

Extra Project 6.5: The Average Value of a Function**Objective**

To investigate the average value of a function.

Narrative

By definition, the average value of the function $f = f(x)$ over the interval $[a, b]$ is

$$\frac{1}{b-a} \int_{x=a}^b f(x) dx.$$

Task

a) Type the command lines below into Maple in the order in which they are listed. These command lines compute the average value of $f(x) = 18x - x^3$ over the interval $[1, 3]$.

```
> # Project 6.5: The Average Value of a Function
> restart;
> with(plots): with(plottools):
> f := x -> 18*x-x^3;
> a := 1.0; b := 3.0;
> plot0 := plot(f(x),x=a..b):
> display(plot0);
> Av_f := (1/(b-a))*evalf(int(f(x),x=a..b));
> plot1 := rectangle([a,0],[b,Av_f],color=green):
> display({plot0,plot1});
```

At this point, make a hard-copy of your typed input and Maple's responses. Then, ...

b) There is a number $c \in [a, b]$ for which $f(c) = \text{Av}_f$; what is c ? (Estimate c from the graphic you produced in part (a).)

c) Does such a c always exist? (That is, if f is any function defined on a closed interval $[a, b]$, is there always a number $c \in [a, b]$ for which $f(c) = \text{Av}_f$?) Justify your answer.