

Project 17.1d: Vector Fields in the Plane

Objective

The objective of this project is to illustrate some of the concepts associated to vector fields using Maple.

Narrative

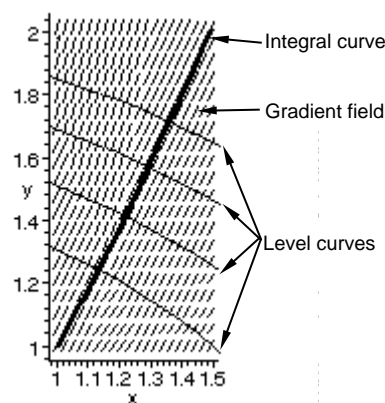
If you have not already done so, read Section 17.1 of the text.

In this project we see that an arbitrary vector field $\mathbf{F}(x, y) = \langle F_1(x, y), F_2(x, y) \rangle$ can be plotted over drawn over the rectangle $[a, b] \times [c, d]$ using the Maple command `fieldplot([F1(x,y),F2(x,y)],x=a..b,y=c..d)`.

We also investigate gradient fields: Associated to every differentiable function $z = f(x, y)$ is a vector field known as the *gradient field* $\nabla f = \langle f_x, f_y \rangle$. The gradient field has the property that at each point P in the domain of f , ∇f is perpendicular to the level curve of f that passes through P . A parametrized curve $x = x(t), y = y(t)$ in the domain of f that follows the gradient field, that is, whose tangent vector has the same direction as ∇f , or for which

$$\frac{dx}{dt} = \frac{\partial f}{\partial x}, \quad \frac{dy}{dt} = \frac{\partial f}{\partial y},$$

is an *integral curve* of ∇f (or a *flow line* or *streamline* of ∇f). Thus each integral curve meets each level curve at right angles.



Task

1. Type the command lines below into Maple in the order in which they are listed. The affect of these command lines is to plot the vector field $\mathbf{F}(x, y) = \langle x^2 - y, y^3 + 1 \rangle$ over the interval $[-1, 1]$.

```
> # Project 17.1d: Vector Fields in the Plane
> restart: with(plots): with(linalg): with(DEtools):
> # Task 1
> fieldplot([x^2-y,y^3+1],x=-1..1,y=-1..1);
```

2. Continue by typing the command lines below into Maple in the order in which they are listed; they plot the gradient of $f(x, y) = x^2 + 3y^2$, draw some integral curves of the associated system of differential equations

$$\frac{dx}{dt} = 2x, \quad \frac{dy}{dt} = 6y,$$

and draw some level curves of f .

```
> # Task 2
> f := (x,y) -> x^2+3*y^2;
> F := grad(f(x,y), [x,y]);
> deq := D(x)(t)=F[1],D(y)(t)=F[2];
> ICs := DEplot([deq], [x,y], t=-2..0.15,
  { [0,1,1], [0,1,-1], [0,-1,1], [0,-1,-1] },
  color=blue,linecolor=black):
> display(ICs);
```

Here we define f .

Here we compute $\mathbf{F} = \nabla f$.

Here we define the above system of differential equations.

Here we use DEplot to identify some integral curves ...

which we now draw.

Next we identify some level curves ...

```

> for i from 1 to 4 do
  LC[i] := implicitplot(f(x,y)=i,x=-4..4,y=-4..4,color=red): od:
                                     and plot both integral and level curves.
> display({ICs,seq(LC[i],i=1..4)},scaling=unconstrained);

```

Note that in the command `DEplot`, the four triples `[t0,x0,y0]` represent initial conditions for four integral curves.

At this time, make a hard-copy of your typed input and Maple's responses. Then, ...

3. On the graphic you drew in Task 1, choose and label any point P in the square $[-0.5, 0.5] \times [-0.5, 0.5]$, and draw by hand the integral curve of \mathbf{F} that passes through P .
4. On the graphic you drew in Task 2, draw by hand the level curve and the integral curve that pass through the point $P(1, 2)$, and label each.