Extra Project 15.8d: Lagrange Multipliers

Objective

The objective of this project is to illustrate how you can use Maple to check your work and answers to your Lagrange Multipliers homework.

Narrative

If you have not already done so, read Section 15.8 in the text.

One of the toughest aspects of Lagrange Multipliers involves solving the systems of equations that the technique leads to. In this project we illustrate how you can use Maple to check your work and answers to your Lagrange Multiplier homework. We work with a specific example: To find the extrema of $f(x, y) = x^2 + y^2$ subject to the constraint (along the curve C whose equation is) $x^4 + y^4 = 1$, we must solve the equation

$$\nabla f = \lambda \nabla g \quad \text{or} \quad \langle 2x, 2y \rangle = \lambda \langle 4x^3, 4y^2 \rangle$$

assuming $x^4 + y^4 = 1$. That is, we must solve the system of 3 nonlinear equations in 3 unknowns:

$$2x = 4\lambda x^3, \quad 2y = 4\lambda y^3, \quad x^4 + y^4 = 1.$$

Task

1. a) Type the following command lines into Maple in the order in which they are listed. They illustrate how Maple can be used to solve the above system of equations.

```
> # Project 15.8d:  Lagrange Multipliers
> restart:
> f := (x,y) -> x^2+y^2;
> g := (x,y) -> x^4+y^4-1;
> eq1 := diff(f(x,y),x)=lambda*diff(g(x,y),x);
> eq2 := diff(f(x,y),y)=lambda*diff(g(x,y),y);
> eq3 := g(x,y)=0;
> solution := solve({eq1,eq2,eq3},{x,y,lambda});    (See the discussion below.)
> f(1,0);
> f(-1,0);
> allvalues(solution[3]);
```

Some observations: The first few lines of code define f and g as well as the equations to be solved. In this example we are using only one constraint for demonstration purposes, but our code would be easy to modify if we were using two or more constraints. Then we ask Maple to solve our system; this is where the magic occurs. It would be nice if we could ask Maple to show its work, but unfortunately that is not possible. But we should be happy we're at least getting the answer: while this problem might have looked a little innocuous at the beginning, it has 8 solutions! In typing `f(1,0);` and `f(-1,0);` we're checking the values of f at the first two solutions to determine what the extrema of f are. When we get to the third solution we find that unknowns are expressed as solutions to equations; by typing `allvalues(solution[3]);` we express all the unknowns of the third solution numerically so we can evaluate f at the third solution. Since we find, however, that one of the unknowns is imaginary (Maple uses I to denote $\sqrt{-1}$), we do not need to evaluate f at the third solution.

b) Evaluate f at the relaining five solutions, and state and classify (on the basis of all the information you have obtained) the absolute maximum and absolute minimum values of f.

2. Do Exercises 3-13, 15-19 on page 990 of the text using Maple.