

Project 7.1c: Inverse Functions

Objective

To investigate inverse functions.

Narrative

If you have not already done so, read Section 7.1 in the text. Some of the key ideas you should take away from this section are:

1. To show that a function f has an inverse, you need to show that f is 1-1. You can do this by showing that either: a) $f'(x) > 0$ for all x in the domain of f , or b) $f'(x) < 0$ for all x in the domain of f .
2. To find f^{-1} for simple functions f , solve the equation $y = f(x)$ for x in terms of y ; the resulting expression in y is $f^{-1}(y)$. (So if you are looking for $f^{-1}(x)$, simply replace y everywhere by x .) Remember that this only works for simple functions f .
3. To check the work you did in part (2) it suffices to verify that: a) $f^{-1}(f(x)) = x$ for all x in the domain of f , and b) $f(f^{-1}(x)) = x$ for all x in the domain of f^{-1} .
4. To find the derivative of f^{-1} , use the fact that $D_x(f^{-1}(x)) = 1/D_y(f(y))$.

Tasks

a) Type the command lines below into Maple in the order in which they are listed. The effect of these commands is to create a table of values and a gridded graph.

```
> # Project 7.1c: Inverse Functions
> restart; with(plots):
> A := array(1..11,1..2);
> for N from 1 to 11 do
  A[N,1] := evalf((N-1)/10): A[N,2] := evalf(sin(Pi*(N-1)/20)): od:
> evalm(A);
> plot0 := plot([seq((N-1)/10,N=1..11)],x=0..1,color=gray):
> plot1 := plot([seq([(N-1)/10,0],[N-1)/10,1]],N=1..11),x=0..1,color=gray):
> display({plot0,plot1},scaling=constrained);
```

At this point, make a hard-copy of your typed input and Maple's responses (both text and graphics). Then continue with the following steps.

- b) What second column entry corresponds to the first column entry of 0.2?
- c) Using only the data in the table you created, estimate to 4 decimal places of accuracy the second column entry that should correspond to a first column entry of 0.275.
- d) What first column entry corresponds to the second column entry of 0.8090169945?
- e) Using only the data in the table you created, estimate to 4 decimal places of accuracy the first column entry that should correspond to a second column entry of 0.8.
- f) Plot the points on the graph whose x -coordinates are the entries in the first column of A , and whose y -coordinates are the corresponding entries in the second column of A . Sequentially connect these points by line segments.

g) Plot the points on the graph whose x -coordinates are the entries in the second column of \mathbf{A} , and whose y -coordinates are the corresponding entries in the first column of \mathbf{A} . Sequentially connect these points by line segments.

h) What geometric relationship is there between the graphs you drew in parts (e) and (f)?

Assuming we use the notation $\mathbf{A}[i, j]$ to denote the number in the i th row and j th column of \mathbf{A} (so $\mathbf{A}[5, 1] = 0.4$, and $\mathbf{A}[3, 2] = 0.3090169945$):

i) compute $(\mathbf{A}[5, 2] - \mathbf{A}[4, 2]) / (\mathbf{A}[5, 1] - \mathbf{A}[4, 1])$ to at least 4 decimal places of accuracy, and

j) compute $(\mathbf{A}[5, 1] - \mathbf{A}[4, 1]) / (\mathbf{A}[5, 2] - \mathbf{A}[4, 2])$ to at least 4 decimal places of accuracy.

k) Your answers to parts (i) and (j) should be reciprocals of one another. What fact would this reflect if, instead of working with functions whose graphs consist of broken line segments as in this project, we had been working with differentiable functions?