

### Extra Project 15.3: Partial Derivatives

#### Objective

In this project we discuss the computation and geometry of partial derivatives.

#### Narrative

If you have not already done so, read Section 15.3 in the text. In this project we discuss the computation and geometry of partial derivatives.

#### Tasks

a) Type the command lines below into Maple; they produce a plot of the graph of  $f(x, y) = -5x/(x^2 + y^2 + 1)$ .

```
> # Project 15.3: Partial Derivatives
> restart: with(plots): with(plottools):
> setoptions3d(axes=normal,orientation=[20,80],grid=[50,50],lightmodel=light1,
    scaling=constrained,style=patchnogrid,shading=zgrayscale);
> f := (x,y) -> (24-x^2-2*y^2)/8;
> plot0 := plot3d(f(x,y),x=-3..3,y=-3..3,color=green): display(plot0);
```

b) Continue by typing the command lines below into Maple; they compute the value  $\text{val}$  of  $f$  at  $P(a, b)$ .

```
> a := 2; b := 2.5;
> val := subs({x=a,y=b},f(x,y));
```

c) Continue by typing the command lines below into Maple; they plot the graph of  $f$  over a slightly smaller domain, plot the  $x$ -curve of  $f$  through  $P(a, b)$  (in blue), compute  $f_x$  and  $f_x(a, b)$ , and then draw the tangent line to the  $x$ -curve of  $f$  through  $P(a, b)$  (in red). (The curve of intersection of the graph of  $z = f(x, y)$  and a plane whose equation is of the form  $x = a$  is called an  $x$ -curve, and the curve of intersection of the graph of  $z = f(x, y)$  and a plane whose equation is of the form  $y = b$  is called an  $y$ -curve.)

```
> plot1 := plot3d(f(x,y),x=-3..3,y=-3..b,color=green):
> curve1 := spacecurve([t,b,f(t,b),t=-3..3],color=blue,thickness=2):
> f1 := (x,y) -> diff(f(x,y),x);
> slope1 := subs({x=a,y=b},f1(x,y));
> tanline1 := spacecurve([t+a,b,slope1*t+val,t=-3..3],color=red,thickness=2):
> display({plot1,curve1,tanline1});
```

d) Continue by typing the command lines below into Maple; they again plot the graph of  $f$  over a slightly smaller domain, plot the  $y$ -curve of  $f$  through  $P(a, b)$  (in blue), compute  $f_y$  and  $f_y(a, b)$ , and then draw the tangent line to the  $y$ -curve of  $f$  through  $P(a, b)$  (in red).

```
> plot2 := plot3d(f(x,y),x=-3..a,y=-3..3,color=green):
> curve2 := spacecurve([a,t,f(a,t),t=-3..3],color=blue,thickness=2):
> f2 := (x,y) -> diff(f(x,y),y);
> slope2 := subs({x=a,y=b},f2(x,y));
> tanline2 := spacecurve([a,t+b,slope2*t+val,t=-3..3],color=red,thickness=2):
> display({plot2,curve2,tanline2});
```

e) Finally, type the command line below into Maple; it plots both the  $x$ -curve and its tangent line at  $P(a, b)$ , and the  $y$ -curve and its tangent line at  $P(a, b)$ . Adjust the graphic to get a good view.

```
> display({plot0,curve1,tanline1,curve2,tanline2});
```

At this point, make a hard copy of your typed input and Maple's responses. Then, ...

f) By hand, label each  $x$ -curve on the graphics you created as " $x$ -curve", the tangent to each  $x$ -curve as "tangent to  $x$ -curve", each  $y$ -curve on the graphics you created as " $y$ -curve", and the tangent to each  $y$ -curve as "tangent to  $y$ -curve".