MATH 164, Spring 2001 **Due Date:**____________ **Name(s):** ________________________

Honors Project 6: Finite Differences

Introduction

In this project we view a sequence $\{a_n\}$ to be a function $a : Z \to R$ which associates to *every* integer $n \in Z$ the real number $a_n \in R$. Thus, for example, if $a_n = n^3$ then some of the values of a are:

$$\begin{array}{rccccccccc} n: & \ldots & -3 & -2 & -1 & 0 & 1 & 2 & 3 & \ldots \\ a_n = n^3: & \ldots & -27 & -8 & -1 & 0 & 1 & 8 & 27 & \ldots \end{array}$$

We define the *forward difference operator* Δ_+ as the function which associates to each sequence a the sequence $\Delta_+ a$ whose nth term is

$$(\Delta_+ a)_n = a_{n+1} - a_n,$$

the *backward difference operator* Δ_- as the function which associates to each sequence a the sequence $\Delta_- a$ whose nth term is

$$(\Delta_- a)_n = a_n - a_{n-1},$$

and the *sum operator* Σ as the function which associates to each sequence a the sequence Σa whose nth term is

$$(\Sigma a)_n = \sum_{i=0}^{n} a_i.$$

Questions

1. If

$$a_n = \begin{cases} 0 & \text{if } n < 0 \\ \dfrac{1}{2^n} & \text{if } n \geq 0 \end{cases},$$

find, in simplest form: a) $(\Delta_+ a)_n$, b) $(\Delta_- a)_n$, and c) $(\Sigma a)_n$.

2. What general rules apply to computing Δ_+, Δ_-, and Σ? (For example, what can you say about $\Delta_+ ca$ where c is a constant and a is a sequence? What can you say about $\Delta_+(a \pm b)$ where a and b are sequences? How about $\Sigma 1$, Σn, Σn^2? *Hint*: Consider Sections 3.3 and 5.2 of the text.)

3. What:

a) is the composition $\Sigma \circ \Delta_+$? That is, what is the value of

$$((\Sigma \circ \Delta_+)\, a)_n = (\Sigma\,(\Delta_+ a))_n = \sum_{i=0}^{n} (\Delta_+ a)_i = \sum_{i=0}^{n} (a_{i+1} - a_i)$$

for any sequence a, in simplest form? How about $\Sigma \circ \Delta_-$?

b) is the composition $\Delta_+ \circ \Sigma$? That is, what is the value of

$$((\Delta_+ \circ \Sigma)\, a)_n = (\Delta_+\,(\Sigma a))_n = (\Sigma a)_{n+1} - (\Sigma a)_n = \sum_{i=0}^{n+1} a_i - \sum_{i=0}^{n} a_i$$

for any sequence a, in simplest form? How about $\Delta_- \circ \Sigma$?

c) result (from MATH 163) do parts (a) and (b) remind us of?