MATH 261, Spring 2001 **Due Date:**_____ **Name(s):** _____

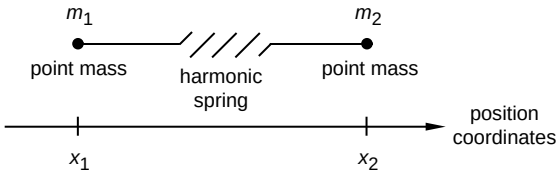
Honors Project 13: Mechanical Energy of a Coupled Spring-Mass System

Quantities associated with a physical system can sometimes be simplified by changing the frame of reference. For example:

The total mechanical energy H of the coupled spring-mass system illustrated below is given by

$$H(x_1, x_2, p_1, p_2) = \frac{p_1^2}{2m_1} + \frac{p_2^2}{2m_2} + \frac{1}{2}(x_2 - x_1 - x_0)^2$$

where x_0 is the equilibrium (unstretched) length of the spring, and $p_i = m_i v_i$ where $v_i = dx_i/dt$, for $i = 1, 2$.



Problem: Assuming we have defined two new coordinates

$$r \equiv x_2 - x_1 - x_0 \quad \text{and} \quad s \equiv \frac{m_1 x_1 + m_2 x_2}{m_1 + m_2}$$

and that

$$p_r \equiv \frac{m_1 m_2}{m_1 + m_2} v_r \quad \text{and} \quad p_s \equiv (m_1 + m_2) v_s,$$

where $v_r = dr/dt$ and $v_s = ds/dt$, express H as a function of r, s, p_r, and p_s in a simplified form.

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