MATH 261, Spring 2001 **Due Date:**____________ **Name(s):** ________________________

Honors Project 19: Optimization for Functions of Three Variables

In MATH 163 we developed a technique for finding (and classifying) the extrema of a differentiable function $f = f(x)$ of one variable. In MATH 261 we developed a technique for finding (and classifying) the extrema of a differentiable function $f = f(x, y)$ of two variables. Develop a technique for finding (and classifying) the extrema of a differentiable function $f = f(x, y, z)$ of three variables.

Comments

To many people, the concept of "dimension" is embodied in three dimensions of space and one of time, and discussions of spaces of more than four dimensions tend to be either philosophical or like science fiction. In mathematics, science, and egnineering, however, spaces of greater than four dimensions arise frequently, and handling them (and operations in them — such as finding the extrema of a differentiable function) is a routine issue. The trick is to understand that a "dimension" can be interpreted as a "degree of freedom" in a very real way. Consider, for example, your arm: Relative to your torso, there are 2 degrees of freedom to the position of your upper-arm. And relative to your upper arm, there are 2 degrees of freedom to the position of your fore-arm. Thus there are $2 * 2 = 4$ degrees of freedom to the position of your fore-arm relative to your torso, or *the position of your forearm relative to your torso* is a point in a four dimensional space. And the *motion* of your forearm relative to your torso is a curve in a five dimensional space! Imagine the dimensionality of a mechanical arm with N joints!