Honors Project 10: Iterative Processes and Periodicity Revisited

In MATH 163, we briefly discussed iterative processes and periodicity in the context of single variable calculus. Not only does much of the discussion of iterative processes extend to the context of several variables, but the theory becomes considerably more interesting.

The following Maple code generates a graphic of the Lorenz system: this system is associated to the system of first-order differential equations

$$\frac{dx}{dt} = a(y - x)$$
$$\frac{dy}{dt} = b(x - y - xz)$$
$$\frac{dz}{dt} = xy - cz$$

where $a = 10, b = 28$, and $c = 8/3$ and $x_0 = 1, y_0 = 1$, and $z_0 = 10$, or, equivalently, to the first-order iterative process

$$x_{n+1} = a(y_n - x_n)\Delta + x_n$$
$$y_{n+1} = b(x_n - y_n - x_n z_n)\Delta + y_n$$
$$z_{n+1} = (x_n y_n - cz_n)\Delta + z_n$$

for some small real number Δ.

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> # Honors Project 10:  The Lorenz System
> restart:  with(plots):
> a := 10; b := 28; c := 8/3;
> Delta := 0.01;
> f := (x,y,z) -> a*(y-x)*Delta+x;
> g := (x,y,z) -> (b*x-y-x*z)*Delta+y;
> h := (x,y,z) -> (x*y-c*z)*Delta+z;
> xold := 1.0; yold := 1.0; zold := 10.0;
> mylist := [xold,yold,zold]:
> for N from 1 to 4000 do
    xnew := f(xold,yold,zold):  ynew := g(xold,yold,zold):  znew := h(xold,yold,zold):
    mylist := mylist,[xnew,ynew,znew]:
    xold := xnew:  yold := ynew:  zold := znew:
  od:
> spacecurve([mylist],color=blue,axes=boxed);
```

Problem: Type the command lines listed above into Maple in the order in which they are listed. (*Note:* You might find it useful to limit the range of N until you are sure your code has been typed correctly.) Then, after experimenting with this code, discuss the effects that changing a, b, c and x_0, y_0, z_0 have on this system. Illustrate your discussion with Maple graphics.