MATH 164, Spring 2001 **Due Date:**____________ **Name(s):** ________________________

Honors Project 10: An Epidemic

We assume an epidemic begins when a small number of infected individuals carrying a disease are introduced into a community, that no one in the community is immune to the disease, and that the disease is spread by direct contact of an infected individual with a non-infected, or susceptible, one. We assume, in fact that during the epidemic there are three basic types of individual: those who are susceptible to the disease (of which there are $S = S(t)$), those who are currently infected (of which there are $I = I(t)$), and those who are post-infected — those who are neither susceptible nor infected (of which there are $P = P(t)$). As the epidemic runs its course, susceptible individuals catch the disease making them infected, and infected individuals eventually become post-infected. Thus individuals move from one group to the other:

$$S \Rightarrow I \Rightarrow P,$$

and as long as the epidemic is of a relatively short duration,

$$S + I + P = S(t) + I(t) + P(t) = N$$

where N is a constant.

Three differential equations govern the transition of individuals from one group to the other:

$$\frac{dS}{dt} = -k_1 SI, \qquad \frac{dI}{dt} = k_1 SI - k_2 I, \qquad \frac{dP}{dt} = k_2 I$$

The first equation states that the rate at which the number of susceptible individuals decreases is proportional to the number SI of contacts between susceptible individuals and infected ones. The second equation says that the rate at which the number of infected individuals changes increases because of susceptible individuals becoming infected, and decreases because of infected individuals becoming post-infected. If the rate at which infected individuals becomes post-infected is proportional to the number of infected individuals, then the number of infectives decreases by the same amount the number of post-infectives increases; this is the third equation.

Tasks

1. Using the fact that
$$\frac{dS}{dP} = \frac{dS/dt}{dP/dt} = \frac{-k_1 SI}{k_2 I} = -\frac{k_1}{k_2} S$$
express S as a function of P.

2. Using the fact that
$$\frac{dI}{dP} = \frac{dI/dt}{dS/dt} = \frac{k_1 SI - k_2 I}{k_2 I} = \frac{k_1}{k_2} S - 1$$
and your solution to part (1), express I as a function of P.

3. How would an epidemiologist determine k_1 and k_2 in practice?

4. Since the epidemic will "be over" when there is no further growth in the number of post-infectives, the epidemic will be over when $dP/dt = k_2 I = 0$ or $I = 0$. Using your solution to part (2), find P when the epidemic is over — the *extent* of the epidemic — assuming that $k_1 = 10^{-6}$, $k_2 = 0.98$, and $N = 10^6$.

Comments

1. Observe that although we could not solve the three differential equations that describe the epidemic in closed form, we could use the chain rule
$$\frac{dy}{dx} = \frac{dy/dt}{dx/dt}$$
to solve for S and I in terms of P, and that this led to our computation of a useful quantity: the extent of the epidemic.

2. Also observe that the same type of model we used in this project to study the spread of an epidemic could be adapted to the study of the spread of information, a topic of concern in a field such as advertising.