MATH 164, Spring 2001 **Due Date:**____________ **Name(s):** ________________________

Extra Project 10.5d: Differential vs. Difference Equations

(Value: 10 pts)

Suppose you're interested in studying the differential equation

$$\frac{dx}{dt} = 4x\left(\frac{3}{4} - x\right), \quad x(0) = x_0 \in (0,1) \tag{1}$$

and you've forgotten that it is just the logistic equation. (See the Epilog below.) If the only tools at your disposal are numerical (read MatLab here, as opposed to Maple) then you might be tempted to approximate solutions to equation (1) by solutions to the corresponding difference equation

$$\Delta x_n = x_{n+1} - x_n = 4x_n\left(\frac{3}{4} - x_n\right), \quad x_0 \in (0,1) \tag{2}$$

which can be obtained via the iterative scheme

$$x_{n+1} = 4x_n\left(\frac{3}{4} - x_n\right) + x_n = 4x_n(1 - x_n), \quad x_0 \in (0,1)$$

or

$$x_{n+1} = 4x_n(1 - x_n), \quad x_0 \in (0,1). \tag{3}$$

Unfortunately we're now faced with a dilemma: On one hand, we know that *all* solutions $x = x(t)$ to equation (1) approach 3/4 as $t \to \infty$. (See Section 10.5 of the text.) On the other hand, in MATH 163's Project 0.6b we observed, in discussing equation (3), that, "for some values of $x_0 \in [0,1]$, x_n converges [as $n \to \infty$], for some it repeats, and for others it wanders aimlessly over the interval $[0,1]$." exhibiting chaos!

Problem: Explain what's going on here.

Epilog: The same issues raised in this project might well arise with equations other than the logistic equation. The reason we singled out the logistic equation in this project is that we know what its solutions look like, so it's easy to pose the dilemma posed above.