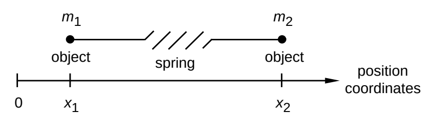
MATH 164, Spring 2001 **Due Date:**____________ **Name(s):** ________________________

Honors Project 5: Coupled Spring-Mass Systems

In Honors Project 4 we considered spring-mass systems in which a single object was connected to a ground by a single spring. There are numerous variations on such systems, and these variations are important in applications. In this project we consider two such variations.

We begin with the situation in which two objects of mass m_1 and m_2 are connected by a single spring whose spring constant is k and whose unstretched equilibrium length is L. We assume the objects move horizontally with no friction or damping due to the environment.



Applying Newton's Second Law of Motion to each object, we find that

$$m_1 \frac{d^2 x_1}{dt^2} = F_1 \quad \text{and} \quad m_2 \frac{d^2 x_2}{dt^2} = F_2$$

where F_i is the force acting on object $i, i = 1, 2$. Since the only force acting on each object is (according to Hooke's law) proportional to the distance the spring is stretched out of equilibrium, it follows that

$$m_1 \frac{d^2 x_1}{dt^2} = k(x_2 - x_1 - L) \quad \text{and} \quad m_2 \frac{d^2 x_2}{dt^2} = -k(x_2 - x_1 - L).$$

We now make two observations:

First,

$$m_1 \frac{d^2 x_1}{dt^2} + m_2 \frac{d^2 x_2}{dt^2} = \frac{d^2}{dt^2}(m_1 x_1 + m_2 x_2) = 0,$$

so the center of mass of our system

$$\frac{m_1 x_1 + m_2 x_2}{m_1 + m_2}$$

does not accelerate: it moves at constant velocity (and hence obeys Newton's First Law of Motion).

Second, if $z = x_2 - x_1 - L$ then it follows that

$$\frac{d^2 z}{dt^2} = -k \left(\frac{1}{m_1} + \frac{1}{m_2} \right) z \qquad (*)$$

so the stretching of the spring exhibits simple harmonic motion. Indeed, if we let

$$\frac{1}{m} = \frac{1}{m_1} + \frac{1}{m_2},$$

so the *reduced mass*

$$m = \frac{m_1 m_2}{m_1 + m_2}$$

then

$$\frac{d^2 z}{dt^2} = -\frac{k}{m} z.$$

Problems:

1. Prove equation (*).

2. Consider the situation in which an object of mass m is connected to one wall by a spring whose spring constant is k_1 and whose unstretched equilibrium length is L_1 and to an opposite wall by a spring whose spring constant is k_2 and whose unstretched equilibrium length is L_2. (See the figure below.) Observe that if D is the distance between the walls, then D is *not* necessarily equal to $L_1 + L_2$. (Can you see why?) We finally assume the objects move horizontally with no friction or damping due to the environment. What can you say about the motion of the object? Specifically, does it illustrate simple harmonic motion? Explain.