MATH 261, Spring 2001 **Due Date:**____________ **Name(s):** ________________________

Extra Project 17.6e: Different Strokes ...

Objective

In this project we illustrate that you can often learn something about a surface from one parametrization that you cannot from another.

Narrative

Some surfaces can be parametrized in more than one way, and you can often learn something about a surface from one parametrization that you cannot from another.

Tasks

1. a) Using Maple, draw the graph of $z = \mathrm{Tan}^{-1}(y/x)$ over the region $(x, y) \in [-1, 1] \times [-1, 1]$ using the command `plot3d(f(x,y),x=a..b,y=c..d)`.

   b) Verify that $x = s\cos t, y = s\sin t, z = t$ is a parametrization of $z = \mathrm{Tan}^{-1}(y/x)$.

   c) Using Maple and the parametrization in part (b) — with $(s, t) \in [-1, 1] \times [0, 2\pi]$ — draw the graph of $z = \mathrm{Tan}^{-1}(y/x)$.

   d) What does the graph of part (c) reveal that the graph of part (a) does not?

2. a) Using Maple, draw the graph of the hyperbolic paraboloid $z = x^2 - y^2$ over the region $(x, y) \in [-1, 1] \times [-1, 1]$ using the command `plot3d(f(x,y),x=a..b,y=c..d)`.

   b) Verify that $x = s + t, y = s - t, z = 4st$ parametrizes the hyperbolic paraboloid $z = x^2 - y^2$.

   c) Using Maple and the parametrization in part (b) — with $(s, t) \in [-1, 1] \times [-1, 1]$ — graph the hyperbolic paraboloid $z = x^2 - y^2$.

   A ruled surface is a surface that can be described by a family of lines. For example, a cone is a ruled surface since it can be described by revolving a line that passes through the vertex of the cone around the axis of the cone. The hyperbolic paraboloid $z = x^2 - y^2$ is a *doubly ruled surface*: there are two sets of lines on it. Note that these lines can be seen in the graphic of part (c) but not in the graphic of part (a)!

3. a) Verify that the hyperboloid of one sheet $x^2 + y^2 = 1 + z^2$ can be parametrized $x = \sqrt{1+s^2}\cos t, y = \sqrt{1+s^2}\sin t, z = s$.

   b) Using Maple and the parametrization in part (a) — with $(s, t) \in [-1, 1] \times [0, 2\pi]$ — graph the hyperboloid of one sheet $x^2 + y^2 = 1 + z^2$.

   c) Repeat part (a) using the parametrization $x = \cos t - s\sin t, y = \sin t + s\cos t, z = s$.

   d) Using Maple and the parametrization in part (c) — with $(s, t) \in [-1, 1] \times [0, 2\pi]$ — graph the hyperboloid of one sheet $x^2 + y^2 = 1 + z^2$.

   The hyperboloid of one sheet $x^2 + y^2 = 1 + z^2$ is also a doubly ruled surface. One set of these lines can be seen in the graphic of part (d) but not in the graphic of part (b).

   e) Find a parametrization that illustrates the *second* set of lines, and graph the hyperboloid of one sheet illustrating these lines.

Comments

The bottom line is that coming up with a parametrization of a surface is not always easy, and once you have found one it can be tempting to call off the search for more. However, sometimes a continued search is well worth the time and effort since it can lead to insights you otherwise might miss!