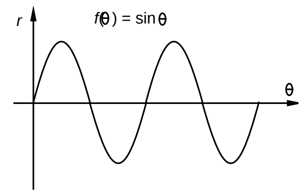
MATH 164, Spring 2001 Due Date:____________ Name(s): ________________________

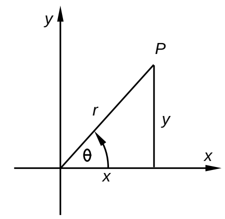
Honors Project 11: Rectangular vs. Polar $r\theta$-Coordinates

Objective

In trigonometry, you studied *rectangular* $r\theta$-coordinates. These are the coordinates with respect to which you drew graphs of the trig functions, such as the one to the right of $f(\theta) = \sin\theta$.



In calculus, we study *polar* $r\theta$-coordinates. These are defined by the scheme illustrated by the figure to the right.



These are different coordinate systems. In this project we investigate the relationship between them.

Narrative

Rectangular and polar $r\theta$-coordinates are related by a transformation (a topic to be discussed in detail in later calculus courses). A *transformation* $T : R^2 \to R^2$ is a mapping from one copy of R^2 to another copy of R^2 which may be written — at least in *our* case — in any of the following ways:

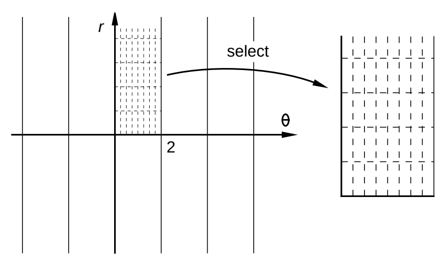
$$T := (r, \theta) \to (x, y) = (r\cos\theta, r\sin\theta)$$

$$T(r, \theta) = (x, y) = (r\cos\theta, r\sin\theta)$$

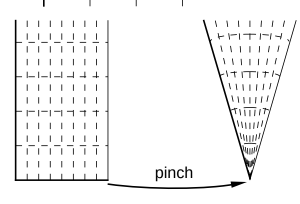
$$x = r\cos\theta \quad y = r\sin\theta$$

In this project, we discuss the geometry of this transformation.

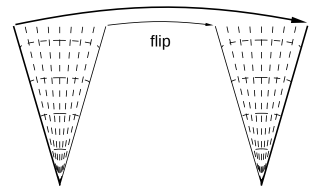
You can think of a transformation of the plane as a function that distorts the plane. The specific transformation T described above can be described by first selecting a semi-infinite strip 2π units wide,



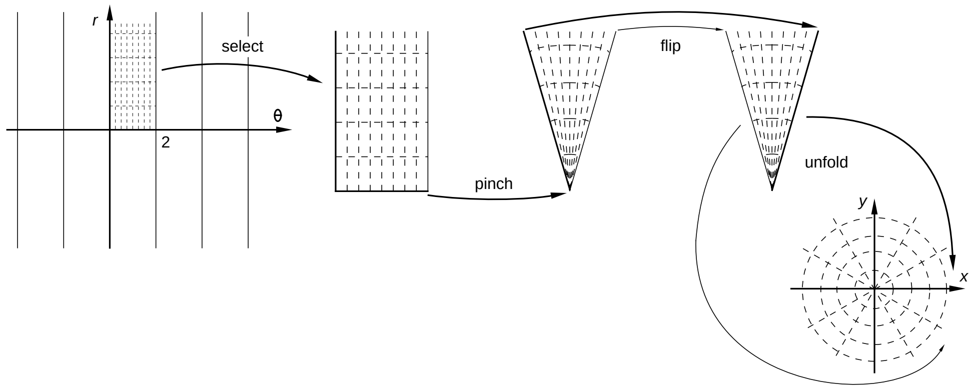
and then pinching the interval on the θ-axis to a point.



Next, flip the resulting "fan"-shaped region over.



Finally, unfold the resulting “fan”.



Note how the vertical lines in the rectangular $r\theta$-coordinate plane — lines whose equations are of the form $\theta = \theta_0$ — are transformed by T into rays that emanate from the origin, and how the horizontal lines in the rectangular $r\theta$-coordinate plane — lines whose equations are of the form $r = r_0$ — are transformed by T into circles whose centers are the origin.

Task

To test your understanding of this transformation, see if you can understand why the graphs of $r = a \sin n\theta$ and $r = a \cos n\theta$ in rectangular $r\theta$-coordinates (sine and cosine waves) are transformed into roses that have $2n$-leaves, as θ goes from 0 to 2π (only n of which are visible if n is odd). (*Hint*: How is the graph of $r = \sin 3\theta$ transformed by T? How about the graph of $r = \sin 3\theta$? To get the complete picture, you must use the fact that every semi-infinite strip 2π units wide in the rectangular $r\theta$-coordinate plane can be transformed by T.)

Present your work in the form of an essay supported by computer- and/or hand-drawn graphics.