MATH 261, Spring 2001 **Due Date:**_____ **Name(s):** _____

Honors Project 1: Olbers' Black Sky Paradox

Heinrich Olbers was an astronomer who, in 1826, drew an interesting conclusion about the darkness of the nighttime sky. Olbers assumed that the universe is infinite in size, that it is populated uniformly with stars and galaxies, and that it is static in time and space (so that nothing moves or evolves). Using the fact that the light intensity I at a distance ρ from a light source is proportional to the inverse of the square of ρ:

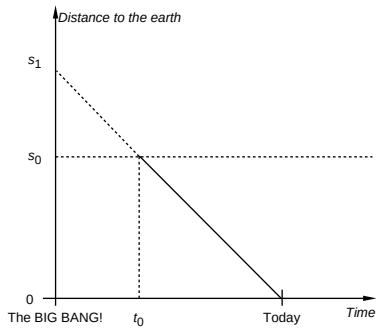
$$I = I(\rho) = \frac{k}{\rho^2}$$

for some positive constant k, Olbers concluded that the total amount of light seen by an observer standing on the face of the earth should be infinite; that is,

$$\int\int\int_{R^3} I \, dV = \infty.$$

Prove this result by writing the triple integral as an improper iterated triple integral in spherical coordinates, and evaluating it.

The nighttime sky is not bright, however: it is dark. Thus at least one of the assumptions must be wrong. Which? Well, today astronomers believe it is a combination of factors. On one hand, since the universe is expanding, distant stars contribute less light than near stars. More importantly, when we look into the sky, we are looking back in time, and we can look back in time only so long (see the figure to the right): If you see a star in the nighttime sky tonight, a star that is and always has been at a fixed distance s_0 from the earth then you will be looking at light emitted by the star at some time t_0 in the past. You might also be able to see light emitted by a star prior to t_0, a star at a distance of between s_0 and s_1 from the earth, but you could not see light from a star at any further distance: light from a star at such a distance would have had to have been emitted prior to the big bang.



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