MATH 164, Spring 2001 **Due Date:____________** **Name(s): ________________________**

Honors Project 7: Ballistics with Air Resistance

In MATH 163, we found that if — neglecting air resistance — a projectile of mass m is fired vertically upward then its height y above the ground at any time is governed by the equation $my'' = -mg$ where g is the force due to gravity. It followed that $v = y'(t) = -gt + y_0$, and that $y = y(t) = -\frac{1}{2}gt^2 + v_0 t + y_0$. Thus, if we use metric units — so $g = 9.8$ m/sec^2 — then $v = -4.9t + v_0$. A little reflection on this formula for velocity might lead you to question the assumption that we are neglecting air resistance: as t increases without bound, $v = -4.9t + v_0$ increases without bound, and this is impossible. On one hand, practical experience illustrates that objects can *not* fall at ever increasing speed; there is, in fact, a speed dubbed *terminal velocity* beyond which a projectile cannot travel. On the other hand, if we accept the fact that no object can move beyond the speed of light then this formula leads to a contradiction. In this project, we investigate the affects of air resistance.

One way to incorporate air resistance into our discussion of the motion of a projectile, is to use the equation

$$my'' = -mg - cy' \tag{1}$$

where the term cy' represents drag due to air resistance. Observe that cy' is proportional to y': the larger the velocity the greater the air resistance. The constant c is known as the *drag coefficient*; c depends on the shape the projectile: the larger c the greater the air resistance.

To solve this equation, we treat it as a *first* order differential equation in $v = y'$: we may rewrite it

$$mv' = -mg - v$$

and solve it for v by separation of variables; and having expressed $v = x'(t)$ as a function of t, we can integrate to find $y = y(t)$.

Tasks

1. Type the following command lines into Maple in the order in which they are listed. The affect of these commands is solve equation (1) assuming that $m = 1.0$, $c = 0.2$, $g = 9.8$, $y_0 = 0.0$, and $v_0 = 128$.

```
> # Honors Project 7:  Ballistics with Air Resistance
> restart:  with(DEtools):
> m := 1.0; c := 0.2; g := 9.8; y0 := 0.0; v0 := 128;
> y2 := diff(y1(t),t);
> dsolve(m*diff(y1(t),t) = -m*g-c*y1(t),y1(0)=v0, y1(t));
> y1 := unapply(rhs(%),t);
> dsolve(diff(y(t),t) = y1(t),y(0)=y0,y(t));
> y := unapply(rhs(%),t);
```

2. Continue by typing the following command line into Maple. The affect of this command is to plot the solution to equation (1) for $t \in [0, 10]$.

```
> t0 := 0.0; t1 := 10.0; plot(y(t),t=t0..t1); y(t1);
```

3. By using trial-and-error, change the value of `t1` in the last line you typed until you obtain the value of `t1` *greater than* `t0` for which `y(t1)` is within 2 decimal places of 0.

4. Observe that the curve you get is *not* symmetric. Explain why it is not.

5. Find when the projectile achieves its maximum altitude, and find the maximum altitude.
6. Find the terminal velocity of the projectile.