MATH 261, Spring 2001 **Due Date:**_____ **Name(s):** _____

Extra Project 15.2a: Limits and Continuity

Objective

If you have not already done so, read Section 15.2 of the text. In this project we investigate why limits of functions of two variables may fail to exist.

Narrative

It may not be easy to see why $\lim_{(x,y)\to(a,b)} f(x,y)$ fails to exist even though $f(x,y)$ exists for every value of (x,y) close to (a,b), but in some cases Maple can help.

Task

1. Type the command lines below into Maple in the order in which they are listed. They draw the graph of a function $f(x,y)$ for which $\lim_{(x,y)\to(0,0)} f(x,y)$ fails to exist — since $\lim_{x\to 0} f(x,0) \neq \lim_{y\to 0} f(0,y)$ — even though $f(x,y)$ exists for every value of (x,y) close to $(0,0)$.

```
> # Project 15.2a:  Limits and Continuity
> restart:  with(plots);
> # Task 1
> f := (x,y) -> (2*x^2+y^2)/(x^2+y^2);
> plot3d(f(x,y),x=-10..10,y=-10..10);
```

2. Continue by typing the command lines below into Maple in the order in which they are listed. They draw the graph of a function $g(x,y)$ for which $\lim_{(x,y)\to(0,0)} g(x,y) = 0$ when approaching the origin along any line that passes through the origin, but $\lim_{(x,y)\to(0,0)} g(x,y)$ fails to exist since, if we approach the origin along the curve $x = t^3, y = t$, we find that $\lim_{(x,y)\to(0,0)} g(x,y) = 0.5$.

```
> # Task 2
> g := (x,y) -> x*y^3/(x^2+y^6);
> plot3d(g(x,y),x=-10..10,y=-10..10);
```

   At this time, make a hard-copy of your typed input and Maple's responses. Then, ...

3. On the basis of the graphic you drew in Task 1, explain (in words) why, $\lim_{(x,y)\to(0,0)} f(x,y)$ fails to exist even though $f(x,y)$ exists for every value of (x,y) close to $(0,0)$.

4. On the basis of the graphic you drew in Task 2, explain (in words) why, $\lim_{(x,y)\to(0,0)} g(x,y)$ fails to exist even though $g(x,y)$ exists for every value of (x,y) close to $(0,0)$.

   (You may need to move around the graphs you created in Tasks 1 and 2 to do Tasks 3 and 4. You may also want to zoom-in on the origin by changing the ranges of x and y.)