

Project 4.5b: Graphing Functions

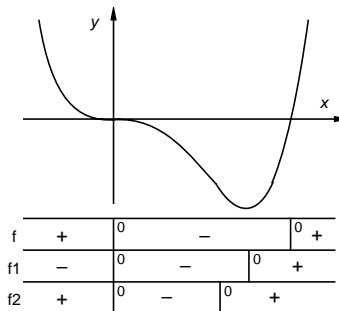
Objective

To investigate how we can relate where a function f and its first and second derivatives are positive, negative, zero, and do not exist, and the graph of f .

Narrative

The author provides one mechanism for gathering information to be used in graphing a function. In this project we present another.

To record where a function f and its first and second derivatives $f' = f1$ and $f'' = f2$, respectively, are positive, negative, zero, and do not exist, we use three “recording strips” below the graph of f as illustrated in the figure at the right; here $f(x) = (x^4 - 4x^3)/10$. (See Example 6, p. 245 of the text.) Study this figure carefully. Note how the vertical bars are drawn where f' , f'' , and f''' are 0 or do not exist, and the spaces between the vertical bars are labelled + or - depending on the sign of f' , f'' , and f''' over those intervals.



Tasks

1. Type the command lines below into Maple in the order in which they are listed. They initialize Maple for this project.

```
> # Project 4.5b: Graphing Functions
> restart;
> with(plots):
```

2. Type the command lines below into Maple in the order in which they are listed. They produce a graph of the function $f(x) = 4x/(1+x^2)$, and three recording strips below the graph of f .

```
> # Task 2
> f := x -> 4*x/(1+x^2);
> plot0 := plot({-6,-5,-4,-3,0},x=-6..6,y=-6..3):
> plot1 := textplot({[-6,-3.5,'f'],[-6,-4.5,'f1'],[-6,-5.5,'f2']});
> plot2 := plot(f(x),x=-6..6):
> display({plot0,plot1,plot2});
```

3. a) Type the command lines below into Maple in the order in which they are listed. They determine where $f(x) = 3x^4 - 16x^3 + 18x^2$ and its first and second derivatives are positive, negative, zero, and do not exist; this information will be used later in this project.

```
> # Task 3
> f := x -> 3*x^4-16*x^3+18*x^2;
> evalf(solve(f(x)=0,x));
> f1 := D(f);
> evalf(solve(f1(x)=0,x));
> f2 := D(f1);
> evalf(solve(f2(x)=0,x));
```

b) Type the command line below into Maple. It produces an empty graph and three recording strips.

```
> display({plot0,plot1});
```

At this time, make a hard-copy of your input and Maple's responses. Then, ...

4. Fill in the recording strips on the graphic you produced in Task 2 using the graph of f in that graphic as a guide.
5. a) Fill in the recording strips on the graphic you produced in Task 3(b) using the information you computed in Task 3(a) as a guide.
b) Use the information you recorded in (a) to sketch the graph of f in the space provided.

Comments

To find the intercepts and vertical asymptotes of the graph of f we need to find the values of x for which $f(x) = 0$ and $f(x)$ does not exist, to find the (possible) critical values of f we need to find the values of x for which $f'(x) = 0$ and $f''(x)$ does not exist, and to find the (possible) inflection points of the graph of f , we need to find the values of x for which $f''(x) = 0$ and $f''(x)$ does not exist. Since for every polynomial function f the values of $f(x)$, $f'(x)$, and $f''(x)$ exist for all x , in the above problem we only had to use information about where $f(x) = 0$, $f'(x) = 0$, and $f''(x) = 0$. If, for a function in a more general class of functions, $f(x)$, $f'(x)$, and/or $f''(x)$ did not exist for certain values of x , we would have to find these values and label vertical bars in recording strips with $\pm\infty$. (See Extra Project 4.5c.)