

Chapter 2 supplement

At the operational level hundreds of decisions are made in order to achieve local outcomes that contribute to the achievement of the company's overall strategic goal. These local outcomes are usually not measured directly in terms of profit, but instead are measured in terms of quality, cost-effectiveness, efficiency, productivity, and so forth. Achieving good results for local outcomes is an important objective for individual operational units and individual operations managers. However, all these decisions are interrelated and must be coordinated for the purpose of attaining the overall company goals. Decision making is analogous to a great stage play or opera, in which all the actors, the costumes, the props, the music, the orchestra, and the script must be choreographed and staged by the director, the stage managers, the author, and the conductor so that everything comes together for the performance.

For many topics in operations management, there are quantitative models and techniques available that help managers make decisions. Some techniques simply provide information that the operations manager might use to help come to a decision; other techniques recommend a decision to the manager. Some techniques are specific to a particular aspect of operations management; others are more generic and can be applied to a variety of decision-making categories. These different models and techniques are the "tools" of the operations manager. Simply having these tools does not make someone an effective operations manager, just as owning a saw and a hammer does not make someone a carpenter. An operations manager must know how to use decision-making tools. How these tools are used in the decision-making process is an important and necessary part of the study of operations management. In this supplement and others throughout the text, we examine several different aspects of operational decision making.

Decision Analysis

In this supplement we demonstrate a quantitative technique called **decision analysis** for decision-making situations in which uncertainty exists. Decision analysis is a generic technique that can be applied to a number of different types of operational decision-making areas.

Many decision-making situations occur under conditions of *uncertainty*. For example, the demand for a product may not be 100 units next week but may vary between 0 and 200 units, depending on the state of the market, which is uncertain. Decision analysis is a set of quantitative decision-making techniques to aid the decision maker in dealing with a decision situation in which there is uncertainty. However, the usefulness of decision analysis for decision making is also a beneficial topic to study because it reflects a structured, systematic approach to decision making that many decision makers follow intuitively without ever consciously thinking about it. Decision analysis represents not only a collection of decision-making techniques but also an analysis of logic underlying decision making.

Decision-Making Without Probabilities

A decision-making situation includes several components--the decisions themselves and the events that may occur in the future, known as *states of nature*. Future states of nature may be high demand or low demand for a product or good economic conditions or bad economic conditions. At the time a decision is made, the decision maker is uncertain which state of nature will occur in the future and has no control over these states of nature.

When probabilities can be assigned to the occurrence of states of nature in the future, the situation is referred to as *decision making under risk*. When probabilities cannot be assigned to the occurrence of future events, the situation is called *decision making under uncertainty*. We discuss this latter case next.

To facilitate the analysis of decision situations, they are organized into **payoff tables**. A payoff table is a means of organizing and illustrating the payoffs from the different decisions, given the various states of nature, and has the general form shown in Table S2.1.

TABLE S2.1 Payoff Table

<i>Decision</i>	<i>States of Nature</i>	
	<i>a</i>	<i>b</i>
1	Payoff 1a	Payoff 1b
2	Payoff 2a	Payoff 2b

Each decision, 1 or 2, in Table S2.1 will result in an outcome, or **payoff**, for each state of nature that will occur in the future. Payoffs are typically expressed in terms of profit, revenues, or cost (although they may be expressed in terms of a variety of quantities). For example, if decision 1 is to expand a production facility and state of nature *a* is good economic conditions, payoff 1a could be \$100,000 in profit.

Once the decision situation has been organized into a payoff table, several criteria are available to reflect how the decision maker arrives at a decision, including maximax, maximin, minimax regret, Hurwicz, and equal likelihood. These criteria reflect different degrees of decision-maker conservatism or liberalism. On occasion they result in the same decision; however, they often yield different results. These decision-making criteria are demonstrated by the following example.

EXAMPLE S2.1

Decision-Making Criteria Under Uncertainty

The Southern Textile Company is contemplating the future of one of its plants located in South Carolina. Three alternative decisions are being considered: (1) Expand the plant and produce lightweight, durable materials for possible sales to the military, a market with little foreign competition; (2) maintain the status quo at the plant, continuing production of textile goods that are subject to heavy foreign competition; or (3) sell the plant now. If one of the first two alternatives is chosen, the plant will still be sold at the end of the year. The amount of profit that could be

earned by selling the plant in a year depends on foreign market conditions, including the status of a trade embargo bill in Congress. The following payoff table describes this decision situation.

Decision	States of Nature	
	Good Foreign Competitive Conditions	Poor Foreign Competitive Conditions
Expand	\$800,000	\$500,000
Maintain status quo	1,300,000	-150,000
Sell now	320,000	320,000

Determine the best decision using each of the decision criteria.

1. Maximax
2. Maximin
3. Minimax regret
4. Hurwicz
5. Equal likelihood

SOLUTION:

1. Maximax

The decision is selected that will result in the maximum of the maximum payoffs. This is how this criterion derives its name--the maximum of the maxima. The **maximax criterion** is very optimistic. The decision maker assumes that the most favorable state of nature for each decision alternative will occur. Thus, for this example, the company would optimistically assume that good competitive conditions will prevail in the future, resulting in the following maximum payoffs and decisions:

Expand: \$ 800,000
 Status quo: 1,300,000 ← Maximum
 Sell: 320,000

Decision: Maintain status quo

2. Maximin

The **maximin criterion** is pessimistic. With the maximin criterion, the decision maker selects the decision that will reflect the *maximum* of the *minimum* payoffs. For each decision alternative, the decision maker assumes that the minimum payoff will occur; of these, the maximum is selected as follows:

Expand: \$ 500,000 ← Maximum
 Status quo: - 150,000
 Sell: 320,000

Decision: Expand

3. Minimax Regret

The decision maker attempts to avoid *regret* by selecting the decision alternative that minimizes the maximum regret. A decision maker first selects the maximum payoff under each state of nature; then all other payoffs under the respective states of nature are subtracted from these amounts, as follows:

Good Competitive Conditions	Poor Competitive Conditions
$\$1,300,000 - 800,000 = 50,000$	$\$500,000 - 500,000 = 0$
$1,300,000 - 1,300,000 = 0$	$500,000 - (-150,000) = 650,000$
$1,300,000 - 320,000 = 980,000$	$500,000 - 320,000 = 180,000$

These values represent the regret for each decision that would be experienced by the decision maker if a decision were made that resulted in less than the maximum payoff. The maximum regret for *each decision* must be determined, and the decision corresponding to the minimum of these regret values is selected as follows:

Regret Value

Expand: \$ 500,000 ← Minimum

Status quo: 650,000

Sell: 980,000

Decision: Expand

4. Hurwicz

A compromise between the maximax and maximin criteria. The decision maker is neither totally optimistic (as the maximax criterion assumes) nor totally pessimistic (as the maximin criterion assumes). With the **Hurwicz criterion**, the decision payoffs are weighted by a **coefficient of optimism**, a measure of the decision maker's optimism. The coefficient of optimism, defined as α , is between 0 and 1 (i.e., $0 < \alpha < 1.0$). If $\alpha = 1.0$, then the decision maker is completely optimistic, and if $\alpha = 0$, the decision maker is completely pessimistic. (Given this definition, $1 - \alpha$ is the *coefficient of pessimism*.) For each decision alternative, the maximum payoff is multiplied by α and the minimum payoff is multiplied by $1 - \alpha$. For our investment example, if α equals 0.3 (i.e., the company is slightly optimistic) and $1 - \alpha = 0.7$, the following decision will result:

Expand: \$ $800,000(0.3) + 500,000(0.7) = \$590,000$ ← Maximum

Status quo: $1,300,000(0.3) - 150,000(0.7) = 285,000$

Sell: $320,000(0.3) + 320,000(0.7) = 320,000$

Decision: Expand

5. Equal Likelihood

The **equal likelihood** (or **LaPlace**) **criterion** weights each state of nature equally, thus assuming that the states of nature are equally likely to occur. Since there are two states of nature in our example, we assign a weight of 0.50 to each one. Next, we multiply these weights by each payoff for each decision and select the alternative with the maximum of these weighted values.

Expand: \$ 800,000(0.50) + 500,000(0.50) = \$650,000 ← **Maximum**

Status quo: 1,300,000(0.50) – 150,000(0.50) = 575,000

Sell: 320,000(0.50) + 320,000(0.50) = 320,000

Decision: Expand

The decision to expand the plant was designated most often by four of the five decision criteria. The decision to sell was never indicated by any criterion. This is because the payoffs for expansion, under either set of future economic conditions, are always better than the payoffs for selling. Given any situation with these two alternatives, the decision to expand will always be made over the decision to sell. The sell decision alternative could have been eliminated from consideration under each of our criteria. The alternative of selling is said to be *dominated* by the alternative of expanding. In general, dominated decision alternatives can be removed from the payoff table and not considered when the various decision-making criteria are applied, which reduces the complexity of the decision analysis.

Different decision criteria often result in a mix of decisions. The criteria used and the resulting decisions depend on the decision maker. For example, the extremely optimistic decision maker might disregard the preceding results and make the decision to maintain the status quo, because the maximax criterion reflects his or her personal decision-making philosophy.

Decision Making with Probabilities

For the decision-making criteria we just used we assumed no available information regarding the probability of the states of nature. However, it is often possible for the decision maker to know enough about the future states of nature to assign probabilities that each will occur, which is decision making under conditions of *risk*. The most widely used decision-making criterion under risk is **expected value**, computed by multiplying each outcome by the probability of its occurrence and then summing these products according to the following formula:

$$EV(x) = \sum_{i=1}^n p(x_i) x_i$$

where

x_i = outcome i

$p(x_i)$ = probability of outcome i .

**EXAMPLE
S2.2****Expected Value**

Assume that it is now possible for the Southern Textile Company to estimate a probability of 0.70 that good foreign competitive conditions will exist and a probability of 0.30 that poor conditions will exist in the future. Determine the best decision using expected value.

SOLUTION:

The expected values for each decision alternative are computed as follows.

$$EV(\text{expand}) = \$ 800,000(0.70) + 500,000(0.30) = \$710,000$$

$$EV(\text{status quo}) = 1,300,000(0.70) - 150,000(0.30) = 865,000 \leftarrow \text{Maximum}$$

$$EV(\text{sell}) = 320,000(0.70) + 320,000(0.30) = 320,000$$

The decision according to this criterion is to maintain the status quo, since it has the highest expected value.

Expected Value of Perfect Information

Occasionally additional information is available, or can be purchased, regarding future events, enabling the decision maker to make a better decision. For example, a company could hire an economic forecaster to determine more accurately the economic conditions that will occur in the future. However, it would be foolish to pay more for this information than it stands to gain in extra profit from having the information. The information has some maximum value that is the limit of what the decision maker would be willing to spend. This value of information can be computed as an expected value--hence its name, the **expected value of perfect information (EVPI)**.

To compute the expected value of perfect information, first look at the decisions under each state of nature. If information that assured us which state of nature was going to occur (i.e., perfect information) could be obtained, the best decision for that state of nature could be selected. For example, in the textile company example, if the company executives knew for sure that good competitive conditions would prevail, they would maintain the status quo. If they knew for sure that poor competitive conditions will occur, then they would expand.

The probabilities of each state of nature (i.e., 0.70 and 0.30) indicate that good competitive conditions will prevail 70 percent of the time and poor competitive conditions will prevail 30 percent of the time (if this decision situation is repeated many times). In other words, even though perfect information enables the investor to make the right decision, each state of nature will occur only a certain portion of the time. Thus, each of the decision outcomes obtained using perfect information must be weighted by its respective probability:

$$\$1,300,000(0.70) - (500,000)(0.30) = \$1,060,000$$

The amount of \$1,060,000 is the expected value of the decision *given perfect information*, not the expected value of perfect information. The expected value of perfect information is the maximum amount that would be paid to gain information that would result in a decision better than the one made *without perfect information*. Recall that the expected-value decision without perfect information was to maintain status quo and the expected value was \$865,000.

The expected value of perfect information is computed by subtracting the expected value without perfect information from the expected value given perfect information:

$$\text{EVPI} = \text{expected value given perfect information} - \text{expected value without perfect information.}$$

For our example, the EVPI is computed as

$$\text{EVPI} = \$1,060,000 - 865,000 = \$195,000$$

The expected value of perfect information, \$195,000, is the maximum amount that the investor would pay to purchase perfect information from some other source, such as an economic forecaster. Of course, perfect information is rare and is usually unobtainable. Typically, the decision maker would be willing to pay some smaller amount, depending on how accurate (i.e., close to perfection) the information is believed to be.

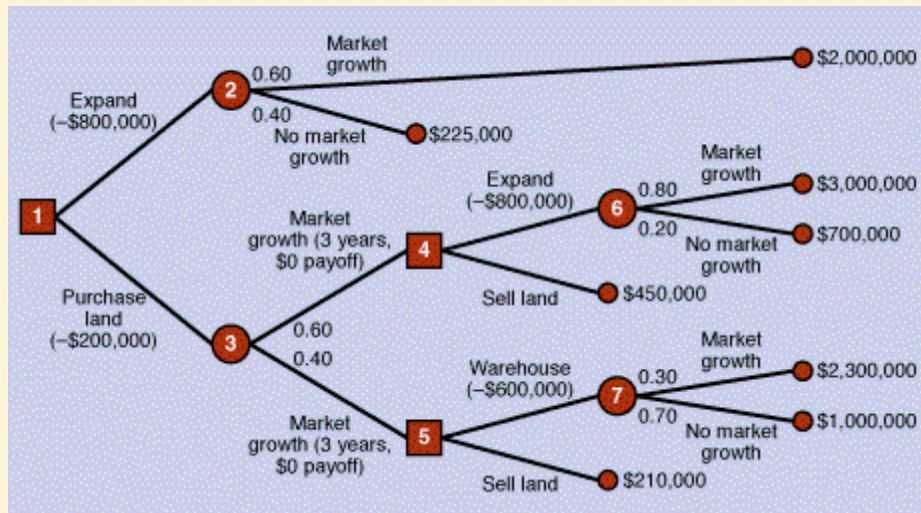
Sequential Decision Trees

A payoff table is limited to a single decision situation. If a decision requires a series of decisions, a payoff table cannot be created, and a **sequential decision tree** must be used. We demonstrate the use of a decision tree in the following example.

EXAMPLE S2.3

A Sequential Decision Tree

The Southern Textile Company is considering two alternatives: to expand its existing production operation to manufacture a new line of lightweight material; or to purchase land to construct a new facility on in the future. Each of these decisions has outcomes based on product market growth in the future that result in another set of decisions (during a ten-year planning horizon), as shown in the following figure of a sequential decision tree. In this figure the square nodes represent decisions and the circle nodes reflect different states of nature and their probabilities.



The first decision facing the company is whether to expand or buy land. If the company expands, two states of nature are possible. Either the market will grow (with a probability of 0.60) or it will not grow (with a probability of 0.40). Either state of nature will result in a payoff. On the other hand, if the company chooses to purchase land, three years in the future another decision will have to be made regarding the development of the land.

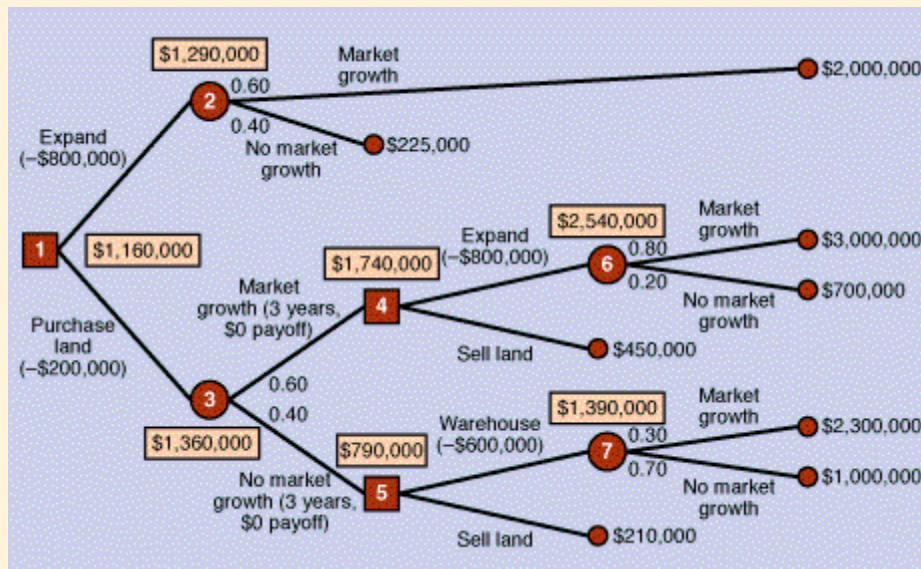
At decision node 1, the decision choices are to expand or to purchase land. Notice that the costs of the ventures (\$800,000 and \$200,000, respectively) are shown in parentheses. If the plant is expanded, two states of nature are possible at probability node 2: The market will grow, with a probability of 0.60, or it will not grow or will decline, with a probability of 0.40. If the market grows, the company will achieve a payoff of \$2,000,000 over a ten-year period. However, if no growth occurs, a payoff of only \$225,000 will result.

If the decision is to purchase land, two states of nature are possible at probability node 3. These two states of nature and their probabilities are identical to those at node 2; however, the payoffs are different. If market growth occurs for a three-year period, no payoff will occur, but the company will make another decision at node 4 regarding development of the land. At that point, either the plant will be expanded at a cost of \$800,000 or the land will be sold, with a payoff of \$450,000. The decision situation at node 4 can occur only if market growth occurs first. If no market growth occurs at node 3, there is no payoff, and another decision situation becomes necessary at node 5: A warehouse can be constructed at a cost of \$600,000 or the land can be sold for \$210,000. (Notice that the sale of the land results in less profit if there is no market growth than if there is growth.)

If the decision at decision node 4 is to expand, two states of nature are possible: The market may grow, with a probability of 0.80, or it may not grow, with a probability of 0.20. The probability of market growth is higher (and the probability of no growth is lower) than before because there has already been growth for the first three years, as shown by the branch from node 3 to node 4. The payoffs for these two states of nature at the end of the ten-year period are \$3,000,000 and \$700,000.

If the company decides to build a warehouse at node 5, then two states of nature can occur: Market growth can occur, with a probability of 0.30 and an eventual payoff, of \$2,300,000, or no growth can occur, with a probability of 0.70 and a payoff of \$1,000,000. The probability of market growth is low (i.e., 0.30) because there has already been no market growth, as shown by the branch from node 3 to node 5.

SOLUTION:



We start the decision analysis process at the end of the decision tree and work backward toward a decision at node 1.

First, we must compute the expected values at nodes 6 and 7:

$$EV(\text{node 6}) = 0.80(\$3,000,000) + 0.20(\$700,000) = \$2,540,000$$

$$EV(\text{node 7}) = 0.30(\$2,300,000) + 0.70(\$1,000,000) = \$1,390,000$$

These expected values (as well as all other nodal values) are shown in boxes in the preceding figure.

At decision nodes 4 and 5, a decision must be made. As with a normal payoff table, the decision is made that results in the greatest expected value. At node 4 the choice is between two values: \$1,740,000, the value derived by subtracting the cost of expanding (\$800,000) from the expected payoff of \$2,540,000, and \$450,000, the expected value of selling the land computed with a probability of 1.0. The decision is to expand, and the value at node 4 is \$1,740,000.

The same process is repeated at node 5. The decisions at node 5 result in payoffs of \$790,000 (i.e., \$1,390,000 – 600,000 = \$790,000) and \$210,000. Since the value \$790,000 is higher, the decision is to build a warehouse.

Next the expected values at nodes 2 and 3 are computed:

$$EV(\text{node 2}) = 0.60(\$2,030,030) + 0.40(\$225,000) = \$1,290,000$$

$$EV(\text{node 3}) = 0.60(\$1,740,000) + 0.40(\$790,000) = \$1,360,000$$

(Note that the expected value for node 3 is computed from the decision values previously determined at nodes 4 and 5.)

Now the final decision at node 1 must be made. As before, we select the decision with the greatest expected value after the cost of each decision is subtracted.

$$\text{Expand: } \$1,290,000 - 800,000 = \$490,000$$

$$\text{Land: } \$1,360,000 - 200,000 = \$1,160,000$$

Since the highest *net* expected value is \$1,160,000, the decision is to purchase land, and the payoff of the decision is \$1,160,000.

Decision trees allow the decision maker to see the logic of decision making by providing a picture of the decision process. Decision trees can be used for problems more complex than this example without too much difficulty.

Decision Analysis with POM for Windows, Excel, and Excel OM

Throughout this text we will demonstrate how to solve quantitative models using the computer with POM for Windows, a software package by Howard J. Weiss published by Prentice Hall, Excel, the Microsoft spreadsheet package, and Excel OM, a spreadsheet "add-in," also by Howard J. Weiss published by Prentice Hall.

POM for Windows is a user-friendly, menu-driven package and requires little instruction. POM for Windows can solve the decision analysis problems in Examples S2.1 and S2.2, as well as the decision tree problem in Example S2.3. Following in Exhibit S2.1 and Exhibit S2.2 are the solution output screens for Examples S2.1 and S2.2. The first screen includes the maximax, minimax, Hurwicz, and expected value solutions. The second screen shows the expected value of perfect information of our example.

POH for Windows

File Edit View Module Format Tools Window Help

100%

And 8.2 B I U

Objective:

 Profits (maximize)

 Costs (minimize)

Hurwicz/Alpha: 0.3

Instruction: There are more results available in additional windows. These may be opened by using the WINDOW option in the Main Menu.

Decision Table Results

Examples S2.1 and S2.2: Decision Analysis Solution

	State 1	State 2	EMV	Row Min	Row Max	Hurwicz
Probabilities	0.7	0.3				
Expand	800,000.	500,000.	710,000.	500,000.	800,000.	590,000.
Maintain status quo	1,300,000.	-150,000.	865,000.	-150,000.	1,300,000.	285,000.
Sell now	320,000.	320,000.	320,000.	320,000.	320,000.	320,000.
		maximum	865,000.	500,000.	1,300,000.	590,000.
			Best EV	maximin	maximax	Best

The maximum expected monetary value is 865,000 given by Maintain status quo
 The maximin is 500,000 given by Expand
 The maximax is 1,300,000 given by Maintain status quo

EXHIBIT S2.1

Perfect Information

Examples S2.1 and S2.2: Decision Analysis Solution

	State 1	State 2	Maximum
Probabilities	0.7	0.3	
Expand	800,000.	500,000.	
Maintain status quo	1,300,000.	-150,000.	
Sell now	320,000.	320,000.	
Perfect Information	1,300,000.	500,000.	
Perfect*probability	910,000.	150,000.	1,060,000.
Best Expected Value			865,000.
Exp Value of Perfect Info			195,000.

EXHIBIT S2.2

We will also be using Excel spreadsheets to demonstrate how to solve quantitative operational problems throughout this text. In Exhibit S2.3, the Excel worksheet screen for the determination of the expected values in Example S2.2 is shown. Note that the expected values contained in cells G6, G7, and G8 were computed using the expected value formulas embedded in these cells. For example, the formula for cell G6 is shown on the formula bar on the Excel screen.

Formula for expected value computed in cell G6

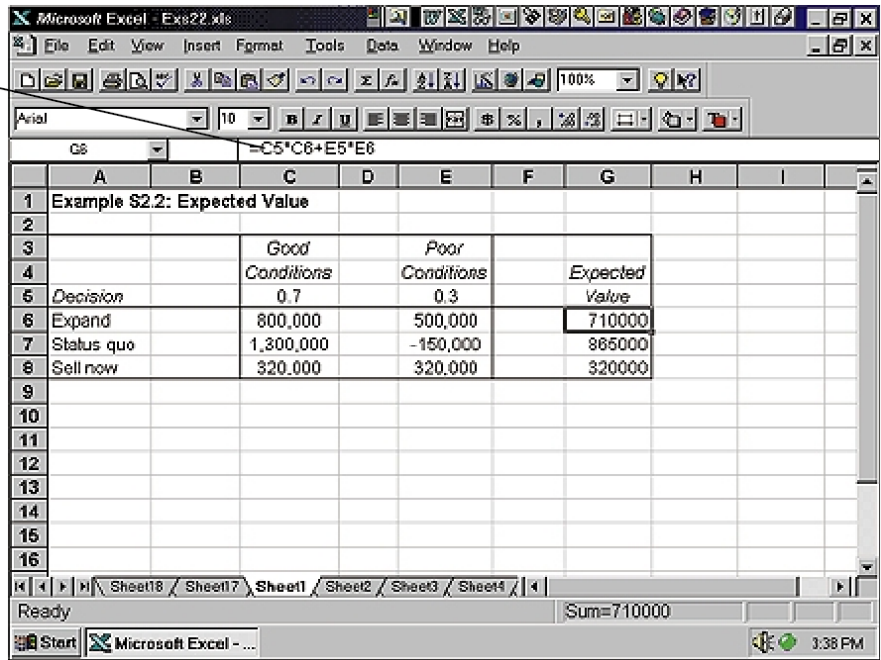


EXHIBIT S2.3

Excel OM is a spreadsheet add-in that includes a number of "macros" for different problem types. These modules provide spreadsheet setups or templates that include all the necessary formulas necessary to solve a particular type of problem. This relieves the user of having to set up and format the spreadsheet with formulas and headings.

Excel OM is normally accessed from the computer's program files after it is loaded. When Excel OM is activated, "OM" will be displayed at the top of the spreadsheet (as indicated in Exhibit S2.4). Clicking on OM will pull down a menu of modules, one of which is "Decision Analysis." Clicking on Decision Analysis will result in a window for spreadsheet initialization where you can enter the problem title and parameters--in this case the number of decision alternatives and states of nature. Clicking on "OK" will result in the spreadsheet shown in Exhibit S2.4. The spreadsheet will initially have example data values in cells B8:C11. Thus, the first step is to replace the existing data values with the values for our Southern Textile Company Example S2.2, which results in the solution values shown in Exhibit S2.4.

Click on "OM" to access Decision Analysis macro.

Enter problem parameters in cells B8:C11.

The screenshot shows an Excel spreadsheet titled "Examples S2.1 and S2.2". It contains several tables and calculations related to decision analysis. A tooltip is visible over the main data table, stating: "Enter the profits or costs in the main body of the data table. Enter probabilities in the first row if you want to compute the expected value."

Outs	Good	Poor	EMV	Minimum	Maximum
Expand	80000	50000	71000	50000	80000
Maintain status quo	130000	-150000	85000	-150000	130000
Sell now	32000	32000	32000	32000	32000
			Maximum	85000	130000

Column best	Expected Value of Perfect Information	Expected Value
Expand	130000	50000
Maintain status quo		105000
Sell now		85000
		135000

Regret	Good	Poor	Expected	Maximum
Expand	50000	0	35000	50000
Maintain status quo	0	85000	195000	85000
Sell now	88000	18000	74000	98000
			Minimum	195000

EXHIBIT S2.4

Summary

In this supplement we have provided a general overview of decision analysis. To a limited extent we have also shown the logic of such operational decisions throughout the organization are interrelated to achieve strategic goals.

Key Formulas

Expected Value

$$EV(x) = \sum_{i=1}^n p(x_i) x_i$$

Expected Value of Perfect Information

EVPI = expected value given perfect information – expected value without perfect information.

Case Problème

CASE PROBLEM 2S.1

Transformer Replacement at Mountain States Electric Service

Mountain States Electric Service is an electrical utility company serving several states in the Rocky Mountain region. It is considering replacing some of its equipment at a generating substation and is attempting to decide whether it should replace an older, existing PCB transformer. (PCB is a toxic chemical known formally as polychlorinated biphenyl.) Even though the PCB generator meets all current regulations, if an incident occurred, such as a fire, and PCB contamination caused harm either to neighboring businesses or farms or to the environment, the company would be liable for damages. Recent court cases have shown that simply meeting utility regulations does not relieve a utility of liability if an incident causes harm to others. Also, courts have been awarding large damages to individuals and businesses harmed by hazardous incidents.

If the utility replaces the PCB transformer, no PCB incidents will occur, and the only cost will be that of the transformer, \$85,000. Alternatively, if the company decides to keep the existing PCB transformer, then management estimates there is a 50-50 chance of there being a high likelihood of an incident or a low likelihood of an incident. For the case in which there is a high likelihood that an incident will occur, there is a .004 probability that a fire will occur sometime during the remaining life of the transformer and a .996 probability that no fire will occur. If a fire occurs, there is a .20 probability that it will be bad and the utility will incur a very high cost of approximately \$90 million for the cleanup, whereas there is a .80 probability that the fire will be minor and a cleanup can be accomplished at a low cost of approximately \$8 million. If no fire occurs, then no cleanup costs will occur. For the case in which there is a low likelihood of an incident occurring, there is a .001 probability that a fire will occur during the life of the existing transformer and a .999 probability that a fire will not occur. If a fire does occur, then the same probabilities exist for the incidence of high and low cleanup costs, as well as the same cleanup costs, as indicated for the previous case. Similarly, if no fire occurs, there is no cleanup cost.

Perform a decision tree analysis of this problem for Mountain States Electric Service and indicate the recommended solution. Is this the decision you believe the company should make? Explain your reasons.¹

¹ This case was adapted from W. Balson, J. Welsh, and D. Wilson, "Using Decision Analysis and Risk Analysis to Manage Utility Environmental Risk," *Interfaces* 22, no. 6 (November-December 1992): 126-39.

References

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