

Extra Project 7.1a: Inverse Functions

Objective

To investigate some properties of inverse functions.

Narrative

If you have not already done so, read Section 7.1 in the text. The key ideas you should take away from this section are:

1. To show that a function f has an inverse, you need to show that f is 1-1. You can do this by showing that either: a) $f'(x) > 0$ for all x in the domain of f , or b) $f'(x) < 0$ for all x in the domain of f .
2. To find f^{-1} for simple functions f , solve the equation $y = f(x)$ for x in terms of y ; the resulting expression in y is $f^{-1}(y)$. (So if you are looking for $f^{-1}(x)$, simply replace y everywhere by x .) Remember that this only works for simple functions f .
3. To check your work, verify that: a) $f^{-1}(f(x)) = x$ for all x in the domain of f , and b) $f(f^{-1}(x)) = x$ for all x in the domain of f^{-1} .
4. To find the derivative of f^{-1} , use the fact that $D_x(f^{-1}(x)) = 1/D_y(f(y))$.

In this project we investigate the function $f(x) = x/(3-x)$ and its inverse. In this project we introduce the command `unapply`: roughly speaking, the effect of the command line `g := unapply(h,y)` is to use the expression `h` to define the function `g`.

Task

a) Type the command lines in the left-hand column below into Maple in the order in which they are listed. The effect of each command is described in the right-hand column for your reference. Your lab report will be a hard copy of your typed input and Maple's responses.

<pre>> # Project 7.1a: Inverse Functions > restart; > f := x -> x/(3-x); > sol := solve(y=f(x),x);</pre>	<p>Clear Maple's memory. Let $f(x) = x/(3-x)$. Let <code>sol</code> denote the expression we get from solving the equation $y = f(x)$ for x in terms of y.</p>
<pre>> g := unapply(sol,y); > plot({f(x),g(x),x},x=0..2,y=0..2, scaling=constrained);</pre>	<p>Use the expression <code>sol</code> to define the function $g = f^{-1}$. Plot the graphs of f, g, and $y = x$.</p>
<pre>> simplify(f(g(x))); > simplify(g(f(x))); > simplify(1/D(f)(x));</pre>	<p>Check that $(f \circ g)(x) = x$. Check that $(g \circ f)(x) = x$. We now verify that $1/D_x(f(x)) = D_y(g(y))$ by first computing $1/D_x(f(x)) \dots$</p>
<pre>> simplify(D(g)(y)); > simplify(subs(y=f(x),%));</pre>	<p>then $D_y(g(y)) \dots$ and observing that they are the same (since $y = x/(3-x)$)!</p>

At this point, make a hard-copy of your typed input and Maple's responses (both text and graphics). Then continue with the following steps.

b) Sketch and label by hand the graph of the line whose equation is $y = x$ on the plot you created in part (a), as well as the graphs of f and g .

c) The point $P(1, \frac{1}{2})$ lies on the graph of f , and the point $Q(\frac{1}{2}, 1)$ lies on the graph of f^{-1} . Plot and label by hand P and Q .

Comments

You might find it instructive to repeat this project for the functions $f(x) = 2x + 1$, $f(x) = \ln x$, and $f(x) = e^x$. In Maple, the natural log function is written `ln(x)`, the natural exponential function is written `exp(x)`, and the common log function is written `log10(x)`. You might also find it instructive to repeat this project for the function $f(x) = x^3$; this example is a little different than the others we have considered so far, however: in this case Maple returns *three* expressions when it solves the equation $y = f(x)$ for x in terms of y . Can you see why? In this case you need to use `g := unapply(sol[1],y)` rather than `g := unapply(sol,y)` to make the rest of the code work.