) and (ii) anything else which has the same property, is strongly

226

self-connected, and overlaps it is once again part of it (maximality).)

Given the mereotopological notion of mssc, we can now characterize the subset of predicates that refer exclusively to integrated wholes. In the following text, I use the symbol P_{MSSC} to refer to such predicates. As stated in (7.5), there is nothing in the denotation of P_{MSSC} that is not an mssc entity.[3] This means that, e.g., *apple*, is P_{MSSC}, whereas *juice*, *rice*, and *furniture* are not since either they do not refer to mssc individuals at all or they also denote entities that are not mssc.

(7.5) Predicate of mssc individuals
$P_{\text{MSSC}} \leftrightarrow \forall x[P(x) \rightarrow \text{MSSC}(P)(x)]$
(Any m-individual of which a P_{MSSC} predicate is true is an mssc m-individual relative to the relevant property.)

Let us now consider the semantics of the English count noun *apple* provided in (7.6). What it says is that *apple* is an expression of type $\langle e, t \rangle$ which yields the truth value True for entities that are mssc with respect to the property APPLE. Given the definition in (7.4), MSSC(APPLE)(x) entails APPLE(x) which means that the discussed predicate denotes a set of apples that are integrated wholes. Therefore, the proposed semantics captures the intuition that referents of count nouns constitute discrete objects without any reference to the notion of atomicity.

(7.6) Count noun
$[\![\text{apple}]\!] = \lambda x[\text{MSSC}(\text{APPLE})(x)]$

Since the main topic of this study concerns subatomic quantification, I will refrain from discussing the exact semantics for different types of mass nouns. Nonetheless, the meaning of all of the expressions mentioned above can be captured in terms of mereotopological distinctions. In particular, substance terms such as *juice* can be modeled in terms of m-individuals having a certain property that are firmly connected to distinct m-individuals with the same property, formally involving the FC relation introduced in (6.28) (see Grimm 2012b, pp. 140–142). On the other hand, granular mass nouns such as *rice* can be

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3. For convenience, I use the term 'mssc entity/individual' to refer to objects that are maximally strongly-self connected relative to a relevant property.

accounted for by treating them as aggregates, i.e., expressions that can denote MSSC individuals, their sums as well as clusters thereof, i.e., configurations of MSSC entities transitively connected via the TC relation defined in (6.32) (see Grimm 2012b, pp. 142–148).[4] Finally, object mass nouns such as *footwear* can be thought of as denoting both MSSC entities and sums thereof though definitely more work needs to be done with respect to this category.

The mereological approach to nominal semantics allows us to distinguish countable nouns without postulating atomicity. As we will see, this will prove crucial in accounting for subatomic quantification. However, before I move to the discussion of the machinery behind measuring and counting, the issue of number morphology on nouns needs to be addressed. In the next section, I will take up the meaning of the plural.

7.2.2   Pluralization

Following Grimm (2012b), I assume that number morphology is sensitive to the mereotopological structure of referents of the basic, i.e., unmarked, form of a noun.[5] Specifically, as we have seen in the previous section in a language such as English only a subset of nouns can be characterized as denoting single objects conceptualized as MSSC individuals. For the purpose of this study, I assume that in general this property corresponds to countability and thus the compatibility of such nouns with the plural morpheme.[6] In particular, the plural marker attaches only to predicates that have exclusively MSSC entities

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4.   Clusters can be further relativized to different types of connection such as external connection, see ET in (6.29), and proximate connection, see PC in (6.33) (Grimm 2012b).

5.   Alternatively, one could argue that this kind of information is already encoded in the semantics of the stem (pace, e.g., Pelletier and Schubert 1989, Borer 2005a). However, for the sake of brevity I will not explore this hypothesis here.

6.   This is, of course, a simplification. As mentioned before, for the sake of simplicity I ignore here collective nouns as well as count abstract nouns. However, I assume that in principle it is possible to extend the core mereotopological concepts to develop a framework based on notions derived from or inspired by MSSC that would account for phrases such as *two ideas* and *three committees* (see also Grimm 2014).

in their extensions.[7] Having this in mind, the plural can be analyzed as an operation which selects a set of MSSC individuals and applies the strict pluralization operation $^+$, see the semantics in (7.8). The definition of $^+$ provided in (7.9) employs the classical * operator introduced by Link (1983) which is defined as a closure under sum formation, see (7.7). Specifically, what $^+$ does is that it selects a predicate, applies * to it and then removes all the MSSC individuals from the pluralized set. The selectional requirement of the plural marker is introduced by what I refer to as the INDIVIDUATION PRESUPPOSITION represented by P_{MSSC} introduced after a dot following immediately the relevant λ.[8] The individuation presupposition requires an argument of the λ operator to be a predicate denoting only MSSC individuals as defined in (7.5).

(7.7)   Algebraic closure (cf. Link 1983)
$$*P \stackrel{\text{def}}{=} \{x | \exists P' \subseteq P[x = \bigsqcup P']\}$$

(7.8)   Plural
$$[\![\text{PL}]\!] = \lambda P \, . \, P_{\text{MSSC}}[^+P]$$

(7.9)   Strict pluralization
$$^+P \stackrel{\text{def}}{=} *P - P$$

As indicated by (7.8), I adopt here the so-called exclusive analysis of the plural. Within this view, plurals are assumed to refer only within the domain of sums, i.e., singularities are excluded from the denotation, and thus the plural is treated as designating *more than one* (cf. Hoeksema 1983, Chierchia 1998a,b, Grimm 2013; see also Wągiel 2017).[9] Hence, what the plural does is that it selects a set including

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7.   Without any additional assumptions, this approach fails to account for inherently plural mass nouns such as *leftovers* (Acquaviva 2008) as well as pluralized mass nouns as observed, e.g., in Greek (Tsoulas 2009). However, since the general topic of the meaning of the plural lies beyond the scope of this study, I leave it for future research.

8.   For notational conventions used in this study see Section 1.3.2.

9.   This decision is mainly motivated by independent reasons and as far as I can see for the most part nothing hinges on this from the perspective of the phenomena I attempt to account for here. However, the exclusive analysis will prove advantageous in the context of set partitives. Notice also that many arguments have been proposed in favor of the inclusive view on which singularities are included in the meaning of

only MSSC individuals and yields a new set consisting of all the sums formed by joining the members of the input set. For instance, let us consider the semantics of the bare plural *apples*, see (7.10). In this case, the plural morpheme *-s* combines successfully with the noun since the individuation presupposition is satisfied, i.e., *apple* is a predicate of MSSC entities. As a result, the set denoted by *apple* is first closed under sum and then all the MSSC individuals are removed. Hence, we obtain a new predicate which is true only of entities that are pluralities of apples.

(7.10)   Plural NP
$$[\![apples]\!] = [\![PL]\!]([\![apple]\!]) = \lambda x \left[^+ \left(\lambda y [\textsc{mssc}(\textsc{apple})(y)] \right)(x) \right]$$

To see how this works, consider the following example. Let us assume a model with three individual apples *a*, *b*, and *c*. Given such a model, the common noun *apple* denotes the set of singular apples as in (7.11a). On the other hand, its plural counterpart *apples* in (7.11b) denotes the set comprising all the sums obtained from the singular individuals, i.e., pluralities of apples. In both cases, the semantic type of the expression in question is $\langle e, t \rangle$.

(7.11)   Plural NP
   a. $[\![apple]\!] = \{a, b, c\}$
   b. $[\![apples]\!] = \{a \sqcup b, a \sqcup c, b \sqcup c, a \sqcup b \sqcup c\}$

An important advantage of such treatment of the plural is that it explains why the plural marker does not occur on mass nouns without triggering any additional semantic effect such as a portion interpretation. Or, alternatively, it is not the plural marking that triggers that effect but rather the application of the Universal Packager is a prerequisite for the plural morpheme to be able to attach to a mass noun.[10]

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the plural mainly based on the data concerning downward-entailing environments (cf. Link 1983, Schwarzschild 1991, Landman 2000, Sauerland et al. 2005, Spector 2007, Zweig 2009). However, see Grimm (2013) for an analysis of such contexts maintaining the exclusive view.

10. I restrain here, from the discussion of the role of the Universal Sorter since it is unclear whether kind-level entities can be considered as subject to topological relations. Potentially, subkind readings constitute a problem for the described approach. For sure, more research is required on this topic.

Since packaging restricts the extension of a mass term in such a way that the resulting portion has properties similar to integrated objects, it is plausible to postulate that it does so by applying an MSSC constraint or at least some related restriction. In any case, non-coerced mass nouns fail to satisfy the individuation presupposition simply because either they do not refer to MSSC individuals at all as in the case of substance terms, or though their denotations include MSSC individuals, they also include other entities such as sums (object mass nouns) and clusters (granulars).

Apart from the individuation presupposition, the approach to the meaning of the plural provided here is essentially one of the two standard analyses. However, there is an important comment to be made concerning the morpho-syntax/semantics interface. Namely, I assume that not all morphological plurals get the interpretation in (7.8). Leaving aside pluralia tantum which would require much unrelated consideration, I posit that only bare plurals are semantic plurals, i.e., expressions on which the plural marker is interpreted. To foreshadow, I will claim that plural morphology in quantificational constructions such as numeral phrases in languages such as English makes by contrast no semantic contribution and is merely triggered syntactically by agreement (cf., e.g., Krifka 1989, 2007).

7.2.3 Consequences

Adopting the mereological account for nominal semantics has some interesting consequences. First, as we have seen in Section 7.2.1 the notion of MSSC allows us to model individuals denoted by singular count nouns without reference to atomicity. Therefore, there is no need to postulate a special category of entities that have no proper parts. To the contrary, unlike atoms MSSC objects are viewed as certain configurations of parts, specifically configurations forming integrated wholes. This is a radically different approach from purely mereological theories. In my opinion, its great advantage is that it succeeds in capturing two ontological intuitions concerning individuals at the same time. As we have seen in Chapter 5, there are good reasons to believe that cognitive structures underlying conceptualization of solid objects differ from those corresponding to scattered entities. On the other hand, human beings are able to perceive wholes simultaneously

as discrete units and as configurations of parts. Unlike atomicity based theories, a mereotopological account enables us to model natural language expressions in such a way that the first cognitive aspect can be captured without sacrificing the latter. From my point of view, it is a great achievement.

The second favorable consequence of mereotopology is that it provides a very simple and intuitive way how to distinguish between integrated sums of parts, i.e., singular individuals, and arbitrary sums, i.e., plural entities. The distinction simply boils down to whether topological notions are involved in the part-whole structure of an entity or not. As MSSC individuals, referents of singular count nouns consist of connected elements, whereas pluralities encode no topological relations holding between their parts. Intuitively, the sum of two apples a and b remains the same entity $a \sqcup b$ irrespective of whether a and b touch each other, stay in proximity, or are situated in distant locations. On the other hand, a would cease to exist if its halves were separated from each other by some contextually significant distance.[11] In other words, while plural entities can be modeled in purely mereological terms, singular individuals require an additional topological layer. It seems that such a distinction accounts for an intuition that contrary to what standard mereology posits singularities and pluralities are very different types of creatures. However, this approach does not exclude yet another ontological possibility, namely the existence of sums of solid entities that are topologically arranged in a particular way. This fact is of considerable significance since there is linguistic evidence indicating that introducing such type of entities might be necessary for the meaning of certain expressions in natural language. As we have seen in Section 2.3, referents of Italian irregular plurals make a good candidate for such a class. [12]

Another welcome result is that the fact that the theory adopted here allows to distinguish between different types of entities in terms

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11. Often, when parts remain in proximity it is still possible to retrieve the original part-whole structure from the context. Though the constraints on such a 'reconstruction' operation are potentially an interesting topic in cognitive science, I refrain here from any speculations on this issue and focus only on clear cases.

12. Another kind of expressions that are arguably of this sort are certain types of Slavic derived collectives (see Dočekal and Wągiel 2018, Grimm and Dočekal to appear).

of the distinction between mereotopological and purely mereological configurations allows us to reduce the number of domains. In particular, there is no need to distinguish between the domain of entities and the domain of portions of matter as postulated by Link (1983). Since referents of count nouns are not considered mereological atoms, i.e., entities without proper parts, but rather mereotopologically constrained sums of parts they have additional properties compared to arbitrary collections of portions of matter. Those properties are formulated in terms of connection, specifically mssc. Therefore, it is sufficient to assume only one domain of concrete things, namely the domain of entities and different types of objects populating this domain can be distinguished in terms of different types of either mereological or mereotopological part-whole structures.

I believe that at this point it is useful to readdress the classical paradox in (7.12) which led Link to distinguish between the domain of entities and the domain of portions of matter. Assume that (7.12a) points at a ring which was recently forged from some old Egyptian gold. If the ring is nothing more than a sum of portions of gold, it is surprising that (7.12b) is not contradictory. To the contrary, in the described scenario it is true. In other words, the problem concerns the fact that an object appears to have different properties than the sum of its parts, but in mereological terms the two are identical (see also Rothstein 2010, 2017 for discussion).

(7.12)   English (Link 1983; adapted)
     a.   This is a gold ring.
     b.   The ring is new but the gold is old.

Mereotopology offers a new explanation for the paradox. What is crucial here is that the ring is not just the sum of portions of gold it is made of. Rather, to be perceived as a ring an entity needs to remain in a particular spatial configuration, i.e., its parts need to be related by a topological relation. Therefore, the reason why (7.12b) is not contradictory is that it says that while the parts are old, the configuration is new. For instance, this intuition could be captured by attributing different topological properties to the definite descriptions

the ring and *the gold* even at a point in time at which of (all of) the gold.[13]

To sum up, the view developed here offers a radically different perspective on nominal semantics compared to standard approaches which utilize the notion of atomicity. Given the above discussion, I conclude that it turns out to be advantageous both from the conceptual and practical point of view because it better captures intuitions concerning the nature of individuals as well as allows for simpler models with less domains. Soon we will see that since it is free from the deficits of atomicity based approaches discussed in Section 7.1, it also proves significantly favorable in modeling subatomic quantification. In the following section, I will make a first step towards developing an analysis along those lines. In particular, I will introduce the theoretical background for a unified analysis of numerical expressions.

7.3   Measure functions

Following Krifka (1989, 1995), I model quantification over both parts and wholes in numeral and measure constructions in terms of extensive measure functions, i.e., operations that directly relate entities to numbers. The core mechanics behind a measure function is that when applied to a plurality of individuals or quantity of substance it maps it onto a real number corresponding to the number of individuals or units making up the plurality or quantity in question. Such operations need to satisfy what we expect from counting, i.e., they need to be additive. The definition in (7.13) ensures that no entity will be counted twice.

(7.13)   Additive measure function
μ is an additive measure function iff it satisfies the following requirement
$$\forall x \forall y \left[\neg x \circ y \to \left[\mu(x \sqcup y) = \mu(x) + \mu(y) \right] \right]$$
(The measure of a sum consisting of non-overlapping parts equals the arithmetic sum of the measures of parts.)

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13. Of course, it would be a topic for another study to work out what these properties are.

234

Assuming supplementation for the join operation as defined in (6.6) in Section 6.1 and repeated here in (7.14) alongside the Archimedean property of measure functions, see (7.15), it can be demonstrated that an additive Archimedean measure function is monotonic, see (7.16) (cf. Schwarzschild 2002).

(7.14)   Supplementation
$\forall x \forall y[x \sqsubset y \rightarrow \exists z[z \sqsubseteq y \wedge \neg(z \circ x)]]$
(Whenever a thing has a proper part, it has more than one.)

(7.15)   Archimedean property
$\forall x \forall y[\mu(x) > 0 \wedge y \sqsubseteq x \rightarrow \mu(y) > 0]$
(If a measure of a thing is greater than zero, then a measure of its part is also greater than zero.)

(7.16)   Monotonicity
$\forall x \forall y[x \sqsubset y \rightarrow \mu(x) < \mu(y)]$
(A measure of a whole is greater than a measure of its proper part.)

Examples of extensive additive measure functions include LITER, METER, and CALORIE, whereas measure functions such as DEGREE-CELSIUS and CARAT are non-additive.[14] For instance, for an entity x the measure function LITER yields a number of liters corresponding to the volume of that entity. In other words, it measures the quantity of space occupied by its argument in units of liters. Similar, METER and CALORIE return the number of meters and calories of a particular entity, respectively.

7.3.1   Counting operation

With this machinery in place, we can now start developing an account for counting. Let us begin with an operation to which I will refer as $\mu_{\#}$. Its formal definition is provided in (7.17).[15] In Section 5.2.2, I have demonstrated that counting and measuring differ in that the first is topology-sensitive, whereas the latter is not. Despite this fact, for

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14. This distinction corresponds to the monotonicity/non-monotonicity distinction in Schwarzschild (2002). In the following part of the chapter, I will focus only on additive/monotonic measure functions.
15. The symbol # in the subscript indicates quantification in terms of number of elements.

convenience I will talk about measure functions when referring to both operations yielding a cardinality of objects as well as operations giving a measure in terms of, e.g., volume. Given the definitions introduced above, the counting operation $\mu_\#$ defined in (7.17) is a kind of measure function. Nevertheless, it is crucial to emphasize that it is a special kind of measure function. Similar to other additive measure functions, it cares about non-overlap, but in addition it is also sensitive to the mereotopological structure of entities it applies to. In particular, as specified in (7.17) it counts exclusively mssc individuals.[16] This fact makes it distinctively different from measure functions such as LITER which ignore whether their arguments refer integrated entities that come in one piece.

(7.17)   Measure function $\mu_\#$
          $\mu_\#$ is an additive measure function standardized by the following requirement
          $\forall x[\mu_\#(x) = 1 \text{ iff } \text{mssc}(x)]$

To see the measure function $\mu_\#$ at work, let us consider the example provided in (7.18). Assume that there are two distinct mssc individuals a and b, i.e., $a \neq b$. In such a case, after we apply $\mu_\#$ to the sum of a and b, the returned value will be 2 which of course is a correct result.

(7.18)   Counting via $\mu_\#$
          $\text{mssc}(a) \wedge \text{mssc}(b) \wedge \neg(a \circ b) \rightarrow \mu_\#(a \sqcup b) = 2$
          (If both a and b are mssc and they do not overlap, i.e., they are not identical, then the measure function $\mu_\#$ applied to the sum of a and b yields the number 2.)

The measure function $\mu_\#$ seems to do what we would expect an operation intended to capture the core intuitions concerning counting to do. However, there is yet another improvement to be implemented. As we have discussed in Section 6.2.1, it is desirable to relativize mssc to a particular property in order to avoid ending up with the single

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16. In the original theory of Krifka (1989) the equivalent of $\mu_\#$ measures in terms of natural units, hence the NU operation. On the other hand, Krifka (1995) introduces the OU and KU operations (for 'object unit' and 'kind unit', respectively), whereas in newer versions of the theory (e.g., Krifka 2007) the AC measure function (for 'atomic count') is defined to count atoms.

integrated whole corresponding to the entire material universe. There-
fore, a proper device intended to account for counting should also
be relativized to a property so that it does not count absolutely MSSC
entities but rather objects that are MSSC with respect to a particular
characteristic, e.g., apples. This is can be ensured by introducing a
new operation to which I will refer as #. As can be seen in the defini-
tion in (7.19), it is similar to $\mu_{\#}$ in that the notion of MSSC plays here a
crucial role. However, # differs in that it takes a property P and yields
a measure function that returns a number of MSSC individuals relative
to P. For instance, assume there are two distinct apples a and b. Then,
can yield a measure function that will count entities that are MSSC
with respect to the property of being an apple. As a result, we obtain
the number 2.

(7.19)   Measure function #(P)
         #(P) is an additive measure function standardized by the
         following requirement
         $\forall P \forall x [\#(P)(x) = 1 \text{ iff } \text{MSSC}(P)(x)]$

Before we move on to introducing another measure function that
will prove useful for modeling multiplier phrases, let us discuss the
mechanism of contextual conditioning which will allow us to account
for proportional partitive words.

7.3.2   Contextual conditioning

The evidence presented in Chapter 2 shows that cross-linguistically
partitive words can appear both in entity and mass partitives on the
one hand and set partitives on the other. In general, in the first two
cases the partitive word quantifies over portions of matter constituting
either a particular individual or a quantity of substance, whereas in
the third situation it quantifies over integrated wholes making up
a plurality. To account for this double compatibility a procedure is
required which will determine what is to be measured or counted
when. For this purpose, I build on Bale and Barner (2009)'s proposal
who assume a generalized context-dependent measure function μ.
Such an approach posits a mechanism of contextual conditioning
along the lines defined in (7.20). The core of the idea is that particular

measure functions are ordered and depending on a context those that are ranked higher in the series are favored over those that are ranked lower.

(7.20)   Contextual conditioning
          μ is interpreted as one of the measure functions m_z in the series $\langle m_1, m_2, m_3 \ldots m_n \rangle$ such that the argument for μ is in the range of m_z; furthermore, contextually m_z is preferred to m_y if $z < y$

Bale and Barner use the mechanism in (7.20) to account for various meanings of *more* observed in comparative constructions such as *X has more NP than Y*. Specifically, depending on the NP comparison may be specified in terms of cardinality, volume, intensity etc. For the purpose of this study, I will make use of the very same mechanism for a procedure determining that partitive words can quantify over portions of matter in entity and mass partitives and over individuals in set partitives. In particular, I propose a partial ordering of measure functions as provided in (7.21). The first measure function in the series, i.e., the one ranked highest, is the measure function $\#(P)$ which is devised to count integrated wholes. I remain agnostic with respect to the exact position in the ranking other measure functions occupy, but for our purposes establishing the ordering between $\#(P)$ and measure functions measuring in terms of volume is certainly sufficient.

(7.21)   Partial ordering of measure functions
          $m_1 = \#(P) < m_n \in \text{VOLUME}$
          where VOLUME is a set of measure functions measuring entities in terms of volume including M^3, LITER, PINT etc.

To foreshadow, the contextually conditioned generalized measure function μ can cover the meanings of partitive words in entity and mass partitives on the one hand and set partitives on the other. In particular, μ is interpreted as a measure function quantifying over units of volume when the partitive word combines with a DP involving a singular count noun which refers to an individual as well as with a DP with a mass term referring to a quantity of substance. On the other hand, when the partitive word c-commands a DP comprising a regular plural noun, μ is interpreted an operation counting individuated integrated wholes.

238

This is because whenever the context allows for it, $\#(P)$ is preferred over the functions from the set VOLUME, e.g., CM^3. An additional positive consequence of the adopted mechanism of contextual conditioning is that it allows for quantification over individuals in partitives with object mass nouns in the embedded DP (cf. Barner and Snedeker 2005, Bale and Barner 2009).

7.3.3 Counting essential parts

The final component of the analysis related to measure functions has to do with modeling multipliers. As we have seen in Chapter 4, multipliers quantify over cognitively salient parts of individuals. In order to capture this intuition, I propose an additional operation ⊞.[17] This operation takes a property and yields an additive measure function ⊞(P) which counts parts of a whole. The formal definition of ⊞(P) is provided in (7.22). Notice that ⊞(P) quantifies only over parts of MSSC individuals as ensured by the first conjunct of the requirement. Furthermore, the second conjunct imposes that a part ⊞(P) maps onto a number needs to be essential for the whole. The parthood relation captures the intuition that only entities within the part-whole structure of an object are subject to quantification and the the use of improper parthood, i.e., $\sqsubseteq$, rather than $\sqsubset$ is to account for cases of homogeneous entities without any distinguishable salient parts such as standardized portions of coffee or martini. In particular, one might want to talk about a single portion as opposed to a double portion rather than to unrelated two portions since there is a clear semantic contrast between *single coffee* ~ *double coffee* ~ *two coffees*.

(7.22)   Measure function ⊞(P)
⊞(P) is an additive measure function standardized by the following requirement
$\forall P \forall x [⊞(P)(x) = 1$ iff MSSC$(P)(x)$
$\wedge \exists y [y \sqsubseteq x \wedge$ ESSENTIAL$(P)(y) \wedge \#(y) = 1]]$

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17. Here is a mnemonic. The ⊞ symbol is a square constituted by four smaller squares each of which is a part of the whole with comparable properties. Hence, ⊞ represents a type of object multiplier phrases usually refer to.

As witnessed by comparing (7.19) and (7.22), the main difference between #(P) and ⊞(P) lies in what type of objects they map onto numbers. The first simply associates any MSSC object with a number, whereas the latter yields 1 only for what I refer to as an essential part of an entity, i.e., a cognitively salient element. The notion ESSENTIAL in (7.22) is relativized to a property and can be defined as in (7.23). I keep the definition somewhat vague on purpose since what is considered essential for an entity to be perceived as having a certain property is often not entirely clear and may differ with respect to a particular context. For instance, as we have discussed in Chapter 4 multiplier phrases such as *double garage* can denote objects of a different internal structure as long as they can hold two vehicles. Nevertheless, one important feature of an essential part is that it is an integrated and recognizable entity within a whole that can be easily individuated against other parts.

(7.23)   Essential parts
ESSENTIAL(P) is true of an MSSC part of an entity in the extension of P that is essential for that entity to be considered as having a property P

An example of an essential part would be a patty in a hamburger. Intuitively, it makes the most important element of the whole sandwich. Similar, out of the blue it seems that the crucial part of a bed is an area covered with a mattress on which a person can sleep. Finally, in a natural context the most salient piece of, say, a shotgun is its barrel since it guarantees functionality as well as constitutes a significant portion of the weapon. However, notice that the broad definition in (7.23) also covers self-sufficient parts, i.e., parts that have a property comparable, i.e., very similar or identical, to the property of a whole. I argue that a self-sufficient part can be considered an extreme case of an essential part. If for some reason it is considered that what is essential cannot be reduced to one particular element, then ESSENTIAL(P) yields an MSSC part that resembles the object it is a part of. For instance, let us once again consider the case of *double crown* discussed in Chapter 4. Although there can be multiple cognitively salient parts within the make-up of a crown, e.g., jewels, orbs, crosses, or *fleurs-de-lis*, intuitively no such element is essential for a thing to be considered a

crown. On the other hand, an object such as the Pschent or papal tiara counts as a double or triple crown, respectively, because it comprises parts with comparable properties as the whole. Similar cases arguably involve objects such as doors and layers, thus a double door involves two connected door leaves and a double layer is an entity consisting of two merged layers. For the reasons discussed above, I leave the issue concerning when an essential part is a self-sufficient part vague and dependent on how a particular artifact is conceptualized.

All things considered, given (7.22) and (7.23) the $\boxplus(P)$ measure function returns a number of essential parts in each object being a member of a given set. The properties of such a device might help us to account for the interpretation of multiplier phrases. With the set-up and all the relevant ingredients necessary to account for the numerical expressions I am interested here in place, let us now propose the semantics for cardinals and multipliers that will allow us to account for the observed phenomena in count explicit and proportional partitives as well as in multiplier phrases.

7.4 Numerical expressions

In the history of formal semantics spanning over almost half a century there were not that many expressions that received as much attention as numerals. Though a lot of revealing research has been done in this field (see, e.g., Horn 1972, Barwise and Cooper 1981, Scha 1981, Krifka 1999, Landman 2003, 2004, Hofweber 2005, Ionin and Matushansky 2006, Geurts 2006, Nouwen 2010, Kennedy 2013, Rothstein 2013, 2017 to name just a few influential studies), in my opinion there is a shared misconception concerning numerals since they are commonly treated as simplex, i.e., non-decomposable, expressions.[18] On the other hand, it has been observed that in a language such as English cardinals have multiple uses and therefore it is misleading to search for "the" meaning of numerals but rather it is more appropriate to talk about multiple meanings associated with such expressions (e.g., Bultinck 2005, Geurts 2006). The interplay of these two aspects results in treating

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18. A notable exception is the theory of Kennedy (2013) who proposes that numerals are generalized quantifiers over degrees and discusses their possible semantic decomposition.

cardinal numerals either as ambiguous or postulating some shifting mechanism to account for at least some of the uses. However, I believe there are good reasons to claim that the assumption that numerals are simplex is incorrect.

In my opinion, the prevailing view most probably stems from the limited scope of the mainstream research on numerals. In particular, most work on numerical quantification has been done on the basis of English data and since English lacks a rich morphology, certain semantic distinctions are not marked by means of different formal exponents. However, even in English there are complex numericals such as *twice*, *twofold*, *twosome*, and adjectival *two-time* but this set of expressions for the most part has been surprisingly neglected in the study of numerical quantification. On the other hand, evidence from languages that have a broader repertoire of derivational means suggests that numerals are in fact compositional. For instance, recent research on cardinals as well as derivationally more complex numerical expressions in Slavic and Semitic indicates that different numerical affixes correspond to distinct semantic operations responsible for deriving various meanings (see, e.g., Dočekal 2012, 2013, Dočekal and Wągiel to appear for Czech, Wągiel 2014 for Polish, Khrizman 2015 for Russian and Fassi Fehri 2016, 2017 for Arabic).

In the following sections, I posit a compositional account for numerical expressions in natural language. I assume that a prerequisite for such an approach is that it accounts for the three counting principles discussed in Section 5.2.2. In particular, it needs to be guaranteed that the numerical expression ignores overlapping, non-integrated, and non-maximal entities. In other words, it needs to be devised in such a way that it counts right.

7.4.1 Numeral roots

Many numerical expressions in numerous languages are morphologically complex and can be sequenced into separate morphemes including numeral roots and various additional affixes. In this study, I follow my previous work in arguing that it is plausible to treat numeral roots as names of natural numbers, i.e., expressions referring to abstract objects of type n, see (7.24a) (Wągiel 2015b, to appear a,

to appear b). For instance, the English root $\sqrt{tw}$ as in, e.g., *two* and *twenty*, simply names the integer 2, as specified in (7.24b).

(7.24)   Numeral root
      a. $[\![\sqrt{\text{Numeral}}]\!] = n$
      b. $[\![\sqrt{tw}]\!] = 2$

Furthermore, I assume that virtually all meanings numerical expressions can have can be derived from this basic semantics by application of additional operations encoded by different types of classifiers. Those classifiers can be either introduced overtly by a particular morpheme or silent. For the sake of coherence and in order to stay as close to the main argument as possible, in the following sections I will focus on derivations of only two kinds of numerical expressions, namely cardinals and multipliers.[19]

7.4.2 Cardinals

I propose that cardinals are born as singular terms, specifically as names of numbers at type n. This meaning is preserved in contexts clearly calling for numerical arguments such as mathematical statements in (7.25) (see Rothstein 2017 for an extensive discussion of such environments). However, the singular term semantics can also serve as a basis to derive expressions that are more abstract than number-denoting.

(7.25)   English (Rothstein 2013)
      a.   Two plus two is four.
      b.   Two is the only even prime number.

Since this study concerns subatomic quantification, in the following part of this section I will restrict my proposal to the use of numerals as nominal modifiers. In particular, I posit that in attributive position the meaning associated with the numeral root is shifted to an expression of type $\langle\langle e, t\rangle, \langle e, t\rangle\rangle$, i.e., the type of a predicate modifier (cf. Ionin and Matushansky 2006). I postulate that this shift is performed by

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19. For some preliminary proposals of how to treat compositionally other complex numerical expressions in Slavic see Wągiel (2015b, to appear a), to appear b.

a classifier element which I will refer to as $CL_\#$ (see also Sudo 2016 for a similar proposal based on Japanese data). As suggested by the abbreviation and provided in (7.26), $CL_\#$ is an expression of type $\langle n, \langle \langle e, t \rangle, \langle e, t \rangle \rangle \rangle$, i.e., a function from numbers to predicate modifiers, that introduces the # operation. However, it is not the only thing it does. In order to ensure that counting goes right, the classifier imposes a particular constraint on predicates it can select, specifically via the individuation presupposition it selects only predicates that denote exclusively MSSC individuals. Consequently, the resulting cardinal will not be compatible with mass nouns as well as pluralia tantum since they do not fulfill this requirement. Notice also that a predicate P in the first conjunct is pluralized by the classical * operator. Such a semantics guarantees that an integer n will be associated with the number of MSSC individuals making up a plurality in the extension of P, see (7.17)–(7.19). The use of * rather than the strict pluralization operator $^+$ postulated for semantic plurals, see (7.9), is motivated by the existence of the numeral *one* which requires singularities.

(7.26)   Classifier$_\#$
$$[\![CL_\#]\!] = \lambda n \lambda P. \, P_{\text{MSSC}} \, \lambda x [*P(x) \wedge \#(P)(x) = n]$$

Given the semantics of $CL_\#$ proposed above, it is straightforward how particular cardinals are derived. When the classifier combines with a numeral root, e.g., $\sqrt{tw}$, the number variable in (7.26) is saturated by a particular integer and this gives rise to a full-fledged attributive numeral as exemplified in (7.27). Therefore, *two* in a phrase such as *two apples* takes a property of being an apple and yields a set of pairs of objects that are MSSC with respect to that property, i.e., a set of pairs of apples. This is a welcome result, however an important comment is required. For this semantics to work, it needs to be stipulated that the morphological plural on the noun the cardinal modifies is not interpreted semantically since otherwise the pluralized property would fail to satisfy the individuation presupposition. In other words, the source of plurality is the * operator introduced by $CL_\#$, whereas the number marker on the noun is a mere agreement plural triggered syntactically with no semantic contribution (cf. Krifka 1989, Ionin and Matushansky 2006). At first sight this might seem implausible, but there are well-documented cases of mismatches between the plural

244

as a morpho-syntactic category and the semantic notion of plurality (see Nouwen 2016 for an overview) as well as examples of languages such as Finnish, Hungarian, and Turkish that despite having plural morphology do not employ it in numeral phrases.[20]

(7.27)   Cardinal numeral
$$[\![\text{two}]\!] = [\![\text{CL}_\#]\!]([\![\sqrt{tw}]\!]) = \lambda P.\, P_{\text{MSSC}}\, \lambda x[^*P(x) \wedge \#(P)(x) = 2]$$

The treatment postulated above builds on a well-established way of thinking about cardinals since the idea that numerals are in fact names of numbers dates back to Frege (1884). An early formal semantic account for cardinals proposed by Scha (1981) assumes a complex syntactic structure for numerical determiners decomposing them into the bottom-most Number projection which is interpreted in terms of reference to an integer and the Numeral and Det projections above which transform a singular term into a predicate based upon that integer and generalized quantifier, respectively. Finally, in a recent theory of counting and measuring by Rothstein (2012, 2013, 2017) it has been acknowledged that when used as singular terms in examples such as (7.25) cardinals seem to refer to abstract objects rather than anything else and this meaning should be accounted for a special shifting operation.

Apart from an explanation for why cardinals are incompatible with mass terms, an important advantage of the semantics developed here is it also allows for a unified analysis of numerical expressions in classifier and non-classifier languages.[21] In the next section, I will extend the proposed classifier semantics for numerals to multipliers

7.4.3   Multipliers

As we have seen in Chapter 4, multipliers are similar to cardinals in that they both do what a counting expression does, i.e. establish

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20. I simplify here for the sake of brevity. For a general discussion on different strategies languages use to combine cardinals with nouns and a proposal of an alternative approach see Bale et al. (2011). One thing that should be mentioned as a potential problem for the proposed approach concerns collective modification below the numeral, e.g., *two parallel streets* and *two similar objects*. However, since this study does not focus on such expressions, I leave this issue for future research.
21. See, e.g., Kobuchi-Philip (2006) for some arguments why this is desirable.

one-to-one correspondence between entities and natural numbers. On the other hand, the two expressions in question differ significantly in that the first do not count whole individuals but rather quantify over certain parts thereof. Furthermore, they appear to be merged with the modified noun before other quantifiers, and thus their scope is significantly limited since they allow for modification by numerals and by the universal quantifier. In other words, they do not infer a plurality of objects, instead they imply that an object involves a plurality of parts having particular features. These facts suggest that the analysis of multipliers should resemble that of cardinals and at the same time differ in such a way that would capture the relevant distinctions.

The morphological evidence from Slavic presented in Chapter 4 strongly suggests that similar to cardinals multipliers should be treated as complex compositional expressions. Just like in the previous section, I propose that multipliers are decomposable into two elements. In a language such as Polish, the first is the name of a number corresponding to a numeral root present in the morphological make-up of the multiplier, whereas the second is a special classifier introduced by the multiplicative affix. I will refer to that classifier as $CL_{\boxplus}$. As indicated by the superscript, this element introduces the $\boxplus$ operation defined in (7.22) Section 7.3.3 above, see (7.28). Similar to $CL_{\#}$, $CL_{\boxplus}$ is a function from numbers to predicate modifiers, i.e., an expression of type $\langle n, \langle \langle e, t \rangle, \langle e, t \rangle \rangle \rangle$. It also involves the individuation presupposition which accounts for the fact that multipliers can only modify expressions denoting MSSC individuals such as count nouns and mass terms coerced by the Universal Packager. However, unlike $CL_{\#}$ the element $CL_{\boxplus}$ does not pluralize predicates it applies to. This is an important difference between the two classifier since it explains why multiplier phrases do not denote pluralities and allow for numeral modification.

(7.28)   Classifier$_{\boxplus}$
$$[\![CL_{\boxplus}]\!] = \lambda n \lambda P. \, P_{\text{MSSC}} \, \lambda x [P(x) \wedge \boxplus(P)(x) = n]$$

Again, it is straightforward how to get a multiplier by combining the meaning of a numeral root, see (7.24), with the semantics of $CL_{\boxplus}$ in (7.28) above. For instance, consider the denotation of the Polish

multiplier *podwójny* 'double' in (7.29). The number referred to by the numeral root $\sqrt{dw}$ simply saturates the first argument slot in (7.28) and as a result an expression of type $\langle\langle e,t\rangle,\langle e,t\rangle\rangle$ is obtained, see (7.29). The derived predicate modifier selects for a set that has only MSSC individuals in their extensions and returns a subset of such MSSC individuals that have two essential parts. Such a predicate can further serve as an argument for another quantificational expression, e.g., a cardinal numeral.

(7.29)   Polish multiplier
$$[\![\text{podwójny}]\!] = [\![\text{CL}_{\boxplus}]\!]([\![\sqrt{dw}]\!]) =$$
$$= \lambda P.\, P_{\text{MSSC}}\, \lambda x[P(x) \wedge \boxplus(P)(x) = 2]$$

The proposed semantics appears to successfully account for the subset of data I focus on here, namely multiplier phrases involving concrete nouns. Furthermore, as indicated in Section 4.2 it also offers a way of thinking about examples I do not deal here with. In particular, after extending the ontology with primitive semantic types for events and roles a very similar mechanism can be proposed to account for the meaning of NPs like *double murder* and *double president*. Such considerations, however, reach far beyond the main topic of this study and I will refrain from discussing an exact implementation of the idea. Instead, in the following section I will present a brief overview on how the proposed semantics fits into a bigger typological picture and how cross-linguistic data supports the treatment of cardinals and multipliers as complex expressions.

7.4.4   Cross-linguistic support

The structure of cardinal numerals proposed above is further supported by several empirical findings made in linguistic typology. First, it has been observed that cross-linguistically numerals and classifiers are always adjacent (Greenberg 1972). Table 7.1 provides the four attested orderings of the numeral, classifier, and noun out of six possible patterns. Notice that in the missing two the noun would separate the numeral and classifier.

LANGUAGE	ORDERING
Vietnamese	[NUM-CL]-N
Thai	N-[NUM-CL]
Ibidio	[CL-NUM]-N
Bodo	N-[CL-NUM]

Table 7.1: Attested relative orderings of numerals and classifiers (Greenberg 1972)

Second, in classifier languages classifiers are often suffixes on numerals as witnessed, e.g., in (7.30) and (7.31) (see Aikhenvald 2000 for more data).

(7.30)   Japanese (Sudo 2016)
           ichi-rin-no hana
           one-CL-GEN flower
           'one flower'

(7.31)   Yucuna (Aikhenvald 2000, p. 106)
           pajluhua-na yahui
           one-CL   dog
           'one dog'

Furthermore, intriguing data from partly classifier languages such as Mi'gmaq and Chol show that it can be the case that in a single language some cardinals require classifiers in order to combine with nouns whereas others do not, see (7.32) (Bale and Coon 2014). This further suggests that the classifier makes a constituent with the numeral and compensates its semantic deficits.

(7.32)   Mi'gmaq (Bale and Coon 2014)
           a.   na'n (*te's)-ijig ji'nm-ug
                five CL-AGR   man-PL
                'five men'
           b.   asugom *(te's)-ijig ji'nm-ug (Mi'gmaq)
                six   CL-AGR   man-PL
                'six men'

Finally, it has been observed that in genetically and typologically diverse languages there exists an alternation between attributive and specialized so-called counting cardinals, i.e., referential expressions that cannot be used as modifiers, see 7.2 (Hurford 1998, 2001). This fact demonstrates that in some cases an exponent of the name of a number can be formally distinct from a related attributive expression. All things considered, I take the data discussed here to indicate that the analysis postulated for cardinals is on the right track.

LANGUAGE	NUMBER	ATTRIBUTIVE	COUNTING
German	2	zwei	zwo
Maltese	2	żewg	tnejn
Chinese	2	liǎng	èr
Hungarian	2	két	kettö
Basque	2	bi	biga

Table 7.2: Attributive and counting numerals across languages (Hurford 2001)

Moreover, the semantics postulated in (7.29) can be further supported by the fact that in many languages the numeral root can be easily distinguished in the morphological make-up of multipliers. As we have seen in Chapter 4, Slavic multipliers are morphologically complex expressions derived from numeral roots by means of affixation which I argue encodes the classifier element CL⊞. Though many European languages borrowed their multipliers from Latin, the existence of multiplicative affixes is by no means a Slavic idiosyncrasy since similar patterns can be observed in Baltic and Finnic, as witnessed by the correspondences given in Table 7.3.

LANGUAGE	NUMBER	CARDINAL	MULTIPLIER
Russian	2	dva	dvojnoj
Lithuanian	2	du	dvigubas
Finnish	2	kaksi	kaksinkertainen

Table 7.3: Multipliers across languages

I conclude that the proposed account is plausible both in terms of what multipliers mean, i.e., what kind of semantic effects they give rise to, and how they often look like, i.e., their morphological complexity in some languages suggests their semantic decomposability. The system developed here is advantageous in that the same compositional mechanism allows for accounting for different numerical expressions. Though in this study I focus only on cardinals and multipliers, after extending it with additional classifiers a number of other numerals can be derived by its means (see Dočekal 2012, 2013, Wągiel 2014, 2015b, to appear b, Dočekal and Wągiel to appear for discussion of other types of complex numerals in Slavic). This unified treatment also explains why only one ordering of cardinals and multipliers is possible. However, before we discuss how a multiplier phrase modified by a cardinal is composed step by step from the pieces defined here, let us introduce additional tools to account for explicit and set partitives.

7.5 Partitives

The main data set presented in Chapter 2 concerned different types of partitives in various languages. In this study, I do not attempt to provide a detailed analysis of the syntax-semantics interface regarding partitive constructions. Rather, based on the cross-linguistic evidence I focus on general issues concerning the interaction between partitivity, topological sensitivity and countability. In particular, I will postulate a minimal set of ingredients necessary for an attempt to explain what happens in count explicit partitives and different types of proportional partitives involving topology-neutral as well as topology-sensitive partitive words. However, before I propose particular denotations of selected expressions of that type, there are several semantic components to comment on.

7.5.1 Partitivity

An important component of partitive constructions considers what kind of DP can be c-commanded be the partitive word. As already mentioned in Section 2.1, partitives are subject to the so-called Partitive

Constraint which disallows certain expressions from the embedded DP (e.g., Jackendoff 1977, Selkirk 1977, Barwise and Cooper 1981, Ladusaw 1982). I will follow here the semantic reanalysis of the Partitive Constraint, as proposed by de Hoop (1997), see (7.33), which states that the whole downstairs DP is an expression of type *e*.

(7.33)   Partitive Constraint (de Hoop 1997; adapted)
          In a partitive, the embedded DP must be entity-denoting, i.e.,
          definite or specific.

The formulation in (7.33) means that the semantics of the node c-commanding the downstairs DP needs to be devised in such a way that it does not take sets as its input but rather individual things. Consequently, since Barker (1998) it is commonly assumed that the preposition *of* in English partitives is a function from entities to sets of entities, i.e., type $\langle e, \langle e, t \rangle \rangle$, expressing partialness by means of proper parthood.[22] Specifically, the semantics of partitive *of* establishes the $\sqsubset$ relation between two entities stating that one is a proper part of another.

     However, as we have seen in Chapter 2 and 3 there are languages that do not employ prepositions in partitive constructions. For instance, Russian, Hungarian, and Basque make use of case marking instead, whereas Japanese and Chinese can wield a number of strategies to express partitivity including classifier structures. Moreover, in German and Portuguese NPs can be modified by adjectival *half*-words in order to yield proportional meaning. These facts seem to suggest that from the cross-linguistic perspective the mapping between semantics and morphology is not that obvious. After all, given that $\sqsubset$ is introduced by some other element, it is not clear what the exact semantic contribution of *part*-words is since they seem to do precisely the same thing. Therefore, for convenience I incorporate the general partitive semantics into the meaning of partitive words themselves, see (7.34). Since I will focus here on the composition of German and Polish partitives, the reason behind this move is mainly to avoid an unnecessary detour into the semantics of case.[23] However, if required

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22. But see Ionin et al. (2006) for a treatment based on improper parthood.
23. See Kagan (2013) for a detailed discussion of some aspects of the meaning of genitive in Russian.

the distribution of particular components to be discussed below could be rearranged so that, say, the genitive case in a language such as German and Polish is associated with the semantics in (7.34), whereas particular partitive words contribute additional meaning or are treated as void.

(7.34)   Partitivity
         $[\![\text{PART}]\!] = \lambda y \lambda x [x \sqsubset y]$

Notice that since I assume only one domain, i.e., the domain of entities, with one parthood relation $\sqsubseteq$ defined over it, the partitivity semantics in (7.34) simply employs the basic mereological notion of proper parthood which unlike $\sqsubseteq$ is not reflexive. As a result, there is no need to distinguish between entity and set partitives in terms of different parthood relations. Consequently, it is straightforward to expect that in general partitive words are able to combine both with expressions referring to singularities and those denoting pluralities. As witnessed in Section 2.2, this kind of behavior is exactly what is observed cross-linguistically.

As we have seen, the semantic re-formulation of the Partitive Constraint in (7.33) urges us to ensure that the downstairs DP position in partitives is occupied by an expression of type *e*. Putting aside the straightforward case of proper names, in the next sections I will discuss two mechanisms how to guarantee adequate extensions of definites and specific indefinites, namely maximization and choice functions, respectively.

7.5.2   Maximization

In order to account for definite DPs, I assume the standard maximization operation MAX, see (7.35) (cf. Sharvy 1980, Link 1983), which yields the unique maximal entity in the denotation of a predicate.[24] In other words, MAX maps any set which may include singularities, pluralities, or both onto the element that all other elements in that set are part of. In cases it selects a pluralized predicate, it returns the supremum, i.e., the largest plurality in the set. On the other hand, if MAX is applied to a singular predicate, it returns a singularity, provided that that

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24. In the literature, the symbol σ is often used instead of MAX.

predicate denotes a singleton, i.e., there is only one relevant entity in a given context. Maximality thus allows us for a unified account for both singular and plural definite DPs.

(7.35) Maximization operator
$\textsc{max}(P) = \forall y[P(y) \rightarrow y \sqsubseteq x]$
defined if there is unique x in P, otherwise undefined
(If defined, the element in P which all other things in P are part of.)

In article languages such as English and German, the $\textsc{max}$ operator is introduced by the definite article, e.g., *the* and *der/die/das/die*, respectively. On the other hand, for article-less languages such as Polish I assume a DP projection over NP headed by a silent determiner that can be interpreted as a definite article (e.g., Veselovská 1995, Progovac 1998, Rutkowski 2002). The semantics for such an overt or covert definite DP head is given in (7.36).

(7.36) Definite article
$\llbracket \text{DEF} \rrbracket = \lambda P[\textsc{max}(P)]$

Let us now contemplate how the proposed semantics delivers the meaning of definite singulars and plurals. Assuming a model for (7.37) with a unique apple a, the noun *apple* denotes a singleton set involving a, see (7.37a). After the NP is merged with the determiner the resulting DP *the apple* in (7.37b) simply refers to a. On the other hand, in a model with three apples a, b, and c the pluralized noun *apples* denotes the set of a, b, c, and all the derived pluralities, see (7.38a), whereas the definite *the apples* has only the maximal sum in its extension, see (7.38b). This shows that the mechanics of $\textsc{max}$ captures the intuition that singular definites such as *the apple* are true of a unique apple, whereas a definite plural DP like *the apples* corresponds to a plurality consisting of all the apples.

(7.37) Definite singular DP
a. $\llbracket \text{apple} \rrbracket = \{a\}$
b. $\llbracket \text{DEF apple} \rrbracket = a$

(7.38)   Definite plural DP

    a. $[\![\text{PL apple}]\!] = \{a \sqcup b, a \sqcup c, b \sqcup c, a \sqcup b \sqcup c\}$

    b. $[\![\text{DEF [PL apple]}]\!] = a \sqcup b \sqcup c$

The MAX operation allows us to model the meaning of both singular and plural definites in partitives in accordance with the Partitive Constraint. In order to account for specific indefinites, I assume choice functions which will be briefly described in the next section.

7.5.3   Choice functions

A choice function is an operator selecting a member from a set (Reinhart 1997, Kratzer 1998). On the adopted view, the choice function variable remains free at LF and its value is provided by the context, i.e., the choice of a particular member of a given set may vary depending on extra-linguistic circumstances. In general, choice functions can be applied to any kind of set. As a result, they have been widely used in the cross-linguistic research on different types of specific indefinites (see, e.g., Matthewson 1998, Kratzer and Shimoyama 2002, Alonso-Ovalle and Menéndez-Benito 2003, Yanovich 2005, Wągiel to appear b). In this study, however, I will limit my focus to the domain of entities. In particular, I embrace an approach that a choice function CH over entities is an expression of type $\langle \langle e, t \rangle, e \rangle$ which if applied to a non-empty set of entities P, it yields a specific entity from P relative to a particular context, see (7.39).

(7.39)   Choice function

    $\text{CH}_{\langle \langle e,t \rangle, e \rangle}$ is a choice function if $P_{\langle e,t \rangle}(\text{CH}(P_{\langle e,t \rangle})) = 1$

I assume that choice functions are introduced by a specificity element which can either be expressed formally or silent. For instance, in Russian it is encoded by a special suffix on indefinite pronouns (see, e.g., Yanovich 2005) but lack any exponent in the case when an indefinite DP including a common noun gets a specific interpretation. In either way, the specificity element takes a predicate as its input and applies CH on that predicate as specified in (7.40).

(7.40)   Specificity element

    $[\![\text{SPEC}]\!] = \lambda P[\text{CH}(P)]$

Provided a model with three apples a, b, and c, the denotations of singulars and plurals are exactly as such cases discussed above, see (7.41a) and (7.42a), whereas the extensions of specific indefinites can be as follows. For instance, in a given context *apple* can be interpreted as referring to a certain apple, say, b in (7.41b). Similar, the plural *apples* can be understood to designate a specific though indefinite plurality of apples, e.g., $a \sqcup b$ in (7.41b).

(7.41)   Specific indefinite singular DP
        a. $[\![\text{apple}]\!] = \{a, b, c\}$
        b. $[\![\text{SPEC apple}]\!] = b$ (in a certain context)

(7.42)   Specific indefinite plural DP
        a. $[\![\text{PL apple}]\!] = \{a \sqcup b, a \sqcup c, b \sqcup c, a \sqcup b \sqcup c\}$
        b. $[\![\text{SPEC [PL apple]}]\!] = a \sqcup b$ (in a certain context)

The discussed mechanism will allow us to account for specific DPs in partitives. However, for the sake of brevity in the following sections I will focus only on constructions involving definite DPs downstairs. For the treatment of specific indefinites in the embedded DP, it is sufficient to substitute the choice function for the maximization operator.

7.5.4   Set partitive constraint

Having provided means to ensure that the complement of the partitive component is of type e, let us return to the meaning of partitive words in different constructions. One of the observations discussed in Chapter 2 concerned the cross-linguistic analogy between entity and set partitives. In particular, in languages such as German and Polish the same partitive word can combine with both singular and plural DPs. In the first case, it gives rise to a part-of-a-singularity reading, whereas in the latter configuration a part-of-a-plurality interpretation is inferred. This suggest that either the meanings of partitive words are general enough to cover both readings or such expressions are ambiguous cross-linguistically. However, the latter option seems unlikely. Furthermore, in Section 2.5 I have shown that at least in German it is implausible to postulate ambiguity since partitive words can give rise to both discussed readings simultaneously in partitives with coordinated structures involving singular and plural DPs. Therefore, the data

suggest that the proper treatment should be formulated in terms of one general meaning. Given that no sorted domains are postulated and singular individuals are distinguished from plural entities in terms of the distinction between mereotopological and purely mereological configurations, the partitivity semantics introduced in (7.34) is exactly what we need. However, utilizing one unified parthood relation for singularities and pluralities in both entity and set partitives turns out to be problematic in one respect.

The issue is the following. If there is only one domain with one parthood relation, there is nothing that constrains set partitives to refer to an arbitrary material part of any singular entity making up a plurality. For instance, given the plurality of apples $a \sqcup b \sqcup c \sqcup d$ a *part*-word taking that plurality as an argument could return some part of, say, b. However, this is not how set partitives are interpreted. To the contrary, under ordinary circumstances they denote a set of individual parts of a plurality, e.g., in the above example the extension could include $a \sqcup b$, $b \sqcup c$, $a \sqcup b \sqcup c$ etc. In order to exclude material part interpretations, I postulate a special restriction on the meaning of set partitive constructions, namely the set partitive constraint as formulated in (7.43). The strict pluralization operation $^+$ in the formula defined in (7.9) is responsible for the exclusive interpretation of the plural. In particular, $^+P_{\text{MSSC}}$ represents a pluralized predicate with all the MSSC individuals removed from its extension, e.g., for the set $\{a, b, c\}$ the $^+$ operation would yield $\{a \sqcup b, a \sqcup c, b \sqcup c, a \sqcup b \sqcup c\}$. Hence, the set partitive constraint formulated in (7.43) ensures that an entity that is part of a plurality is either an MSSC individual or a sum of such individuals. In other words, it excludes all material parts from the denotations of set partitives.

(7.43) Set partitive constraint
$\forall P_{\text{MSSC}} \forall y \forall x [(^+P_{\text{MSSC}}(y) \wedge x \sqsubset y) \rightarrow {}^*P_{\text{MSSC}}(x)]$

The set partitive constraint guarantees the proper interpretation of partitives involving plural DPs. It seems that postulating this restriction is a necessary move for an analysis of partitive constructions building on the unified parthood relation. However, I believe that it is not a big

price to pay compared to the gains such an approach can offer.[25] In the next section, I will readdress the issue of partitioning discussed briefly in Section 7.1 in order to provide a background for analysis of particular types of partitive words. The notion of partition will turn out useful in modeling continuous parts.

7.5.5 Partitions

As we the evidence discussed in this study indicates, quantification over both wholes and parts can only operate on sets consisting of disjoint members, i.e., entities that do not share a part. However, since there are numerous ways how to divide a thing into parts, many of such divisions would yield multiple entities that do overlap. But such divisions are undesirable since only those sets can serve as the domain of quantification that comprise separate, i.e., non-overlapping, continuous parts. For instance, if one counted one half of a teddy bear as one, another half as two, and its head and right paw as three and four, one would count some things multiple times. By doing so they would violate one of the crucial counting principles and, as a result, fail to establish a one-to-one correspondence between discrete entities and numbers. In order to avoid that, a mechanism ruling out undesirable divisions is required. Such a mechanism is supposed to guarantee that the domain of quantification does not include overlapping parts. In other words, what we want is an operation that will be able to carve up a whole into distinct separate entities. For this purpose, I will make use of a device well-known in the study of pluralities, namely partitions (see, e.g, Schwarzschild 1996, Chierchia 2010).

The partitioning operation π is a function of type $\langle\langle e,t\rangle,\langle e,t\rangle\rangle$ which selects a set of entities, i.e., a predicate P, and yields its subset $\pi(P)$, i.e., a set of those elements in P that do not overlap. Given the standard mereological definition of overlap, see (6.5), partitioning imposes a condition that no two members of a partition $\pi(P)$ share a

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25. One consequence worth noting is that adopting (7.43) we might have to abandon the unconditioned transitivity of parthood. Though it is definitely not a mainstream assumption, it might turn out empirically correct as far as natural language is concerned (see also Moltmann 1997 for related arguments).

part. The formal definition of the partitioning operation π is provided in (7.44).[26]

(7.44)   Partitioning function π
π is a function such that for any property P and any x and y in $\pi(P)$ the following requirement is satisfied
$\neg \exists z [z \sqsubseteq x \wedge z \sqsubseteq y]$
(No two members of a partition overlap.)

To see how partitioning works let us consider what happens when π is applied to the set A in (7.45a). As a result, we get (7.45b) which includes only those members of A that do not share a part. On the other hand, the set B in (7.45c) does not satisfy the condition defined in (7.44) since $a \sqcup b$ and $a \sqcup c$ share a part, namely a. Therefore, although B is a subset of A, it is not its partition.

(7.45)   Partitioning
a. $A = \{a \sqcup b, a \sqcup c, a \sqcup d, b \sqcup c, b \sqcup d, c \sqcup d, a \sqcup b \sqcup c,$
$a \sqcup b \sqcup d, a \sqcup c \sqcup d, b \sqcup c \sqcup d, a \sqcup b \sqcup c \sqcup d\}$
b. $\pi(A) = \{a \sqcup b, c \sqcup d\}$
c. $B = \{a \sqcup b, a \sqcup c\}$

It is important to emphasize that (7.45b) represents only one of numerous possible partitions. For many sets there are multiple subsets that would satisfy the non-overlap requirement after applying π. Depending on a context some of those possible partitions may be considered more natural than others since a particular arrangement of parts making up an entity may suggest a particular division. For instance, a teddy bear consists of cognitively salient elements recognizable as legs, paws, a head, and a body. The fact that we perceive the whole in such a structured way may result in partitioning the set of material parts of the toy in question accordingly. However, different partitions are not excluded, e.g., into its left and right half or its brown and white part. Crucially, once a particular partition is established in a given context it is fixed. This seems to be a welcome result since it satisfies

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26. Notice that this definition diverges from what is usually expected from partitions, i.e., not only non-overlap but also relative atomicity. In that sense, π does not deliver proper partitions. However, I will continue to use the term for the sake of convenience.

the intuition that despite multiple possible divisions of a whole into parts once a choice concerning a particular division has been made, it cannot be altered. Figuratively speaking, one cannot change what's being counted while already counting.

7.5.6 Individuation

The partitioning function π guarantees that a reference set is free of entities sharing a part. This is an important improvement since it accounts for the ban of overlap in the domain of quantification. However, as discussed in Chapter 5 there are two more principles that need to be accommodated within a theory of countability, namely the conditions of integrity and maximality. Since in this study I am primarily concerned with subatomic quantification, I will focus on what makes part *a part*, i.e., on how to model the contrast between an arbitrary share and a stable integrated and recognizable entity within a whole. In order to do so, we need a more fine-grained device than mere partitioning. For this purpose, I assume an additional semantic element which I will refer to as the individuating element (IND). This operation is a function of type $\langle\langle e, t\rangle, \langle e, t\rangle\rangle$ which selects a set of entities and returns a subset of the input consisting of non-overlapping integrated objects. This can be achieved by incorporating into the semantics of IND two components, specifically the partitioning function π and the mereotopological MSSC relation. To be precise, I propose that IND introduces the MSSC restriction relative to a partitioned property, as defined in (7.46). Consequently, IND turns a set consisting of different types of entities into a set including only MSSC individuals.

(7.46)   Individuating element
$$\llbracket\text{IND}\rrbracket = \lambda P \lambda x [\text{MSSC}\big(\pi(P)\big)(x)]$$

In general, an idea of an individuating operation is by no means new. In fact, theories postulating similar operators in order to account for the mass/count distinction date back at least to Sharvy (1979). For instance, one of the most prominent and innovative systems was developed by Borer (2005a) who proposes that count syntax involves a classifier element which is absent in mass expressions. It is introduced by a null head merged in the region between the NP and DP and

expresses division by partitioning the reference domain of encoded by the nominal stem. Similar, Bale and Barner (2009) posit a count noun functional head interpreted as a function from semi-lattices to individuated semi-lattices. Admittedly, a closely related idea to IND was proposed by Scontras (2014, p. 97) for atomizers such as *grain* in, e.g., *grain of rice*. On his view, partitioning is defined in such a way that it applies to a kind and the result is a set of MSSC instantiations of that kind. However, what is new about the notion I propose is that as we will see it allows for the individuation of material parts of individuals, something that Scontras's system fails to account for unless postulating kinds corresponding to denotations of particular partitive words which seems both conceptually undesirable and empirically inappropriate.[27] Given the linguistic relevance of subatomic quantification discussed in Chapter 2, 3, and 4 I argue that the semantics proposed in (7.46) proves to be more advantageous.

Despite the fact that in many cases the individuating element is silent, postulating it is well motivated by the restricted meaning of partitive words in count explicit and proportional partitives as we have seen from the contrasts provided in Chapter 2. Though topology-neutral expressions of that kind can denote both contiguous and discontiguous entities such as scattered portions of matter and arbitrary sums of individuals, the moment they combine with the cardinal numeral they can refer exclusively to integrated objects. Furthermore, as discussed in Chapter 3 Polish and German provide evidence that there are expressions that encode the individuating element formally. In particular, as witnessed by the morphological complexity of Polish topology-sensitive partitive words such as *połówka* and *ćwiartka* as opposed to topology-neutral *połowa* and *ćwierć* there are good reasons to postulate that the suffix *-k-* is the exponent of IND. Similar, German stacked *eine* in proportional partitives of the the type *die eine Hälfte* DP also appears to express the semantics in (7.46).

The next section will provide the semantics of some of the various types of partitive words discussed in Chapter 2 and 3.

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27. An independent problem for approaches relating countability to kinds is that at least in some languages predicates that according to standard tests proposed by Carlson (1977, 1980) do not correspond to kinds, e.g., predicates with certain indexicals, do not seem to behave differently when it comes to counting.

7.5.7 Partitive words

Having defined the generalized contextually conditioned measure functions μ in (7.20) as well as the individuating element IND in (7.46), we are ready to propose the semantics of both topology-neutral and topology-sensitive partitive words. Let us start with German *Teil*. As discussed in Section 7.5.1, for convenience I assume that parthood or, more precisely, proper parthood is encoded in the semantics of partitive words. Thus, the meaning of *Teil* is given in (7.47). The purely mereological nature of $\sqsubset$ makes it a topology-neutral expression which can denote a set of scattered entities, arbitrary sums, or integrated individuals. This characteristic corresponds to the compatibility of *Teil* with explicit, set, and mass partitives reported in Section 2.2. On the other hand, the constraint on set partitives postulated in (7.43) guarantees that when it selects a plural genitive DP, it does not yield an arbitrary material part or parts of an mssc individual making up a plurality. However, when *Teil* combines with a count singular DP, the resulting set can consist of various parts of the relevant object since the proposed semantics bears no topological commitments with respect to the to spatial arrangement of portions of matter the whole phrase denotes. In other words, explicit entity partitives can refer to continuous as well as discontinuous parts.

(7.47)   German topology-neutral *part*-word *Teil*
           $[\![\text{Teil}]\!] = \lambda y \lambda x [x \sqsubset y]$

However, as we have seen in Section 2.2.2, in count explicit partitives the numeral in accordance with the counting principle of integrity can quantify only over contiguous portions of matter. This means that in such a syntactic configuration the possible denotation of *Teil* is significantly restricted. In other words, the *part*-word that is normally topology-neutral is shifted into a topologically sensitive expression that yields a set of those parts of the relevant whole that are spatially integrated. I propose that this shift is performed by a silent IND element. As specified in (7.46), IND applies mssc to a partition of the set denoted by the input predicate. As a result, all of those parts denoted by the phrase c-commanded by *Teil* that are overlapping as well as discontinuous are excluded from the extension of the shifted expression.

All the details concerning composition will be discussed in Section 7.6.

Let us now turn to different types of Polish *half*-words. In general, the semantics of topology-neutral *połowa* given in (7.48) is quite similar to the one of *Teil*. Both are expressions of type $\langle e, \langle e, t \rangle \rangle$, i.e., functions that take an entity and yield a set of its parts. Furthermore, similar to *Teil* nothing restricts *połowa* from selecting any particular type of thing, i.e., it is perfectly compatible with individuals, scattered entities, and pluralities of objects. Finally, the resulting set involves both continuous and discontinuous halves. The main difference between the two is that *połowa* is a proportional expression which denotes only those parts that constitute a certain portion of a whole. This is captured by the generalized measure function μ. Given the mechanism of contextual conditioning defined in (7.20), μ returns an integer corresponding to a number of individuals if its argument is a plurality, whereas when applied to a singular individual or scattered entity, it yields a value corresponding to the measure of its volume in some contextually salient units, e.g., cm^3. Since as discussed in Section 3.1.2 *half*-words are inherently vague and can indicate parts that are either smaller or greater than an exact half, the approximately equal relation $\approx$ is preferred over $=$ in the formula in (7.48). Overall, what *połowa* does is that it selects an entity of any sort and returns a set of its continuous and discontinuous parts such that they constitute approximately 50% of the total cardinality or volume of that entity.

(7.48)   Polish topology-neutral *half*-word *połowa*
$$[\![\text{połowa}]\!] = \lambda y \lambda x [x \sqsubseteq y \wedge \mu(x) \approx \mu(y) \times 0.5]$$

Compared to *połowa*, its topology-sensitive counterpart *pół* fails to combine with with predicates denoting scattered entities and arbitrary sums of individuals. Given the framework developed here, it is possible to account for its distributional restrictions discussed in Section 3.1.1 in terms of mereotopological distinctions. In particular, I propose that the semantics of *pół*, see (7.49), is almost identical to (7.48) with the exception that it involves a special selectional requirement which I refer to as the integrated individual presupposition, see (7.50). Specifically, the symbol y_{MSSC} in (7.49) stands for an individual that is MSSC relative to some property. This explains why *pół* does not com-

bine with plural and mass DPs since these expressions do not refer to mssc individuals in their extensions. Therefore, *pół* can only head proportional entity partitives and after it takes an integrated object, it yields a set consists of various continuous as well as discontinuous halves of that object.

(7.49)   Polish topology-sensitive *half*-word *pół*
$$[\![\text{pół}]\!] = \lambda y \,.\, y_{\text{MSSC}}\, \lambda x[x \sqsubset y \wedge \mu(x) \approx \mu(y) \times 0.5]$$

(7.50)   Integrated individual presupposition
$$x_{\text{MSSC}} \leftrightarrow \forall y[y = x \rightarrow \exists P[\text{mssc}(P)(y)]]$$

Finally, it is time to account for the most formally as well as semantically complex of the Polish *half*-words, namely *połówka*. As we have seen in Section 3.1.1, it is a morphologically complex expression involving a special suffix *-k-* shared with a number of derived partitive words in Polish such as *ćwiartka* 'quarter' and *cząstka* 'part'. All of those expressions are similar to *pół* in that they require mssc individuals as their inputs. However, they differ in that their denotation consist only of continuous parts. Therefore, I propose that the suffix *-k-* in fact introduces the semantics of the IND element specified in (7.46), see (7.51).[28] Thus, the *half*-word *połówka* is a complex expression derived from a simpler topology-sensitive partitive word.

(7.51)   Polish individuating suffix *-k-*
$$[\![\text{-k-}]\!] = [\![\text{IND}]\!] = \lambda P \lambda x[\text{mssc}\big(\pi(P)\big)(x)]$$

Given both the morphological complexity as well as semantic behavior of *połówka* discussed in detail in Section 3.1, I postulate that it is derived from *pół* via the derivational suffix *-k-*.[29] The resulting expression is a topology-sensitive *half*-word, see (7.52), that not only selects entities

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28. An additional argument suggesting that such an analysis is on the right track is the fact that the suffix *-k-* also appears in the make-up of Polish denumeral group nouns such as *dwójka* 'group of two' (see Wągiel 2015b). However, I leave the exact nature of the relationship between *-k-* in *połówka* and *-k-* in *dwójka* for future investigations.

29. More precisely, I assume that *połówka* is derived from *pół* rather than from *połowa* and that the element *-ów-* in *połówka* is semantically vacuous. Such an analysis seems to be corroborated by the fact that the morpheme *-ów-* appears sometimes as a linking element in Polish deadjectival nominalizations such as *złoty* 'golden; złoty (Polish currency)' *sim złotówka* '1-złoty coin'. Notice the ungrammaticality of *złotowy*.

that are MSSC individuals but also denotes only parts that are spatially continuous. The first aspect is ensured by the integrated individual presupposition encoded in *pół*, whereas the semantics of *-k-* guarantees the latter. In particular, the partitioning function π provides a contextually salient partition of the set delivered by *pół*. Thus, the MSSC operation is relativized to a property from whose denotation all overlapping parts have been eliminated as the result of partitioning. Ultimately, the resulting set involves only disjoint integrated parts exactly as desired.

(7.52)   Polish topology-sensitive *half*-word *połówka*
          $[\![\text{połówka}]\!] = [\![\text{-k-}]\!]([\![\text{pół}]\!])$

Having developed all the machinery necessary to account for subatomic quantification in natural language, let us see how the pieces fit together. In the next section, I will walk through the derivation of different types of partitives including German explicit and set partitives and Polish proportional partitives as well as multiplier phrases. The section will be concluded by a proposal how the system developed here could be extended to Italian count explicit partitives involving irregular plurals.

7.6   Composition

In the previous sections, I have provided all the ingredients necessary for an analysis of natural language expressions utilizing subatomic quantification. Now, let us see how those elements interact. I will start with the simplest cases of explicit entity and set partitives. Next, I will walk through the most complex case of count explicit partitives which involves the interplay of almost all postulated components. Subsequently, I will discuss the difference between topology-neutral and topology-sensitive proportional partitives. Finally, I will present the derivation of multiplier phrases modified by numerals. Similar to the previous sections, I will illustrate how the proposed system works on the German and Polish data. For all partitives, I provide examples involving embedded definite DPs. Though for the sake of brevity I do not provide explicit derivations, I assume that for specific DPs nothing
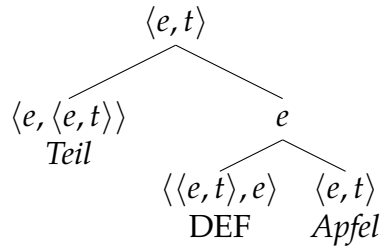
changes except from the use of a choice function, see (7.39), instead of the maximization operator.

7.6.1 Explicit entity partitives

I begin with a relatively simple example of a German explicit entity partitive as the one provided in (7.53). The analysis of this case will allow us to see how the essential components introduced in previous sections interact with each other and as such it will serve us as a reference point for other more complex cases. All things considered, I assume that what gets interpreted at LF is the structure in (7.54).[30] The whole phrase is an expression of type $\langle e, t \rangle$ which gets a part-of-a-singularity reading. The description of the whole derivation is provided in (7.55).

(7.53)   Explicit entity partitive
Teil des   Apfels
part the$_{\text{GEN}}$ apple$_{\text{GEN}}$
'part of the apple'

(7.54)   Structure

$$
\begin{array}{c}
\langle e, t \rangle \\
\diagdown \\
\langle e, \langle e, t \rangle \rangle \qquad e \\
Teil \qquad\qquad \\
\langle \langle e, t \rangle, e \rangle \quad \langle e, t \rangle \\
\text{DEF} \qquad Apfel
\end{array}
$$

(7.55)   Interpretation
a. $[\![\text{Apfel}]\!] = \lambda x[\text{MSSC}(\text{APPLE})(x)]$
b. $[\![\text{DEF}]\!] = \lambda P[\text{MAX}(P)]$
c.   Function Application
$[\![\text{DEF Apfel}]\!] = \text{MAX}([\![\text{Apfel}]\!]) =$
$= \text{MAX}(\lambda x[\text{MSSC}(\text{APPLE})(x)])$
d. $[\![\text{Teil}]\!] = \lambda y \lambda x[x \sqsubset y]$
e.   Function Application
$[\![\text{Teil [DEF Apfel]}]\!] = \lambda x[x \sqsubset [\![\text{DEF Apfel}]\!]] =$
$= \lambda x[x \sqsubset \text{MAX}(\lambda y[\text{MSSC}(\text{APPLE})(y)])]$

---

30. For the sake of simplicity, I ignore the genitive case marking at LF.

The predicate *Apfel* in (7.55a) denotes a set of MSSC individuals that have a property of being an apple. If in a given context this set is a singleton, the maximization operation MAX introduced by the definite article in (7.55b) selects the unique maximal element from that set, e.g., it turns $\{a\}$ into a, see (7.55c). Subsequently, the *part*-word *Teil* with the semantics specified in (7.55d) picks that entity and returns a set of its material parts. Since there are no additional restrictions, the resulting phrase represented in (7.55e) denotes all possible divisions of the apple in question including various overlapping continuous as well as discontinuous parts. This is exactly what we expect from the extension of a topology-neutral explicit entity partitive.

7.6.2 Explicit set partitives

The next example to be discussed is the German explicit set partitive (7.56). An analysis of such a construction should account for the fact that it can get only a part-of-a-plurality interpretation. I assume the semantic structure in (7.57) for the phrase in question. The only difference between (7.57) and (7.54) from the previous section is that the set partitive tree involves the PL node which introduces the pluralization of the predicate whereas the entity partitive tree does not. How the structure corresponding to the phrase in (7.56) is interpreted is demonstrated in (7.58).

(7.56)   Explicit set partitive
          Teil der   Äpfel
          part the_GEN apples_GEN
          'some of the apples'

(7.57) Structure

$$\langle e, t \rangle$$

$$\langle e, \langle e, t \rangle \rangle \qquad e$$
$$\textit{Teil}$$

$$\langle \langle e, t \rangle, e \rangle \qquad \langle e, t \rangle$$
$$\text{DEF}$$

$$\langle \langle e, t \rangle, \langle e, t \rangle \rangle \qquad \langle e, t \rangle$$
$$\text{PL} \qquad \textit{Apfel}$$
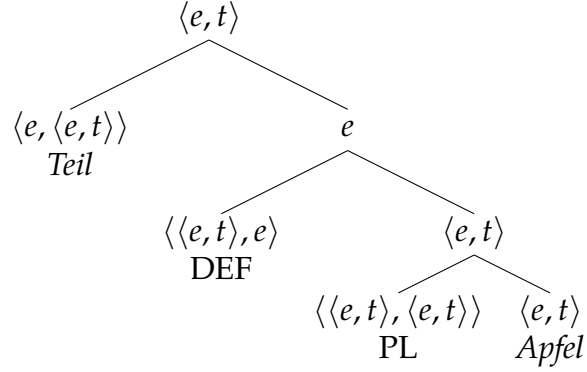
(7.58) Interpretation

    a. $\llbracket \text{Apfel} \rrbracket = \lambda x [\text{MSSC}(\text{APPLE})(x)]$

    b. $\llbracket \text{PL} \rrbracket = \lambda P \, . \, P_{\text{MSSC}} \, \lambda x [^{+}P(x)]$

    c.   Function Application: presupposition satisfied
$\llbracket \text{PL Apfel} \rrbracket = \lambda x [^{+} \llbracket \text{Apfel} \rrbracket (x)] =$
$= \lambda x [^{+} (\lambda y [\text{MSSC}(\text{APPLE})(y)])(x)]$

    d. $\llbracket \text{DEF} \rrbracket = \lambda P [\text{MAX}(P)]$

    e.   Function Application
$\llbracket \text{DEF} [\text{PL Apfel}] \rrbracket = \text{MAX}(\llbracket \text{PL Apfel} \rrbracket) =$
$= \text{MAX} \left(\lambda x [^{+} (\lambda y [\text{MSSC}(\text{APPLE})(y)])(x)] \right)$

    f. $\llbracket \text{Teil} \rrbracket = \lambda y \lambda x [x \sqsubset y]$

    g.   Function Application
$\llbracket \text{Teil} [\text{DEF} [\text{PL Apfel}]] \rrbracket = \lambda x [x \sqsubset \llbracket \text{DEF} [\text{PL Apfel}] \rrbracket] =$
$= \lambda x \left[x \sqsubset \text{MAX} \left(\lambda y [^{+} (\lambda z [\text{MSSC}(\text{APPLE})(z)])(y)] \right) \right]$

Similar to (7.55), the noun in (7.58a) denotes a set of individuals that are MSSC relative to the property APPLE. Since the plural NP in (7.56) is semantically interpreted, the plural marker contributes the meaning in (7.58b). Specifically, it takes the predicate APFEL and applies the strict pluralization operation $^{+}$. As a result, the extension of (7.58c) consists of sums of apples. Next, the definite article yields the maximal sum plurality from the set which will serve as the argument of *Teil*, see (7.58d)–(7.58f). Finally, after the entity variable is saturated as in (7.58g), the explicit set partitive denotes a set of overlapping parts of the largest plurality of apples in a given context. The set partitive constraint defined in (7.43) ensures that this set consists only of parts

that are either mssc entities or sums thereof, i.e., material proper parts of particular individuals are not included. This is a welcome result.

7.6.3 Count explicit partitives

The last type of the explicit partitive constructions to be considered here concerns the significantly more complex example in (7.59). In general, I assume the semantic structure along the lines in (7.60) for count explicit partitives. Notice that in accordance with the semantics of cardinals proposed in Section 7.4.2, the numeral root and $CL_{\#}$ form a constituent. Furthermore, unlike in the explicit partitive examples in (7.54) and (7.57) in order to ensure counting is possible, I postulate that there is a special IND node present in the tree at LF, see Section 7.5.6. In the discussed German example, IND has no formal exponent. However, I argue that the semantic evidence implies that though null it is interpreted. In particular, count explicit partitives refer only to pluralities of integrated parts. This of course is in accordance with the counting principles discussed in Chapter 5 and requires a formal account. The exact step-by-step derivation is given in (7.61).

(7.59) Count explicit partitive
        zwei Teile des   Apfels
        two parts the_GEN apple_GEN
        'two parts of the apple'

(7.60)   Structure
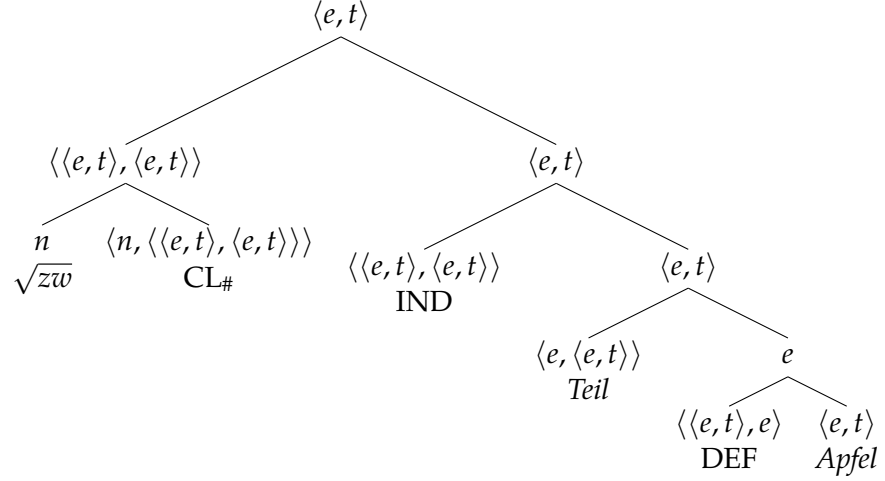
(7.61)   Interpretation
a. $[\![\text{Apfel}]\!] = \lambda x[\text{MSSC}(\text{APPLE})(x)]$
b. $[\![\text{DEF}]\!] = \lambda P[\text{MAX}(P)]$
c.   Function Application
$[\![\text{DEF Apfel}]\!] = \text{MAX}([\![\text{Apfel}]\!]) = \text{MAX}\big(\lambda x[\text{MSSC}(\text{APPLE})(x)]\big)$
d. $[\![\text{Teil}]\!] = \lambda y \lambda x[x \sqsubset y]$
e.   Function Application
$[\![\text{Teil [DEF Apfel]}]\!] = \lambda x[x \sqsubset [\![\text{DEF Apfel}]\!] =$
$= \lambda x\big[x \sqsubset \text{MAX}\big(\lambda y[\text{MSSC}(\text{APPLE})(y)]\big)\big]$
f. $[\![\text{IND}]\!] = \lambda P \lambda x[\text{MSSC}(\pi(P))(x)]$
g.   Function Application
$[\![\text{IND [Teil [DEF Apfel]]}]\!] =$
$= \lambda x\big[\text{MSSC}\big(\pi([\![\text{Teil [DEF Apfel]}]\!])\big)(x)\big] =$
$= \lambda x\Big[\text{MSSC}\big(\pi\big(\lambda z[z \sqsubset \text{MAX}(\lambda y[\text{MSSC}(\text{APPLE})(y)])(z)]\big)\big)(x)\Big]$
h. $[\![\sqrt{\text{zw}}]\!] = 2$
i. $[\![\text{CL}_{\#}]\!] = \lambda n \lambda P.\, P_{\text{MSSC}}\, \lambda x[{}^{*}P(x) \wedge \#(P)(x) = n]$
j.   Function Application
$[\![\sqrt{\text{zw}}\ \text{CL}_{\#}]\!] = \lambda P.\, P_{\text{MSSC}}\, \lambda x[{}^{*}P(x) \wedge \#(P)(x) = [\![\sqrt{\text{zw}}]\!]] =$
$= \lambda P.\, P_{\text{MSSC}}\, \lambda x[{}^{*}P(x) \wedge \#(P)(x) = 2]$
k.   Function Application: presupposition satisfied
$[\![[\sqrt{\text{zw}}\ \text{CL}_{\#}]\ \text{[IND [Teil [DEF Apfel]]]}]\!] =$
$= \lambda x\big[{}^{*}[\![\text{IND [Teil [DEF Apfel]]}]\!](x)$
$\wedge \#([\![\text{IND [Teil [DEF Apfel]]}]\!])(x) = 2\big] =$
$= \lambda x\Big[{}^{*}\big(\lambda w[\text{MSSC}\big(\pi\big(\lambda z[z \sqsubset \text{MAX}(\lambda y[\text{MSSC}(\text{APPLE})(y)])(z)]\big)\big)]\big)(w)\big)(x)$
$\wedge \#\big(\lambda w[\text{MSSC}\big(\pi\big(\lambda z[z \sqsubset \text{MAX}(\lambda y[\text{MSSC}(\text{APPLE})(y)])(z)]\big)\big)](w)\big)(x) = 2\Big]$

The steps (7.61a)–(7.61e) are exactly the same as in the case of the explicit entity partitive derived in (7.55), i.e., starting from the predicate of mssc individuals we end up with the node denoting a set of overlapping continuous and discontinuous parts of a contextually unique apple. Given the counting principles of non-overlap, integrity, and maximality, that expression cannot serve as the domain of quantification since its extension resembles those of mass terms. In particular, it includes both scattered entities and entities that are not disjoint. In Chapter 5, I argued that the deeply rooted mechanism of counting is based upon an algorithm that establishes one-to-one correspondence with numbers only for discrete objects, i.e., non-overlapping integrated entities in their mereological maximality, and that the same applies to subatomic quantification. Therefore, if parts are to be counted, it needs to be ensured that they constitute entities of that sort. This is what the individuating element IND introduced in (7.61f) is for. IND combines with the set of overlapping parts and partitions it in such a way that the new set consists only of disjoint parts. However, non-overlap itself does not guarantee that parts are integrated. This is why IND restricts the denotation of (7.61e) even further by applying the mssc constraint. As a result, the expression in (7.61g) denotes a set of objects that are mssc with respect to the property of being a disjoint proper part of a contextually unique apple. In other words, (7.61g) refers to a set of integrated parts that can be put in one-to-one-correspondence with numbers.

As proposed in Section 7.4.2, the cardinal numeral *zwei* is a complex expression consisting of a numeral root simply naming integer 2, see (7.61h), which serves as an argument for the classifier element $CL_{\#}$, see (7.61i). As indicated in (7.61j), the whole cardinal is a predicate modifier that for a predicate of mssc entities returns a set of pairs of such individuals. This is ensured by the following elements. The individuation presupposition restricts the predicate arguments to those that denote only mssc entities. On the other hand, the first conjunct of the assertion of $CL_{\#}$ pluralizes the predicate by means of the classical * operator, and thus provides sums for the $\#(P)$ measure function which yields the number of individuals that are mssc relative to the property the whole cardinal modifies. After the predicate variable gets saturated by the entity partitive individuated by IND, the resulting count explicit partitive in (7.61k) returns a set of pairs of non-overlapping

integrated parts of a contextually unique apple. And that is exactly what we want.

Though the computation in (7.61) gets quite complex, I believe that the underlying mechanism deriving count explicit partitives is quite simple. Its great advantage is that it allows us to account for the semantic subtleties concerning topological issues highlighted by the novel data I presented in Chapter 2 and 3 via the interplay of the semantics of partitives, the individuating element, and cardinals. In the following sections, I will go in detail through the proposed account for the contrast between Polish topology-neutral and topology-sensitive *half*-words discussed in Section 7.5.7.

7.6.4 Topology-neutral proportional partitives

Let us now turn to Polish proportional partitives. First, I will discuss the step-by-step derivation of the least complex case, i.e., entity partitives involving the topology-neutral *half*-word *połowa* exemplified by the phrase in (7.62). In general, such expressions are very similar to German explicit entity partitives such as those discussed in Section 7.6.1. The semantic tree corresponding to (7.62) is provided in (7.63) and (7.64) demonstrates how the structure is interpreted.[31]

(7.62)   Topology-neutral proportional entity partitive
          połowa jabłka
          half$_2$ apple$_{\text{GEN}}$
          'half of the apple'

(7.63)   Structure

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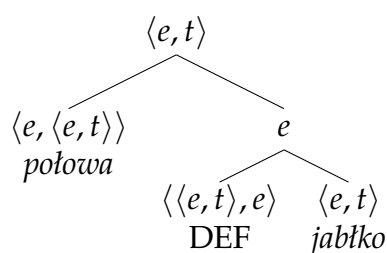
31. Similar to German explicit partitives, I ignore case marking at LF.

(7.64) Interpretation

    a. $[\![\text{jabłko}]\!] = \lambda x[\text{MSSC}(\text{APPLE})(x)]$

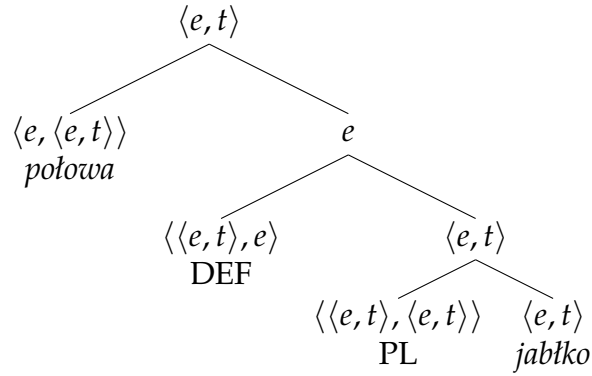   b. $[\![\text{DEF}]\!] = \lambda P[\text{MAX}(P)]$

    c. Function Application

      $[\![\text{DEF jabłko}]\!] = \text{MAX}([\![\text{jabłko}]\!]) = \text{MAX}(\lambda x[\text{MSSC}(\text{APPLE})(x)])$

    d. $[\![\text{połowa}]\!] = \lambda y \lambda x[x \sqsubset y \wedge \mu(x) \approx \mu(y) \times 0.5]$

    e. Function Application

$$[\![\text{połowa [DEF jabłko]}]\!] = \lambda x[x \sqsubset [\![\text{DEF jabłko}]\!]$$
$$\wedge \, \mu(x) \approx \mu([\![\text{DEF jabłko}]\!]) \times 0.5] =$$
$$= \lambda x \Big[x \sqsubset \text{MAX}(\lambda y[\text{MSSC}(\text{APPLE})(y)])$$
$$\wedge \, \mu(x) \approx \mu\Big(\text{MAX}(\lambda y[\text{MSSC}(\text{APPLE})(y)])\Big) \times 0.5 \Big]$$

Similar to German *Apfel*, the predicate *jabłko* in (7.64a) is true of entities that are MSSC with respect to the property APPLE. Though Polish lacks articles, as indicated in Section 7.5.2 I assume the silent definite element DEF specified in (7.64b) which applies the maximization operator MAX to the set denoted by *jabłko*. If it is a singleton, then the definite DP refers to the unique individual from that set, see (7.64c). The denotation of the *half*-word *połowa* is very similar to that of a *part*-word with the exception that it in indicates how big the parts it denotes are, see (7.64d). It takes the MSSC entity denoted by the definite DP as it first argument and the resulting proportional partitive denotes a set of parts of a contextually unique apple. The mechanism of contextual conditioning accounts for the fact that the generalized measure function μ measures its argument in terms of volume. Thus, each part in the denotation of the whole proportional partitive constitutes approximately 50% of the volume of that apple. Notice that among the members of the set there are continuous as well as discontinuous halves many of which overlap. Such a denotation is incompatible with cardinal numerals. In order to deliver countable entities, the individuating element IND needs to be applied similar as in count explicit partitives discussed in Section 7.6.3.

Since the *half*-word *połowa* is topologically neutral, there are no restrictions concerning its distribution. Hence, it can also combine with plurals as in (7.65). I assume the semantic structure of the phrase in (7.66) and interpretation in (7.67).

(7.65)   Topology-neutral proportional set partitive
połowa jabłek
half$_2$ apples$_{\text{GEN}}$
'half of the apples'

(7.66)   Structure

(7.67)   Interpretation

a. $[\![\text{jabłko}]\!] = \lambda x[\text{MSSC}(\text{APPLE})(x)]$

b. $[\![\text{PL}]\!] = \lambda P . P_{\text{MSSC}} \lambda x[^+P(x)]$

c.   Function Application: presupposition satisfied
$[\![\text{PL jabłko}]\!] = \lambda x[^+[\![\text{jabłko}]\!](x)] =$
$= \lambda x[^+(\lambda y[\text{MSSC}(\text{APPLE})(y)])(x)]$

d. $[\![\text{DEF}]\!] = \lambda P[\text{MAX}(P)]$

e.   Function Application
$[\![\text{DEF [PL jabłko]}]\!] = \text{MAX}([\![\text{PL jabłko}]\!]) =$
$= \text{MAX}\left(\lambda x[^+(\lambda y[\text{MSSC}(\text{APPLE})(y)])(x)]\right)$

f. $[\![\text{połowa}]\!] = \lambda y \lambda x[x \sqsubset y \wedge \mu(x) \approx \mu(y) \times 0.5]$

g.   Function Application
$[\![\text{połowa [DEF [PL jabłko]]}]\!] = \lambda x[x \sqsubset [\![\text{DEF [PL jabłko]}]\!]$
$\wedge \mu(x) \approx \mu([\![\text{DEF [PL jabłko]}]\!]) \times 0.5] =$
$= \lambda x\Big[x \sqsubset \text{MAX}\left(\lambda y[^+(\lambda z[\text{MSSC}(\text{APPLE})(z)])(y)]\right)$
$\wedge \mu(x) \approx \mu\Big(\text{MAX}(\lambda x[^+(\lambda y[\text{MSSC}(\text{APPLE})(y)])(x)])\Big) \times 0.5\Big]$

The sole difference between topology-neutral proportional entity
and set partitives is that in the case of the latter the noun is plural-
ized. The plural marker in (7.67b) applies to the predicate of MSSC
individuals in (7.67a) and yields a set consisting of sums formed from
those singularities. Subsequently, given the set partitive constrained

and the mechanism of contextual conditioning the measure function μ in (7.67g) is interpreted as #, i.e., counts the number of MSSC parts of its arguments. As a result, the whole construction denotes plural entities whose cardinality corresponds to approximately 50% of the total number of individuals of the maximal sum apples.

Despite the straightforward differences resulting from distinct semantics of *part-* and *half*-words, the denotations of topology-sensitive explicit and proportional partitives are very much alike. If not modified by a cardinal, both types of expressions can refer to overlapping continuous and discontinuous parts of a singularity as well as to overlapping parts of a plurality. On the other hand, topology-sensitive partitives are significantly more restricted. In the following section, I focus on demonstrating how exactly.

7.6.5 Topology-sensitive proportional partitives

An important insight concerning the Polish topology-sensitive *half*-word *pół* presented in Section 3.1.1 is that it does not appear in set and mass partitives. It is not a weird property of one partitive word since the same behavior is shared, e.g., with the *quarter*-word *ćwierć*. In any case, I argue that this distributional constraint results from a particular topological restriction imposed on the arguments of *pół*. An exemplary topology-sensitive proportional partitive is provided in (7.68), whereas the structure I assume at LF is given in (7.69). The complete derivation is presented in (7.70).

(7.68)   Topology-sensitive proportional entity partitive
         pół   jabłka
         half₁ apple_GEN
         'half of the apple'

(7.69)   Structure

274

(7.70)   Interpretation

a. $[\![\text{jabłko}]\!] = \lambda x[\textsc{mssc}(\textsc{apple})(x)]$

b. $[\![\text{DEF}]\!] = \lambda P[\textsc{max}(P)]$

c.   Function Application
$[\![\text{DEF jabłko}]\!] = \textsc{max}([\![\text{jabłko}]\!]) =$
$= \textsc{max}(\lambda x[\textsc{mssc}(\textsc{apple})(x)])$

d. $[\![\text{pół}]\!] = \lambda y \,.\, y_{\textsc{mssc}} \, \lambda x[x \sqsubset y \wedge \mu(x) \approx \mu(y) \times 0.5]$

e.   Function Application: presupposition satisfied
$[\![\text{pół [DEF jabłko]}]\!] = \lambda x[x \sqsubset [\![\text{DEF jabłko}]\!]$
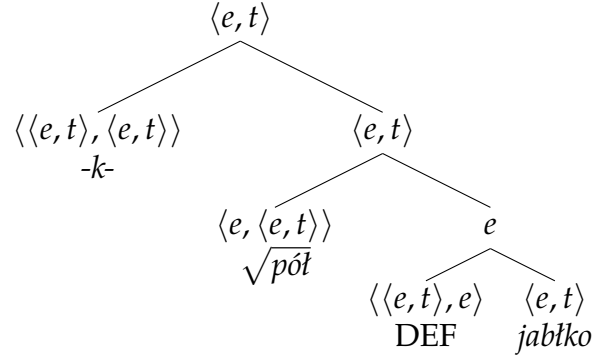$\wedge \, \mu(x) \approx \mu([\![\text{DEF jabłko}]\!]) \times 0.5] =$
$= \lambda x\Big[x \sqsubset \textsc{max}(\lambda y[\textsc{mssc}(\textsc{apple})(y)])$
$\wedge \, \mu(x) \approx \mu\Big(\textsc{max}(\lambda y[\textsc{mssc}(\textsc{apple})(y)])\Big) \times 0.5\Big]$

The derivation in (7.70) is almost identical to its twin counterpart in (7.64). However, notice that in (7.70d) there is a tiny but crucial difference in the semantics of *pół* as proposed in (7.49). Specifically, *pół* incorporates the integrated individual presupposition defined in (7.50) which requires its first argument to be an integrated entity. This constraints the distribution of *pół* to count singulars since plurals and mass terms denote arbitrary sums and scattered entities, respectively, i.e., things distinct from mssc objects. However, *pół* does not impose any topological restriction on parts of the relevant individual it yields. The semantics of the whole partitive in (7.70e) thus states that the denoted set includes both continuous and discontinuous halves. However, as we have seen in Chapter 3 there are also partitive words that are restrictive with respect to the topological properties of entities they deliver.

This brings us finally to the most intriguing case of Polish proportional partitives headed by the topology-sensitive *half*-word *połówka* such as the one in (7.71). Though I restrict my focus here to the semantic structure and interpretation of this example, see (7.72) and (7.73), respectively, as we have seen in Section 3.2 there are also other expressions that behave similarly, e.g., the *part*-word *cząstka* and *quarter*-word *ćwiartka*.

(7.71)   Topology-sensitive individuating proportional partitive
połówka jabłka
half$_3$ apple$_{\text{GEN}}$
'half of the apple'

(7.72)   Structure

(7.73)   Interpretation
a. $\llbracket$jabłko$\rrbracket = \lambda x[\text{MSSC}(\text{APPLE})(x)]$
b. $\llbracket\text{DEF}\rrbracket = \lambda P[\text{MAX}(P)]$
c.   Function Application
$\llbracket\text{DEF jabłko}\rrbracket = \text{MAX}(\llbracket\text{jabłko}\rrbracket) = \text{MAX}(\lambda x[\text{MSSC}(\text{APPLE})(x)])$
d. $\llbracket\sqrt{\text{pół}}\rrbracket = \lambda y \,.\, y_{\text{MSSC}} \,\lambda x[x \sqsubset y \wedge \mu(x) \approx \mu(y) \times 0.5]$
e.   Function Application: presupposition satisfied
$\llbracket\sqrt{\text{pół}}\,[\text{DEF jabłko}]\rrbracket =$
$= \lambda x[x \sqsubset \llbracket\text{DEF jabłko}\rrbracket \wedge \mu(x) \approx \mu(\llbracket\text{DEF jabłko}\rrbracket) \times 0.5] =$
$\lambda x\Big[x \sqsubset \text{MAX}\big(\lambda y[\text{MSSC}(\text{APPLE})(y)]\big)$
$\wedge\, \mu(x) \approx \mu\Big(\text{MAX}\big(\lambda y[\text{MSSC}(\text{APPLE})(y)]\big)\Big) \times 0.5\Big]$
f. $\llbracket\text{-k-}\rrbracket = \llbracket\text{IND}\rrbracket = \lambda P \lambda x[\text{MSSC}\big(\pi(P)\big)(x)]$
g.   Function Application
$\llbracket\text{IND}\,[\sqrt{\text{pół}}\,[\text{DEF jabłko}]]\rrbracket =$
$= \lambda x[\text{MSSC}\big(\pi(\llbracket\sqrt{\text{pół}}\,[\text{DEF jabłko}]\rrbracket)\big)(x)] =$
$= \lambda x\Big[\text{MSSC}\Big(\pi\big(\lambda z[z \sqsubset \text{MAX}\big(\lambda y[\text{MSSC}(\text{APPLE})(y)]\big)$
$\wedge\, \mu(x) \approx \mu\Big(\text{MAX}\big(\lambda y[\text{MSSC}(\text{APPLE})(y)]\big)\Big) \times 0.5\big]\big)\Big)(x)\Big]$

The only difference between (7.70) and (7.73) lies in that in the first the *half*-word *pół* is used whereas the latter involves derivationally complex *połówka*. Thus, the composition in (7.73a)–(7.73c) remains

unchanged and I will start the discussion from the step (7.73d). As proposed in (7.52), the morphological complexity of *połówka* translates into its semantic complexity. In particular, I assume that the root $\sqrt{p\ell}$ is a topologically sensitive expression introducing the integrated individual presupposition, see (7.73d), which ensures that that the *half*-word combines only with MSSC individuals. On the other hand, the suffix *-k-* is interpreted as the individuating element IND, see (7.73e), i.e., it restricts the denotation of the partitive to non-overlapping continuous parts. As a result, the whole phrase refers to integrated halves of a contextually unique apple that can be subject to counting.

This concludes the analysis of distinct types of partitive constructions. The relevance of Polish topology-sensitive individuating proportional partitives is that they show that IND can be formally expressed. As demonstrated in Section 3.4, there are more constructions in different natural languages that can be considered as involving formal exponents of IND. Though I will refrain here from further investigation, if the analysis proposed here is on the right track, I expect to find even more such expressions cross-linguistically. In the next section, I will move on to another type of construction involving subatomic quantification, namely multiplier phrases.

7.6.6 Multiplier phrases

In the previous sections, I have discussed the composition of different kinds of partitives. The only construction involving numerical expressions discussed so far was the German count explicit partitive in (7.59) where the cardinal modified an entity partitive headed by the *part*-word *Teil*. In this section, I will discuss in detail the semantics of constructions involving derived numerical expressions specialized precisely for subatomic quantification, namely multipliers. In order to present the overall picture, I will consider multiplier phrases modified by cardinals exemplified by (7.74). I posit that the semantic structure underlying such expressions is essentially as in (7.75), whereas in (7.76) I provide an exact step-by-step derivation of the main example.

(7.74)   Multiplier phrase modified by a cardinal
          trzy   podwójne hamburgery
          three double$_{PL}$ hamburgers

'three double hamburgers'

(7.75)   Structure
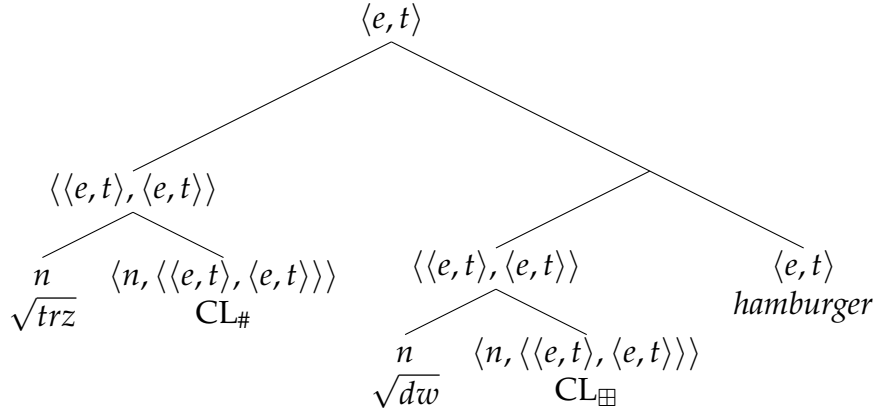
(7.76)   Interpretation

a. $[\![\text{hamburger}]\!] = \lambda x[\text{MSSC}(\text{HAMBURGER})(x)]$

b. $[\![\sqrt{\text{dw}}]\!] = 2$

c. $[\![\text{CL}_{\boxplus}]\!] = \lambda n \lambda P. \, P_{\text{MSSC}} \, \lambda x[P(x) \wedge \boxplus(P)(x) = n]$

d.   Function Application
$[\![\sqrt{\text{dw}} \; \text{CL}_{\boxplus}]\!] = \lambda P. \, P_{\text{MSSC}} \, \lambda x[P(x) \wedge \boxplus(P)(x) = [\![\sqrt{\text{dw}}]\!]] =$
$= \lambda P. \, P_{\text{MSSC}} \, \lambda x[P(x) \wedge \boxplus(P)(x) = 2]$

e.   Function Application: presupposition satisfied
$[\![[\sqrt{\text{dw}} \; \text{CL}_{\boxplus}] \; \text{hamburger}]\!] =$
$= \lambda x[[\![\text{hamburger}]\!](x) \wedge \boxplus([\![\text{hamburger}]\!])(x) = 2] =$
$= \lambda x[\text{MSSC}(\text{HAMBURGER})(x) \wedge \boxplus(\text{MSSC}(\text{HAMBURGER}))(x) = 2]$

f. $[\![\sqrt{\text{trz}}]\!] = 3$

g. $[\![\text{CL}_{\#}]\!] = \lambda n \lambda P. \, P_{\text{MSSC}} \, \lambda x[^*P(x) \wedge \#(P)(x) = n]$

h.   Function Application
$[\![\sqrt{\text{trz}} \; \text{CL}_{\#}]\!] = \lambda P. \, P_{\text{MSSC}} \, \lambda x[^*P(x) \wedge \#(P)(x) = [\![\sqrt{\text{trz}}]\!]] =$
$= \lambda P. \, P_{\text{MSSC}} \, \lambda x[^*P(x) \wedge \#(P)(x) = 3]$

i.   Function Application: presupposition satisfied
$[\![[\sqrt{\text{trz}} \; \text{CL}_{\#}] \; [[\sqrt{\text{dw}} \; \text{CL}_{\boxplus}] \; \text{hamburger}]]\!] =$
$= \lambda x \big[^*[\![[\sqrt{\text{dw}} \; \text{CL}_{\boxplus}] \; \text{hamburger}]\!](x)$
$\wedge \#([\![[\sqrt{\text{dw}} \; \text{CL}_{\boxplus}] \; \text{hamburger}]\!])(x) = 3] =$
$\lambda x \Big[^*\big(\lambda y[\text{MSSC}(\text{HAMBURGER})(y) \wedge \boxplus(\text{MSSC}(\text{HAMBURGER}))(y) = 2]\big)(x)$
$\wedge \#\big(\lambda y[\text{MSSC}(\text{HAMBURGER})(y) \wedge \boxplus(\text{MSSC}(\text{HAMBURGER}))(y) = 2]\big)(x) = 3\Big]$

In analogy to the previous cases, the noun *hamburger* is a predicate of type $\langle e, t \rangle$ denoting a set of entities that are MSSC with respect to the property HAMBURGER, i.e., singular hamburgers. On the other hand, as suggested by morphology the multiplier in (7.76d) consists of the classifier element $\text{CL}_{\boxplus}$ of type $\langle n, \langle \langle e, t \rangle, \langle e, t \rangle \rangle \rangle$ whose number argument, see (7.76c), is saturated by the number 2 referred to by the numeral root in (7.76b). Due to the individuation presupposition in the semantics of $\text{CL}_{\boxplus}$ the resulting numerical expression is a predicate modifier selecting only predicates of MSSC individuals. Since the noun *hamburger* satisfies this requirement, it can serve as the argument for the multiplier. After the two combine in (7.76e), the resulting multiplier phrase, i.e., a function from entities to truth values, denotes a set of singular hamburgers such that each includes two patties. This meaning stems from the fact that the $\boxplus(P)$ measure function defined in (7.22) returns 2 as the number of parts that are essential for the property HAMBURGER and given the relevant extra-linguistic factors such a part happens to be a patty. Notice also that the fact that the first conjunct in (7.76e) is not pluralized, unlike in cardinals, results in that the multiplier phrase is a predicate of MSSC entities, i.e., it denotes singular double hamburgers. As such it can be modified by the cardinal numeral.

Given the compatible expression, the cardinal in (7.76h) is ready for the saturation of the predicate variable. It itself is a complex expression of type $\langle \langle e, t \rangle, \langle e, t \rangle \rangle$ consisting of the numeral root in (7.76f) whose denotation, i.e., the number 3, serves as the argument for the classifier element $\text{CL}_{\#}$. After the cardinal composes with the multiplier phrase in (7.76i), what we obtain is a set of triples of hamburgers with two essential parts each. Counting is done by the measure function $\#(P)$ which puts MSSC individuals in one-to-one correspondence with numbers and yields the integer corresponding to the total number of objects in a plurality delivered by the * operator introduced in the first conjunct. The result is exactly what we wanted to derive, i.e., a set of pluralities consisting of three double hamburgers.

This concludes the analysis of some of the expressions involving subatomic quantification. In this study, I have focused on several different types of partitives as well as multiplier phrases. I have proposed that though both types of constructions quantify over parts of entities, they employ different means to achieve the goal. While partitives em-

ploy the proper parthood relation $\sqsubset$ and in more complicated cases also the individuating element IND and cardinals, multipliers utilize a special measure function $\boxplus(P)$ dedicated to counting essential parts of integrated wholes. In the next section, I will briefly suggest how the analysis I proposed could be extended to cover more peculiar examples such as Italian partitives involving irregular plurals.

7.7 Possible extensions

The last part of this chapter concerns a brief informal discussion of possible extensions of the system developed here to other types of constructions in different languages. In principle, a number of distinct kinds of proportional partitives discussed in Section 3.4 could be accounted for by incorporating the meaning of the individuating element IND into the semantics of some morpheme. For instance, if the approach proposed here is on the right track and there are no additional facts I have not reported here concerning phrases such as (7.77), a straightforward application of the analysis appears to be plausible. It seems that the morpheme *eine* is what encodes IND which accounts for the reported results of the flag test.

(7.77) German topology-sensitive proportional partitive
　　　　die eine Hälfte der　　Fahne
　　　　the a　　half　　the_{GEN} flag
　　　　'the half of the flag'

Similar, in principle the account developed here could be extended to English constructions such as (7.78). However, most probably this case would be more challenging since it might be necessary to capture interactions between the partitive word and determiner on the one hand as well as with the preposition *of* on the other.

(7.78) English topology-sensitive proportional partitive
　　　　a half of the flag

Furthermore, it might turn out that a similar analysis is appropriate for the Mandarin classifier construction in (7.79). Here, the classifier *miàn* is arguably responsible for introducing IND.

(7.79)   Mandarin topology-sensitive proportional partitive
         bàn-miàn guó   qí
         half-CL   national flag
         'a half of the national flag'

Of course, much more work needs to be done to verify whether the proposed in fact accounts for the data from German, Mandarin, and other languages displaying a similar pattern. However, I believe that at this stage of the research it is a plausible hypothesis to postulate that what makes topology-sensitive partitives different from their topology-neutral counterparts is the fact that the first involve a formal exponent of IND.

The next section will be dedicated to the discussion of how the proposed analysis could be applied to yet another type of expression.

7.7.1   Italian partitives with irregular plurals

The last case of a possible extension of the developed system to be discussed here concerns count explicit partitives involving irregular plurals in Italian such as those in (7.80). In Chapter 2, a considerable amount of attention was dedicated to the intriguing effect these expressions give rise to. We have seen that at least a subset of Italian irregular plurals does not merely encode plurality but also implies a certain spatial configuration that the individual entities making up a sum remain in. Arguably, this kind of behavior can be captured in terms of the topological notion of connection. The Italian data served as a crucial source of evidence for the role of integrity in quantification over parts. In particular, the evidence strongly suggests that entities are countable as long as they are conceptualized as spatially continuous despite whether they are singularities or pluralities.

(7.80)   Italian partitive with irregular plurals
         due parti delle mura
         two parts of-the wall_COLL
         'two parts of the walled complex'

Given the arguments in favor of the connected nature of pluralities denoted by the relevant class of Italian irregular plurals, I postulate that it is plausible to model them in terms of clusters (Grimm 2012b,

p. 144). A cluster is a special type of individual defined in terms of the TC relation formulated in (6.32) and repeated here as (7.81). Specifically, the definition in (7.82) states that an entity is a cluster if it involves a plurality of individuals that are transitively connected, i.e., form a connected sequence. Intuitively, clusters are entities that are conceptualized as bundled pluralities whose minimal units are cognitively not salient enough or insignificant to be considered objects in their own rights. For instance, a bundle of rice or a pile of gravel constitute clusters since each minimal unit, i.e., a grain or a pebble, in a bundle or a pile is connected to another minimal unit. Since TC is relativized to a connection, clusters can involve bundled pluralities of entities that are externally or even proximately connected.[32] In the case of Italian irregular plurals, the first would correspond, e.g., to the referents of *mura* 'walls (in a complex)', whereas the latter might be useful to model the denotation of, e.g., *ossa* 'bones (in a skeleton)'.

(7.81)   Transitively connected

$$\text{TC}(x, y, P, C, Z) \overset{\text{def}}{=} \forall z \in Z[P(z) \wedge (x = z_1 \wedge y = z_n) \wedge Cz_1z_2 \wedge Cz_2z_3 \cdots \wedge Cz_{n-1}z_n]$$

where $Z = \{z_1, z_2, \ldots z_n\}$

(Entities x and y are transitively connected relative to a property P, a connection relation C, and a set of entities Z, when all members of Z satisfy P and x and y are connected through the sequence of z_is in Z.)

(7.82)   Cluster

$$\text{CLUSTER}(x, P, C) \overset{\text{def}}{=} \exists Z[x = \bigsqcup Z \wedge \forall z \forall z' \in Z \exists Y[\text{TC}(z, z', P, C, Y)]]$$

(An entity x is a cluster relative to a property P and a connection relation C iff x is a sum of entities falling under the same property which are all transitively connected relative to some set Y under the same property and connection relation.)

If the proposed analysis of Italian irregular plurals in terms of clusters is correct, one would probably need to elaborate on the meaning of cardinal numerals. Notice that clusters can consist of parts that are MSSC entities, e.g., individual bones, as well as sequences of individuals,

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32. Those connection can be formally captures by the notions of EC and PC introduced in (6.29) and (6.33), respectively.

e.g., pluralities of transitively connected bones. This might require to develop a more general devise able to capture more fine-grained shades of integrity than the current formulation of the individuation presupposition. Though such an enterprise lies beyond the scope of this study and I leave it for future research, I believe that the account proposed here provides an inspiring background for such an attempt.

7.8 Summary

In this chapter, I have presented a formal account for subatomic quantification in natural language grounded in the conceptual framework described in Chapter 5. I rejected atomicity as a useful concept for that purpose and instead proposed an analysis based on notions developed within mereotopology, i.e., the theory of wholes extending standard mereology with topological distinctions, and introduced in Chapter 6. The mereotopological account for nominal semantics allows us to get rid of atomicity as an unintuitive and undesirable notion as well as to distinguish between singularities and pluralities in terms of the distinction between part-whole structures that either involve or do not involve the topological notion of connectedness. In particular, I adopted a view on which singular count nouns denote sets of integrated objects modeled in terms of MSSC individuals that can be pluralized in order to yield sets of pluralities of such entities. On the other hand, I postulated a compositional semantics for numerical expressions such as cardinals and multipliers that involves numeral roots treated here as names of numbers as well as classifiers that turn expressions of type n into predicate modifiers. In particular, I proposed that those classifier elements employ measure functions, i.e., operations that relate entities with integers. While cardinals utilize a measure function that simply counts MSSC entities, multipliers involve quantification over essential parts of an individual. Compatibility of the numerical expressions in question is ensured by the individuation presupposition that requires the nodes cardinals and multipliers combine with to be predicates of MSSC objects.

Another component of the analysis regarded the semantics of partitive words. Given the advantages of mereotopology, it was possible to model subatomic quantification without postulating sorted domain

with different parthood relations defined over them. Instead, particular partitive words were treated as expressions employing the unified parthood relation. Specifically, what they do is that they select an entity and return a set of its parts. Different types of partitive words differ with respect to the nature of those parts. For instance, *part*-words denote sets of any proper parts, whereas *half*-words refer to parts that constitute approximately 50% of the cardinality of a plurality or the volume of an mssc individual or a scattered entity such as substance. On the other hand, topology-neutral partitive words denote all kinds of overlapping continuous as well as discontinuous parts, and thus are compatible with singular count nouns, mass terms, and plurals, whereas denotations of topology-sensitive partitive words are more restricted. For instance, the Polish *half*-word *pół* introduces the integrated individual presupposition which imposes a constraint that it can only select mssc entities. This fact explains why it does not combine with mass and plural complements. On the other hand, morphologically complex *połówka* is a compositional expression involving the suffix *-k-* that I argue is a formal exponent of the individuating element that partitions the set of parts so that overlapping halves are excluded as well as removes those parts that are discontinuous. As a result, the extension of *połówka* involves only a set of disjoint integrated halves that could serve as an argument for cardinals. Though the individuating element is formally expressed in the morphology of some Polish partitive words, it can also be silent. I argued that that is the case in, e.g., German count explicit partitives since given the counting principles postulated in Chapter 5, one can only count non-overlapping continuous parts. The chapter was concluded by considerations how the proposed system could be extended to some of the other expressions discussed in this study including Italian partitives involving irregular plurals that could be analyzed as clusters, i.e., bundled pluralities of transitively connected entities. However, the precise implementation of the suggested ideas is left for future research.

8 Conclusion

The research on different aspects of quantification in natural language is abundant and since the early days of formal semantics has continually led to better understanding of certain properties of the language faculty. Nevertheless, one perspective seems to be somewhat neglected. The main aim of this thesis was to contribute to our understanding of quantification in natural language by exploring the so far neglected domain of subatomic quantification, i.e., quantification over parts of building blocks of the denotations of singular count nouns. I have provided compelling evidence for the relevance of this phenomenon for natural language semantics. In particular, I have explored various aspects of meaning of a broad range of linguistic expressions such as different types of partitive constructions, certain types of adjectives as well as multipliers from a cross-linguistic perspective. One of the key findings is the significance of the topological notion of integrity with respect to part-whole structures encoded in nominal semantics. The examination of properties of different types of investigated expressions suggests that there is one unified parthood relation for various types of entities. At the same time, different part-whole structures result from distinct topological relations holding between particular elements. Entities conceptualized as integrated wholes, i.e., objects whose parts stick together, differ from those whose parts are not bound by any topological constraints or, alternatively, are arranged in some other type of spatial configuration. This concerns both wholes, e.g., the prototypical referents of singular count nouns and regular plurals, and parts, e.g., spatially continuous pieces as opposed to arbitrary discontinuous portions of matter. The major claim of the thesis is that subatomic quantification is subject to the very same constraints as quantification over wholes. Specifically, only entities that are conceptualized as non-overlapping integrated contiguous objects can be put in one-to-one correspondence with numbers be it either wholes or parts. In order to capture this generalization, I propose a universal mechanism which allows for counting of both entire objects and their parts. Its formal implementation is based on the theory of wholes called mereotopology which extends standard mereological parthood with topological notions such as connectedness. Building

on this framework, I develop a system which derives the predicted semantics of representative constructions discussed in the empirical part of the thesis. Different aspects of their meaning arise as a result of the interaction between topology, partitivity, and numerical quantification. An important advantage of the proposed approach is that it does not define countability in terms of atomicity since devising counting as quantification over mereological atoms, i.e., entities that have no proper parts, is very problematic for subatomic quantification.

This chapter concludes the thesis by summarizing its empirical insights as well as theoretical contributions to the study of quantification in natural language. First, I will provide an overall summary, then I will discuss some of the open questions and possible topics for future investigations.

8.1 Overall summary

As outlined in Chapter 1, this thesis was intended to contribute to the field of formal semantics by exploring subatomic quantification. Throughout its pages, I have provided a broad range of novel evidence indicating its significance in natural language and developed a novel approach accounting for the observed phenomena. The thesis is divided into two conceptual parts. In the first part comprising Chapter 2, 3, and 4, I have provided a broad range of novel evidence indicating that natural language involves elaborate means to express subatomic quantification. The second part involving Chapter 5, 6, and 7 was dedicated to developing a conceptual framework that would provide means to model quantification over both wholes and parts and a formal approach based on it that would account for a representative subset of the observed phenomena.

The data discussed in Chapter 2 provide cross-linguistic evidence that the same partitive word, e.g., *part*, *half*, and *most*, can commonly appear both in entity and set partitives, i.e., constructions that refer to parts of singularities and pluralities and typically combine with singular and plural DPs, respectively. This fact indicates that such expressions are able to operate both at the superatomic and subatomic level depending on what structure is provided by the embedded DP (cf. Moltmann 1997, 1998). On the other hand, explicit partitives mod-

ified by the cardinal numeral can only quantify over material parts of singular individuals irrespective of whether the complement DP is singular or plural, see Table 8.1. This might suggest that since partitive words have distinct properties in different structures, they differ significantly after all (Schwarzschild 1996).

	SINGULARS		PLURALS	
	bare	count	bare	count
subatomic quantification	✓	✓	*	✓
quantification over wholes	*	*	✓	*

Table 8.1: Properties of partitive words

Nonetheless, the crucial piece of data coming from Italian partitives involving irregular plurals, i.e., expression that infer not only plurality but also cohesion or integrity of parts (Ojeda 1995, Acquaviva 2008), provides evidence to the contrary. In particular, when such a partitive is modified by the cardinal numeral, the whole phrase can get either a part-of-a-singularity reading or a part-of-a-plurality interpretation as long as the parts making up the plurality constitute an integrated entity. Therefore, countability results from the interplay between the meaning of a particular partitive word and the semantic properties of a singular or plural DP it combines with. If a plural expression denotes a plural entity constituted by a cohesive, i.e., spatially related, sum of parts, quantification over continuous parts of such a sum is possible, see Table 8.2.

	SINGULARS		REGULAR PL		IRREGULAR PL	
	bare	count	bare	count	bare	count
subatomic quantification	✓	✓	*	✓	✓	✓
quantification over wholes	*	*	✓	*	✓	✓

Table 8.2: Properties of Italian *parte* 'part'

Furthermore, I have shown that at least in some languages the parallelism between entity and set partitives cannot be explained in terms of semantic ambiguity of partitive words. Instead, I have argued that what allows for the cross-linguistically widely attested

distribution of those expressions is the fact that singulars and plurals actually involve part-whole structures that are based on the same unified parthood relation. Crucially, what distinguishes the two is an additional notion that is responsible for how parts are topologically arranged with respect to each other. In other words, I have posited that the intuitive distinction between integrated wholes and arbitrary sums of parts is reflected in grammar by the syntactic distinction between count singulars and regular plurals, respectively. In addition, the evidence from Italian irregular plurals shows that there are natural language expressions designating entities that are similar to plurals in that they comprise a number integrated objects but at the same time their sum is arranged in such a way that it constitutes a cluster.

Chapter 3 was dedicated to the systematic exploration of the role the topological notion of integrity plays with respect to part-whole structures and subatomic quantification in particular. I have provided novel evidence that shows that the distinction between topology-neutral and topology-sensitive partitive words is lexicalized in Polish. Specifically, Polish has three morphologically distinct *half*-words, i.e., *połowa*, *pół*, and *połówka* all 'half'. When applied to a DP denoting an entity, they all yield its part constituting approximately 50% of the whole. However, where they differ is the type of entity they select for as well as the type of part they return. Given different distributional and referential properties of each expression in question, I have interpreted the alternation in terms of topological sensitivity. While the *half*-word *połowa* is topology-neutral, i.e., it simply measures halves of any type of entity and yields a topologically indeterminate portion of a whole, *pół* and *połówka* are semantically more complex. Similar to its topology-neutral counterpart, in the case of *pół* the outcome of quantification is either a continuous or discontinuous half. However, unlike *połowa* it selects only for integrated wholes, i.e., individuated entities that come in one piece. On the other hand, the meaning of morphologically most complex *połówka* even more restricted. While it shares selectional restrictions with *pół*, in addition it imposes topological constraints on the extension of the resulting partitive construction. As a result, the whole partitive can only be true of a continuous integrated part constituting approximately 50% of the volume of an individuated object, see Table 8.3.

288

	CONTINUOUS PART	DISCONTINOUS PART
połowa	✓	✓
pół	✓	✓
połówka	✓	*

Table 8.3: Denotations of Polish *half*-words

Similar alternations are attested in different classes of other partitive expressions in Polish, e.g., *piece*-words and *quarter*-words. In each of the examined cases, the notion of integrity played a crucial role in predicting possible denotations of distinct types of partitives. Even more interestingly, the cross-linguistic investigation revealed that the phenomenon observed is not a Polish idiosyncrasy. Instead, it appears in several other languages. Though the means particular languages employ to express certain spatial configurations differ, e.g., English utilizes determiner whereas Mandarin classifier semantics, the resulting effect is similar and demonstrates the relevance of the topological notion of integrity in subatomic quantification.

In addition, I have discussed the contrast between two types of Polish *whole*-adjectives (cf. Moltmann 1997, Morzycki 2002), i.e., *cały* 'whole' and *kompletny* 'complete, which emphasizes two distinct aspects of wholeness, namely maximality and integrity. While the first involves both, the latter does imply that no part is missing but does not infer that they remain in a particular topological configuration. The significance of the presented data lays primarily in revealing the relevance of topological relations holding between parts of individuals forcing us to recognize that natural language semantics is sensitive to whether parts come in one piece or constitute discontinuous entities.

	MAXIMALITY	INTEGRITY
cały	✓	✓
kompletny	✓	*

Table 8.4: Interpretations of Polish *whole*-adjectives

The final piece of data has been discussed in Chapter 4. It concerns a neglected class of numerical expressions, namely multipliers such

as English *double*, that display non-trivial quantificational properties. Specifically, I have provided novel evidence demonstrating that multipliers are specialized for subatomic quantification, i.e., they count cognitively salient parts that are essential for a whole to be considered having a certain property. In many cases, those parts are self-sufficient, i.e., have themselves a property comparable to that of the whole, but sometimes they are just intuitively the most salient elements. Though the distribution of multipliers involves also abstract entities denoted by event nominals as well as role nouns, I have argued that the semantic behavior observed in combination with singular count nouns constitutes is the basic quantificational mechanism that can be further extended to other types of entities. Interestingly, when the multiplier combines with the mass term, it forces a portion, i.e., count interpretation of the modified noun.

Furthermore, I have discussed Slavic multipliers that display morphological complexity suggesting semantic compositionality. For instance, the Russian multiplier *dvojnoj* 'double' consists of the numeral root corresponding to the number 2 as well as some additional morphology including a special multiplicative affix. This fact indicates that multipliers share with cardinals reference to integers, but differ in that they are devised to count entities of a different type, see Table 8.5. In particular, cardinals are semantically equipped to count wholes. Though they can be used in subatomic quantification, e.g., in count explicit partitives, it is only possible when they combine with a partitive word. In such a case, however, entities designated by the whole partitive phrase are treated as objects in their own right. Hence, the source of subatomic quantification is the partitive and the cardinal simply counts provided entities in their relative entirety. On the other hand, multipliers always quantify over parts of objects referred to by a modified nominal. Given the cross-linguistically widespread appearance of multipliers, the examined data set shows that similar to cardinals whose purpose is to count wholes natural language developed expressions dedicated to numerical subatomic quantification.

In Chapter 5, I have provided a general conceptual framework intended as a basis for developing a formal account for subatomic quantification in natural language. I have suggested how the linguistic evidence discussed in Chapter 2, 3, and 4 could be linked with insights provided by cognitive psychology. Specifically, extensive research on

	CARDINALS	MULTIPLIERS
subatomic quantification	*	✓
quantification over wholes	✓	*

Table 8.5: Properties of cardinals and multipliers

perception and cognitive development indicates that since early child-hood we conceptualize objects, i.e., integrated wholes, in a different way than other types of entities. Furthermore, even young children possess an ability to simultaneously perceive a whole as an object in their own right as well as a collection of parts and this capacity guides their linguistic development. Finally, human number sense seems to be sensitive to the intuitive notion of integrity. Specifically, young children always count each separate integrated physical entity as one. Based on both linguistic and psychological evidence, I have presented the three claims constituting the conceptual core of the thesis. I have postulated that natural language is sensitive to the topological notion of integrity which manifests itself in how nominal lexicon is classified into different grammatical categories. For instance, count singular nouns designate entities conceptualized as integrated wholes. On the other hand, plural nominals presuppose parts of their referents to be integrated wholes but impose no topological constraints on them, i.e., denote arbitrary sums individuals. The second claim concerns the general counting principles, i.e., a set of rules constraining what can be counted. Specifically, the principle of non-overlap ensures that enumerated entities are disjoint, i.e., each thing can be counted once and once only. The principle of maximality demands that counting involves entities in their mereological entirety, i.e., no part is left out. Finally, the principle of integrity guarantees that an entity that can be put in one-to-one correspondence with numbers needs to be conceptualized as an object that comes in one piece. All things considered, the general counting principles ensure that only sets consisting exclusively of elements that are discrete integrated object can be enumerated. Importantly, there is no reference to atomicity in the postulated quantificational mechanism. Finally, the third claim extends the general counting principles to subatomic quantification. I posit that the set of constraints described above is a universal

mechanism applicable both at the level of wholes and at the level of parts.

In Chapter 6, I have introduced a theory of wholes called mereotopology which enhances standard mereology based on the parthood relation with additional topological features (Casati and Varzi 1999; for linguistic application see Grimm 2012b,a). Within this framework, the primitive notion of connectedness enables us to derive complex notions capturing distinct configurations of parts within a part-whole structure. One of such notions is a property of being maximally strongly self-connected (MSSC) which allows us to capture an intuition on what it means to be an integrated whole. Specifically, an MSSC entity is an object whose every interior part overlaps the whole and anything else that has a relevant property and overlaps it is part of it. Such a perspective seems to correspond neatly to the intuitive view on individuals as configurations of parts arranged in a certain manner rather than atomic elements with no proper parts, and thus constitutes an advantageous alternative for accounts based on atomicity. This alternative view turns out to be of significant value in modeling quantification over both wholes and parts.

Finally, in Chapter 7 I have developed a formal account for subatomic quantification based on the conceptual background and mereotopological notion of MSSC discussed above. In particular, I have argued that an approach building on MSSC entities instead of atoms is more auspicious in modeling quantification over parts. For instance, a significant advantage of a mereotopological account is that it does not require postulating sorted domain with different parthood relations defined over them (Link 1983). The distinction between part-whole structures associated with singular count nouns and regular plurals can be recast in terms of connectedness. While the first encode that the elements of their denotations are MSSC entities, the latter require parts of their referents to be such individuals but impose no topological constraints on pluralities thereof. Furthermore, I have proposed that numerical expressions including cardinals and multipliers are complex semantic expressions derived from names of numbers via different overt or covert classifiers specialized in counting distinct types of objects, e.g., MSSC entities and cognitively salient parts thereof, respectively. The last piece of the puzzle concerned how to capture different semantic interpretations of distinct types of topology-neutral

292

and topology-sensitive partitive words. For this purpose, I have proposed a special individuating element that can be either expressed formally, e.g., as a special suffix on Polish topology-sensitive partitive words, or null. In any case, what this individuating element does is that it partitions the set of entities denoted by a partitive and introduces the MSSC constraint relative to a given partition. This guarantees that the extension of a whole partitive involves only non-overlapping continuous parts. Such integrated parts are conceptualized as objects in their own right within a part-whole structure of an individual, and thus can be quantified over similar to other MSSC entities. With these components in place, I have provided step-by-step derivations of multiplier phrases as well as different types of partitives that account for distinct interpretations of constructions involving topology-neutral and topology-sensitive partitive words. In the proposed system, distinct semantic effects arise as a result of different interactions between partitivity, topology, and quantification. Its great advantage is that it provides explanation for the topological phenomena concerning reference to continuous and discontinuous parts in different types of partitives, an issue that previous approaches failed not only to account for but even to acknowledge (e.g., Chierchia 2010).

8.2 Open questions and future research

The account developed in this thesis provides explanation for a number of phenomena in natural language. I believe its contribution is valuable both from a theoretical perspective and in terms of novel data it covers. However, some questions remain unanswered and many issues require further investigation. One of the most serious challenges concerns the interpretation of count explicit partitives with the embedded plural DP as discussed in Section 2.2.2. Unlike standard explicit set partitives, such expressions only get a part-of-a-singularity reading. I have argued that this effect is due to the general counting principles which require that a counted entity constitutes an integrated whole. Since pluralities fail to satisfy this condition, the only possibility how a relevant phrase could be interpreted involves quantification over material parts. Nonetheless, the issue concerning the interpretation of the morpho-syntactic plural on the downstairs nominal remains

open. The question of what the exact semantic contribution of number in such a structure is (if any) requires further investigation. It might be the case that in such constructions the plural is semantically void. However, there is an intuition that within a set of parts denoted by a count explicit partitive each element is part of a different object. At this point, I can only hypothesize that the mechanism deriving this effect is pragmatic in nature. However, it might also be the case that the plural is interpreted as a distributive operator similar to what has been postulated to account for covariational uses of *different* (see, e.g., Beck 2000, Brasoveanu 2008, Dotlačil 2010). Before a detailed analysis could be proposed, definitely more research is required.

As indicated in Section 7.7, the proposed account can be easily extended to topology-sensitive partitives in other languages than Polish and German as well as to other types of constructions involving subatomic quantification. However, the question whether such an extension is empirically appropriate will in each case require careful consideration. After examining the cross-linguistic data presented in Section 3.4, at first sight it seems plausible that, e.g., in English explicit and proportional partitives it is the determiner that introduces the individuating element. Similar, it appears that the same semantics is associated with a special classifier that follows a partitive word in Mandarin. However, more tests need to be applied to reveal the real nature of the interaction between topology, partitivity, and the determiner or classifier semantics, respectively.

A further issue concerns a detailed analysis of Italian irregular plurals (Ojeda 1995, Acquaviva 2008). Though the data from count explicit partitives involving such expressions were of great significance for capturing the nature of the interaction between partitivity and numerical quantification, the proper semantic treatment of such constructions would require accommodating more complex topological notions. Although this task is suit for another enterprise, I believe this study provides a neat framework for pursuing it, see Section 7.7.1. Similar, a potentially interesting topic concerns investigation into partitives in languages displaying the collective-singulative number alternation (Grimm 2012a,b). Moreover, if the proposed analysis is on the right track, effects similar to what we have observed in partitives with irregular plurals in Italian should also be detectable in Slavic partitives with topologically sensitive derived mass nouns that

denote a plurality of discrete objects forming a cluster (Grimm and Dočekal to appear) as well as swarm nouns (Henderson 2017). A pilot study concerning Czech and Polish suggests that this is in fact the case, yet more work needs to be done before a conclusion can be drawn.

So far, *whole*-adjectives did not receive a lot of attention in the semantic literature. In this thesis, I have argued that different types of Polish expressions of this kind provide evidence for distinguishing between maximality and integrity. However, a detailed analysis of the meaning of *whole*-adjectives awaits for being developed. As a starting point, one could consider an interaction between universal quantification at the subatomic level and the individuating element encoding topological integrity or some similar component. Though the idea that *whole*-adjectives could be interpreted as universal quantifiers over parts has already been proposed (Moltmann 1997). Nevertheless, careful investigations into scope dependencies and related factors are required in order to test whether such treatment is empirically plausible (see Morzycki 2002 for potentially problematic data).

Another area where the proposed perspective could be applied concerns adverbials quantifying over parts of a singularity such as *mostly*, *wholly*, *largely* (cf. Morzycki 2002) as well as *all wet* (cf. Schwarzschild 1996, pp. 162–170). Such explorations would most probably require extending the account advocated here with the insights of the prolific research on scale structures (e.g., Kennedy and McNally 1999, 2005). If both approaches turn out to be compatible, it potentially opens a new perspective on the opposition between partial and total predicates, e.g., *dirty*, *wet*, and *touch* as opposed to *clean*, *dry*, and *eat* (e.g., Yoon 1996, Rotstein and Winter 2004).

A separate subject concerns the interpretation of multipliers with event nominals, role and abstract nouns as well as collectives. In Section 4.2, I have suggested that extending ontology with additional types for events as well as social roles will allow us to extend the basic mechanism proposed to account for the semantic behavior of multiplier modifying concrete nouns to more abstract types of objects. However, the exact implementation of the idea requires careful consideration and rigorous study. It might turn out that a more general notion is necessary or, alternatively, that several derived concepts will prove more advantageous in tackling the distribution of multipliers across the board. In any case, figuring this out seems to be an intrigu-

ing challenge that would almost certainly provide a novel perspective on quantification in natural language.

If the approach I have argued for is correct, I expect that scrupulous cross-linguistic investigation will reveal even more types of constructions indicating the relevance of topological relations within part-whole structures. Though a number of directions worth examining has already been indicated, e.g., adjectival *half*-words in German and Romance, I believe that there are even more parts of the lexicon that are sensitive to topological relations at the subatomic level. One potential example is a class of expressions I refer to as verbs of separation such as *separate*, *dismantle*, and *dissolve* mentioned briefly in Section 2.3.1. At first blush, their meaning seems to affect the topological component of part-whole structures. Examining them from a perspective developed here might be a promising enterprise that would shed light at subatomic quantification in the verbal domain. It seems that an especially auspicious area of research concerns Slavic verbal morphology with its various prefixes and aspectual distinctions (e.g., Filip 1999).

Furthermore, I believe that rigorous experimental investigations could also shed new light on subatomic quantification, as they did in other areas of the field. For instance, what would be of significant interest is the exploration of the relative strength of topological inferences in different types of constructions cross-linguistically. It might be informative to compare structures that encode the individuating element formally such as Polish and German topology-sensitive proportional partitives with constructions where a similar effect arises pragmatically. Similar, I can imagine that testing in multiple contexts to reveal could also provide interesting insights on different aspects of quantification over parts.

To conclude, I hope that this study provides a valuable perspective on previously neglected semantic phenomena in natural language. Though I have presented a number of novel observations concerning distinct types of relatively well-studied constructions as well as genuine insights concerning expressions that so far have not received attention in the study of meaning, I expect that many more obscure regions await to be explored. I believe that a detailed map of the phenomena related to subatomic quantification in natural language can only emerge as a result of careful systematic typological as well as

296

experimental research. This is, however, where other journeys are to begin.

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317

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