

Extra Project 3.7: Implicit Functions and Implicit Differentiation

Objective

To illustrate the practical value of using the implicit description of functions.

Narrative

If you have not already done so, read Section 3.7 of the text.

A function $y = f(x)$ can be described either explicitly — in the form a “recipe” for computing y given x — or implicitly — as the solution to an equation in x and y . While all functions that can be described explicitly can be described implicitly, functions that can be described implicitly cannot always be described explicitly. In this project we illustrate that even in the event that a function that can be described implicitly can be described explicitly, implicit methods — for operations such as differentiation — are sometimes preferable to explicit methods.

Task

a) Type the command lines below into Maple in the order in which they are listed. They instruct Maple to find the derivative of a function $y = f(x)$ defined implicitly by the equation $x^3 + y^3 = xy$ *implicitly*, and to *implicitly* plot the graph of this equation.

```
> # Project 3.7: Implicit Differentiation
> restart;
> eqn0 := x^3+y(x)^3=x*y(x);
> eqn1 := diff(lhs(eqn0),x) = diff(rhs(eqn0),x);
> Diff(y(x),x) = solve(eqn1,diff(y(x),x));
> with(plots):
> implicitplot(eqn0,x=-1..1,y=-1..1,color=red,numpoints=1000);
```

b) Continue by typing the following command line into Maple. They instruct Maple to solve the equation $x^3 + y^3 = xy$ for y *explicitly* in terms of x , and to *explicitly* find the derivative of this function.

```
> solve(x^3+y^3=x*y,y);
> %[1];
> Diff(y(x),x) = D(%);
```

c) Use Maple and implicit differentiation to compute the second derivative of $y = f(x)$ in terms of x and y .

Comments

1. Observe how much simpler Maple’s output appears when our computations are implicit rather than explicit.
2. Observe that there are multiple functions defined by the equation $x^3 + y^3 = xy$ (the graph of this equation produced in part (a) does *not* pass the vertical line test), and hence in computing the derivative of a function defined by this equation explicitly (in part (b)) we must make a choice as to which *branch* we are considering. This is what the “%[1]” command does