## 2.23 Příklady

(1) $X_t(\omega) = A(\omega)\cos\theta t + B(\omega)\sin\theta t$ , $\theta \in [-\pi,\pi]$
$\mathrm{cov}(A,B)=0$, $EA = EB = 0$ , $\sigma_A^2 = \sigma_B^2 = 1$
je stacionární

D.: $\mathrm{Cov}(X_{t+h}, X_t) = \mathrm{Cov}\left((\cos\theta(t+h), \sin\theta(t+h))\begin{bmatrix}A\\B\end{bmatrix}, (\cos\theta t, \sin\theta t)\begin{bmatrix}A\\B\end{bmatrix}\right)$

$= (\cos\theta(t+h), \sin\theta(t+h))\underbrace{\mathrm{cov}\left(\begin{bmatrix}A\\B\end{bmatrix}, \begin{bmatrix}A\\B\end{bmatrix}\right)}_{\mathrm{var}\left(\begin{bmatrix}A\\B\end{bmatrix}\right) = I_2}\begin{pmatrix}\cos\theta t\\ \sin\theta t\end{pmatrix} =$

$= \cos\theta(t+h)\cdot\cos\theta t + \sin\theta(t+h)\sin\theta t = \cos[\theta(t+h) - \theta t] = \cos\theta h$

nezávisí na $t$. $EX_t = \cos\theta t \underbrace{EA}_{0} + \sin\theta t \underbrace{EB}_{0} = 0$

(2) $Z_t \sim IID(0, \sigma^2)$, pak $X_t = Z_t + \theta Z_{t-1}$, $t \in \mathbb{Z}$, $\theta \in \mathbb{R}$ je stacionární

D.: $EX_t = \underbrace{EZ_t}_{0} + \theta \underbrace{EZ_{t-1}}_{0} = 0$

$\mathrm{cov}(X_{t+h}, X_t) = \mathrm{cov}(Z_{t+h} + \theta Z_{t+h-1}, Z_t + \theta Z_{t-1})$

$$= \begin{cases} (1+\theta^2)\sigma^2 & \text{pro } h=0 \\ \theta\sigma^2 & \text{pro } h=\pm 1 \\ 0 & \text{pro } |h|>1 \end{cases}$$

(3) $X_t = \begin{cases} Y_t & \text{pro } t \text{ sudé} \\ Y_t + 1 & \text{pro } t \text{ liché} \end{cases}$, $t \in \mathbb{Z}$, $Y_t \dots$ stacionární

Pak $X_t$ není stacionární, ale je kovariančně stacionární:
$\mathrm{Cov}(X_{t+h}, X_t) = \mathrm{Cov}(Y_{t+h} + j, Y_t + k) = \mathrm{Cov}(Y_{t+h}, Y_t) = \gamma_Y(h)$  $j,k \in \{0,1\}$
$EX_t = EY_t = \mu_Y$ pro $t$ sudé,
$= 1 + EY_t = 1 + \mu_Y$ pro $t$ liché

(4) Uvažujme náhodnou procházku z př. 2.6(3):
$S_0 = 0$, $S_t = X_1 + \dots + X_t$, $t \in \mathbb{N}$ ; $X_t \sim IID(0, \sigma^2)$
Protože: $\mathrm{Cov}(S_{t+h}, S_t) = \mathrm{Cov}\left(\sum_{i=1}^{t+h} X_i, \sum_{j=1}^{t} X_j\right) = \mathrm{Cov}\left(\sum_{j=1}^{t} X_j + \sum_{j=t+1}^{t+h} X_i, \sum_{j=1}^{t} X_j\right) =$

$= \mathrm{Cov}\left(\sum_{j=1}^{t} X_j, \sum_{j=1}^{t} X_j\right) + \mathrm{cov}\left(\sum_{i=t+1} X_i, \sum_{j=1}^{t} X_j\right) = \sum_{i,j=1}^{t} \mathrm{cov}(X_i, X_j) = \sum_{i=1}^{t} \underbrace{\mathrm{cov}(X_i, X_i)}_{\mathrm{var}\, X_i} =$

$\underbrace{\sum_{i=t+1}^{t+h} \sum_{j=1}^{t} \mathrm{cov}(X_i, X_j)}_{0}$ $= t\sigma^2$

Tedy $S_t$ není ani kovariančně stacionární