MATH 261, Spring 2001 **Due Date:**____________ **Name(s):** ________________________

Honors Project 18: Rotation and Quadric Surfaces

If we rotate the xy-coordinate axes counter-clockwise through an angle θ about the origin then we obtain a uv-coordinate system which is related to the xy-coordinate system by

$$u = x\cos\theta - y\sin\theta$$

$$v = x\sin\theta + y\cos\theta$$

One reason such rotations are useful is that they allow us to identify the graph of any quadratic equation

$$Ax^2 + Bxy + Cy^2 + Dx + Ey + F = 0$$

as a conic whose equation in standard form is

$$A'u^2 + C'v^2 + D'u + E'v + F' = 0$$

(that is, whose whiuch has no uv-term). (The required rotation angle is $\theta = \frac{1}{2}\mathrm{Cot}^{-1}\frac{A-C}{B}$.) Thus we may classify the graphs of all second degree equations in x and y as conic sections.

Can the same thing be done for equations of the form

$$Ax^2 + By^2 + Cz^2 + Dxy + Exz + Fyz + Gx + Hy + Iz + J = 0$$

and quadric surfaces in R^3?