

### Extra Project 3.1: Graphing a Function Using a Table

#### Objective

To review the graphing of functions by plotting points and “connecting the dots”, and to investigate the connection between  $f$  and  $f'$  as functions.

#### Narrative

Since  $f'(x) = \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h}$ , it follows that for small values of  $h$ ,

$$f'(x) \approx \frac{f(x+h) - f(x)}{h}. \quad (*)$$

#### Task

a) Type the command lines below into Maple in the order in which they are listed. These command lines produce a table of values for  $f(x) = (1 - x^2)/(1 + x^2)$  and  $f'(x)$ ,  $x \in [-2, 2]$

```
> # Project 3.1: Graphing a Function Using a Table
> restart;
> f := x -> (1-x^2)/(1+x^2);
> f1 := D(f);
> a := -2.0; b := 2.0; N := 10; dx := (b-a)/N;
> M := matrix(N+1,3,(Row,Col)->0):
> M[1,1] := `x`; M[1,2] := `f(x)`; M[1,3] := `f1(x)`;
> Digits := 2;
> for i from 1 to N do
  x := a + (i-1)*dx:
  M[i+1,1] := x: M[i+1,2] := f(x): M[i+1,3] := f1(x): od:
> eval(M);
```

b) Continue by typing the following command lines into Maple. These command lines produce an empty graph (on which you will be asked to graph  $f$  and  $f'$ ).

```
> x := `x`;
> plot(0,x=a..b,y=-1..1);
```

At this time make a hard-copy of your typed input and Maple's responses. Then, ...

c) on the graphic you produced in part (b), use the data you produced in part (a) to graph  $f$  and  $f'$  by hand, each in a different color, and label each graph. (Draw the graphs of  $f$  and  $f'$  by plotting points and “connecting the dots”.)

d) Illustrate (\*) by using the table of values you produced in part (a) to:

1. approximate  $f'(1)$  (you'll be using values in the *second* column of the table in part (a) to do this), and
2. compute the absolute value of the difference between this approximation and the actual value of  $f'(1)$  (which you can obtain from the *third* column of the table in part (a)). This is the *error* in the approximation.