Project 11.1c: Ballistics

Objective

To illustrate the application of parametric curves to ballistics.

Narrative

If a projectile is fired vertically upward into the air with an initial velocity of v_0 m/sec from a point s_0 meters above the ground, then (neglecting air resistance) after t sec the projectile is

$$s = s(t) = -\frac{1}{2}gt^2 + v_0 t + s_0$$

meters above the ground, where $g = 9.8$ m/sec^2 is acceleration due to gravity. If the projectile is fired at an angle of elevation of θ with respect to the horizontal at an initial velocity of v_0 m/sec from a point y_0 meters above the ground (see the figure to the right), then (neglecting air resistance) after t sec the projectile is located at the point whose coordinates are given parametrically by

$$x(t) = (v_0 \cos \theta)t, \quad y(t) = -\frac{1}{2}gt^2 + (v_0 \sin \theta)t + y_0.$$
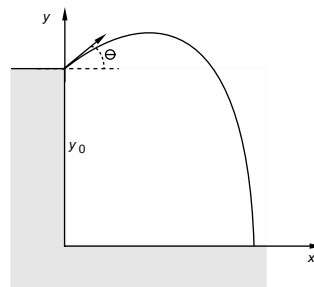
Task

a) Type the command lines below into Maple in the order in which they are listed. They describe the motion of a projectile that is fired at time $t_0 = 0$ at an angle of elevation of $\theta = \pi/4$ with respect to the horizontal at an initial velocity of $v_0 = 128$ m/sec from a point $y_0 = 100$ meters above the ground.

```
> # Project 11.1c:  Ballistics
> restart;
> g := 9.8; t0 := 0; y0 := 100; v0 := 128; theta := Pi/4;
> x := t -> v0*cos(theta)*t;
> y := t -> -0.5*g*t^2+v0*sin(theta)*t+y0;
> vx := D(x); vy := D(y);
> t1 := 1.0; evalf(y(t1)); plot([x(t),y(t),t=t0..t1]);
```

(The quantities vx and vy are known as the x- and y-components of the velocity, respectively. We discuss these quantities further in MATH 261.)

b) By using trial-and-error, change the value of t1 in the last line you typed until you obtain the value of t1 *greater than* t0 for which y(t1) is within 2 decimal places of 0. (In doing this you are estimating the time t1 it takes the projectile to hit the ground.)

c) Continue by typing the command lines in the left-hand column below into Maple in the order in which they are listed.

```
> evalf(x(t1));
```
This is the range of the projectile.

```
> evalf(sqrt(vx(t1)^2+vy(t1)^2));
```
This is the impact velocity. (We discuss impact velocity further in MATH 261.)

```
> t_max := solve(vy(t)=0,t);
```
This is the time at which the vertical component of the velocity is 0; this is the time at which the projectile achieves its maximum altitude.

```
> evalf(y(t_max));
```
This is the maximum altitude.

The *range* of a projectile is the horizontal (or x-) distance the projectile travels before it strikes the ground. To compute the range of a projectile that is fired at an angle of elevation of θ with respect to the horizontal at an initial velocity of v_0 m/sec from a point y_0 meters above the ground in closed form, observe that at the time t_1 the projectile strikes the ground,

$$y(t_1) = -\frac{1}{2}gt_1^2 + (v_0 \sin \theta)t_1 + y_0 = 0.$$

Thus $t_1 = (v_0 \sin \theta + \sqrt{v_0^2 \sin^2 \theta + 2gy_0})/g$, and the range

$$x(t_1) = (v_0 \cos \theta)t_1 = v_0 \cos \theta \frac{v_0 \sin \theta + \sqrt{v_0^2 \sin^2 \theta + 2gy_0}}{g}.$$

If, in particular, $y_0 = 0$, then the range

$$x(t_1) = \frac{v_0^2 \sin 2\theta}{g}.$$

d) What is the maximum range of a projectile that is fired with initial velocity v_0 from ground level $y_0 = 0$? At what angle θ is this maximum range achieved?

e) If a projectile with an initial velocity of 128 m/sec is fired from ground level, can it hit a target 1600 meters away? Justify your answer. If the projectle can hit the target, at what angle of elevation θ must it be fired so that it *will* hit the target?

f) Repeat part (e) for a target that is 1800 meters away.

Your lab report will be a hard copy of your typed input and Maple's responses, as well as your answers to parts (d), (e), and (f).

Comments

There are numerous other questions about ballistics that you are now able to handle and that might be of interest to you. For further information, consult your instructor!