

### Honors Project 3: Kepler's Laws of Planetary Motion

Prior to Johannes Kepler's (1571-1630) statement of his three Laws of Planetary Motion:

1. A planet revolves around the Sun in an elliptical orbit with the Sun at one focus.
2. The line joining the Sun to a planet sweeps out equal areas in equal times.
3. The square of the period of revolution of a planet is proportional to the cube of the length of the major axis of its orbit.

it was thought that the orbits of the planets were circles with the Sun at the center of each.

Kepler's approach to his his three laws was actually quite geometric. (It was Sir Isaac Newton who first showed how Kepler's laws could be deduced from two of his own laws — his Second Law of Motion and his Law of Universal Gravitation — in his *Principia Mathematica* in 1687.) It (Kepler's approach) was based on the fact that if an object travels around a smooth, closed, planar curve (such as a circle or an ellipse) then, to an observer located at a point in the plane of the objects motion but outside the curve, the object will appear to travel to the right, to stop, to travel to the left, to stop, and to travel to the right again, after which the (apparent) motion repeats (see Figure 1). A point such as  $E$  at which the object appears to stop after traveling to the right is called an *easterly elongation*, and a point such as  $W$  at which the object appears to stop after traveling to the left is called a *westerly elongation*.

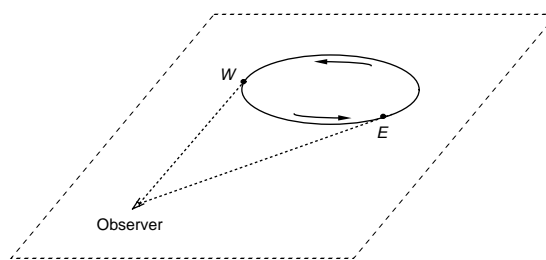


Figure 1

Kepler assumed that the orbit of the earth was a circle with the Sun at its center. He associated to each point of the circle a date of the year (see Figure 2). He then used Tycho Brahe's observations of the easterly and westerly elongations of the planet Mercury — measured as angles as seen from the earth, from the Sun to Mercury — to draw a set of lines to which he concluded the orbit of Mercury must be tangent.

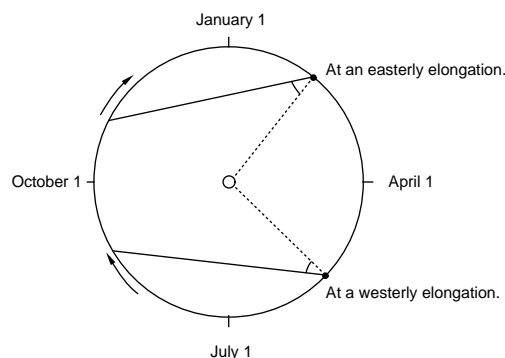


Figure 2

Obtain data from the *Observers' Handbook*, and repeat Kepler's construction for yourself (using as much data as possible). Can you inscribe a circle in the resulting set of lines in such a manner that the circle is tangent to each line? If not, what kind of curve *can* you inscribe? This should give you some insight into why Kepler concluded his first law; can you see how he might have arrived at the other two?

The following Maple code may help: it generates a circle with 12 equally spaced tic-marks:

```
> # CircleWithPoints
> restart: with(plots):
> Circle := implicitplot(x^2+y^2=1,x=-1..1,y=-1..1,scaling=constrained,axes=none):
> P := [0,0];
> N := 12;
> for i from 1 to N do P := P,[cos(2*Pi*i/N),sin(2*Pi*i/N)]: od:
> Points := pointplot([P],color=blue,symbol=circle):
> display(Circle,[Points]);
```