MATH 261, Spring 2001 **Due Date:**____________ **Name(s):** ________________________

Honors Project 4: Propagation of Error in Chemistry

If $f = f(x_1, x_2, \ldots, x_n)$ is a quantity whose value depends on the measurable quantities $x_1, x_2, \ldots, x_n$, then the error df in the measurement of f, in terms of the errors dx_i in the measurements of $x_i, i = 1, 2, \ldots, n$, is given by

$$df = \sum_i \frac{\partial f}{\partial x_i} dx_i.$$

In practice, df is often computed using the fact that

$$|df| = \left(\left(\sum_i \frac{\partial f}{\partial x_i} dx_i \right)^2 \right)^{1/2} = \left(\sum_i \left(\frac{\partial f}{\partial x_i} \right)^2 dx_i^2 + \sum_{i \neq j} \frac{\partial f}{\partial x_i} \frac{\partial f}{\partial x_j} dx_i dx_j \right)^{1/2}.$$

Since the "cross terms"

$$\frac{\partial f}{\partial x_i} \frac{\partial f}{\partial x_j} dx_i dx_j$$

may be positive or negative, their sum is generally significantly smaller than the sum of the squared terms, and their effect on $|df|$ can be ignored. Hence,

$$|df| \approx \left(\sum_i \left(\frac{\partial f}{\partial x_i} \right)^2 dx_i^2 \right)^{1/2}.$$

A 20.00 mL sample of a 0.1250 molarity $CuSO_4$ solution is diluted to 500.0 mL. If the error in measurement of the molarity is ± 0.0002, of the 20.00 mL pipet is ± 0.03 mL, and of the volume of the 500 mL flask is ± 0.15 mL, use the above analysis to determine how the molarity of the resulting solution should be reported. That is, what is the molarity of the resulting solution, *and* what error is there in this measurement? (Recall that the molarity M of a solution is given by $M = n/V$ where n is the number of moles of solute, and V is the volume of the solution in liters. Thus, in our case, $M_{before} V_{before} = M_{after} V_{after}$, or $M_{after} = M_{before} V_{before} / V_{after}$.)

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