MATH 261, Spring 2001 **Due Date:**____________ **Name(s):** ________________________

Honors Project 14: The Energy of Non-Interacting Gas Particles

A general expression for the energy of a collection of N non-interacting gas particles is

$$U = \frac{k}{Q}\frac{\partial Q}{\partial T}$$

where k is a fundamental physical constant (Boltzmann's constant), T is temperature, and Q is a special function called the *partition function*, which is of the form $Q = \frac{1}{N!}q^N$ where q is a *molecular partition function* that must be derived or determined.

Problems:

1. Find U for a collection of N particles if the molecular partition function

$$q = \frac{V}{h^3}(2\pi mkT)^{3/2}$$

   where V is the volume of the gas, m is the mass of a gas particle, and h is a fundamental physical constant (Planck's constant).

2. The heat capacity C_V is the heat energy required to change the temperature of a substance when the volume is constant, and is given by

$$C_V = \frac{\partial U}{\partial T}.$$

   Find C_V for a collection of N particles.

   [*Note on notation*: The heat capacity is sometimes written

$$C_V = \left(\frac{\partial U}{\partial T}\right)_V.$$

   The reason is that when several variables are used in a problem, ambiguity about the meaning of partial derivatives can evolve from ambiguity about which variables are assumed to be independent and which are dependent. For example, if

$$u = x + y \quad \text{and} \quad v = x - y$$

   and x and y are assumed to be independent, then $\partial u/\partial x = 1$. However, if x and v are assumed to be independent then, since $y = x + v$, it follows that $u = 2x + v$ so $\partial u/\partial x = 2$. One way to clarify which variables are assumed to be independent is to use the notation $\left(\frac{\partial u}{\partial x}\right)_y$ to denote the partial derivative of u with respect to x, x and y being the independent variables. Thus in the above example, $\left(\frac{\partial u}{\partial x}\right)_y = 1$ and $\left(\frac{\partial u}{\partial x}\right)_v = 2$. While this notation is useful, it is important to remember when using it that subscripts make a statement about which variables are independent: they *do not* represent partial derivatives.]

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