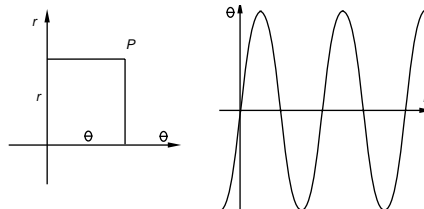## Extra Project 16.9b: Transformation from Rectangular to Polar Coordinates

Objective

In this project we illustrate the geometry of the transformation from rectangular $r\theta$-coordinates to polar $r\theta$-coordinates.
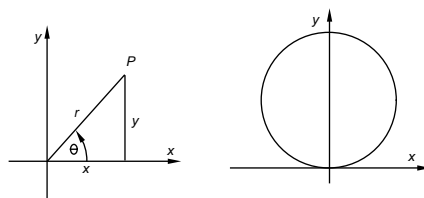
Narrative

In trigonometry we studied rectangular $r\theta$-coordinates, coordinates with respect to which we drew graphs of the trig functions such as $f(\theta) = \sin\theta$.

Rectangular $r\theta$-coordinates.

In second-semester calculus we studied polar $r\theta$-coordinates, coordinates with respect to which the graph of $r = \sin\theta$ looked quite different.

Polar $r\theta$-coordinates.

Rectangular $r\theta$-coordinates are related to polar $r\theta$-coordinates by a transformation $T : R^2 \to R^2$ which may be written in any of the following three equivalent ways:

$$T : (r, \theta) \to (x, y) = (r\cos\theta, r\sin\theta)$$

$$T(r, \theta) = (x, y) = (r\cos\theta, r\sin\theta)$$

$$x = r\cos\theta, \quad y = r\sin\theta$$

In this project, we discuss the geometry of this transformation.

Task

a) Type the command lines below into Maple in the order in which they are listed; they produce two test patterns in rectangular $r\theta$-coordinates, and the images of these test patterns in polar $r\theta$-coordinates.

```
> # Project 16.9b:  Transformation from Rectangular to Polar Coordinates
> restart;
> with(plottools):with(plots):
> f := transform((r,theta) -> [r*cos(theta),r*sin(theta)]):
> TP := plot(0,1,2,3,[1,t,t=0..3],r=0..1,color=blue):
> display(TP); display(f(TP));
> TP := plot(0,1,[1,t,t=0..1],[2,t,t=0..1],[3,t,t=0..1],r=0..3,color=blue):
> display(TP); display(f(TP));
```

b) By hand (*without* using Maple):

(i) sketch the test pattern in rectangular $r\theta$-coordinates determined by $\theta = 1$, $\theta = 2$, $r = 0$, $r = 1$, $r = 2$, $r = 3$, and its image in rectangular xy-coordinates under the transformation T.

(ii) sketch the test pattern in rectangular $r\theta$-coordinates determined by $r = 1$, $r = 2$, $\theta = 0$, $\theta = 1$, $\theta = 2$, $\theta = 3$, and its image in rectangular xy-coordinates under the transformation T.