MATH 163, Spring 2001 **Due Date:**____________ **Name(s):** ________________________

Extra Project 4.1: Maximum and Minimum Values on a Closed Interval

Objective

To illustrate how Maple can be used to find the maximum and minimum values of a function on a closed interval.

Narrative

If you have not already done so, read Section 4.1 of the text. In particular, be sure to read Example 4, p. 224: below we use this example to illustrate how Maple can be used to find the maximum and minimum values of $f(x) = 3x^4 - 16x^3 + 18x^2$ on the closed interval $[-1, 4]$.

Task

a) Type the command lines in the left-hand column below into Maple in the order in which they are listed. The effect of each command is described in the right-hand column for your reference.

`> # Project 4.1:  Maximum and Minimum Values on a Closed Interval`	
`> restart;`	Clear Maple's memory.
`> f := x -> 3*x^4-16*x^3+18*x^2;`	Let $f(x) = 3x^4 - 16x^3 + 18x^2$.
`> plot(f(x),x=-1..4);`	Plot the graph of f over the interval $[-1, 4]$.
`> f1 := D(f);`	Let $f1$ denote the first derivative f' of f.
`> cn:= fsolve(f1(x)=0,x,x=-1..4);`	Find the critical numbers of f on $[-1, 4]$ by solving the equation $f'(x) = 0$.
`> print(cn[1],f(cn[1]));`	The first critical number and the value of f there.
`> print(cn[2],f(cn[2]));`	The second critical number and the value of f there.
`> print(cn[3],f(cn[3]));`	The third critical number and the value of f there.
`> print(-1,f(-1));`	The left-hand endpoint of $[-1, 4]$ and the value of f there.
`> print(4,f(4));`	The right-hand endpoint of $[-1, 4]$ and the value of f there.
`> plot({f(x),f1(x)},x=-1..4,y=-30..40);`	Plot the graphs of f and f' over a y-range large enough to capture all relevant behavior.

At this point make a copy of your typed input and Maple's responses (both text and graphics). Then, ...

b) What is the maximum value of f on the closed interval $[-1, 4]$? What is the minimum value of f on the closed interval $[-1, 4]$?

c) By hand, label the graphs of f and f' in the last plot, and highlight that part of the graph of f over which f is increasing, and that part of the graph of f' over which the values of f' are positive.

Comments

To find the critical numbers of f we must find the values of x in the domain of f for which $f'(x) = 0$ and $f'(x)$ does not exist. We restricted our attention to points at which $f'(x) = 0$ in the above example since $f(x)$ is a polynomial function so $f'(x)$ exists for all x. If $f(x)$ were a quotient, however, then we would need to find not only the values of x for which the numerator of $f'(x)$ — namely `numer(f1(x))` — is zero, but also the values of x for which the denominator of $f'(x)$ — namely `denom(f1(x))` — is zero.