2. Box-Jenkins Methodology - model overview: $\{\underline{X}_t\}_{t\in\mathbb{Z}}$

$m\times 1$, m = dimension

① Models for stationary time series

$\{\underline{X}_t\} \sim MA(q),\ q \le \infty$: $\underline{X}_t = \underline{Z}_t + \Psi_1 \underline{Z}_{t-1} + \dots = \sum_{j=0}^{q} \Psi_j \underline{Z}_{t-j},\ \Psi_0 = I_m$

approximation (no feedback) ... exact

$\{\underline{X}_t\} \sim ARMA(p,q),\ 0 \le p,q < \infty$: $\underline{X}_t = \underbrace{\sum_{j=1}^{p} \Phi_j \underline{X}_{t-j}}_{AR(p)\text{-comp.}} + \underbrace{\underline{Z}_t + \sum_{j=1}^{q} \Theta_j \underline{Z}_{t-j}}_{MA(q)\text{-component}}$

Autoregressive Moving Average Model

Auto Regression = feedback; Moving Average

Special cases:

$\{\underline{X}_t\} \sim MA(q) = ARMA(0,q),\ 0 \le q < \infty$: $\underline{X}_t = \underline{Z}_t + \sum_{j=1}^{q} \Theta_j \underline{Z}_{t-j}$

Moving Average model

$\{\underline{X}_t\} \sim AR(p) = ARMA(p,0),\ 0 \le p < \infty$: $\underline{X}_t = \sum_{j=1}^{p} \Phi_j \underline{X}_{t-j} + \underline{Z}_t$

Autoregressive model

$\{\underline{X}_t\} \sim WN(\underline{0}, \Sigma) = AR(0) = MA(0) = ARMA(0,0)$: $\underline{X}_t = \underline{Z}_t$

White noise

② Models for covariance stationary time series

$\mu_X \ne$ const ... stationarity defects in the mean $\mu_X(t) := E\underline{X}_t$.

a) $\mu_X(t)$ = piecewise (even randomly) polynomial trend

$\{\underline{X}_t\} \sim ARIMA(p,d,q) \equiv \{\Delta^d \underline{X}_t\} \sim ARMA(p,q)$; $\Delta \underline{X}_t := \underline{X}_t - \underline{X}_{t-1}$

$0 \le p,d,q < \infty$ — differencing removes trend

b) $\mu_X(t)$ = trend as above + seasonal (periodic) component with (even random) period $s>1$ and piecewise (even randomly) polynomial trend in amplitudes

$\{\underline{X}_t\} \sim SARIMA(p,d,q,P,D,Q,s) \equiv \{\Delta^d \Delta_s^D \underline{X}_t\} \sim ARMA(p,q) + ARMA(P,Q)$ for $\{\underline{X}_{t+\tau s}\}_\tau$

$\Delta_s \underline{X}_t := \underline{X}_t - \underline{X}_{t-s}$; Δ_s^D ... removes seasonal amplitude trend and the seasonal component itself

Δ^d ... removes residual trend as in a)

(S)ARIMA = (Seasonal) Autoregressive Integrated Moving Average M.

③ Models with external inputs (X) - see [Lju87]

e.g. ARX, ARMAX — autoregression (feedback) at X_t

ARARX, ARARMAX — autoregression at X_t and E_t