MATH 164, Spring 2001 **Due Date:**_____ **Name(s):** _____

Project 12.2b: Sequences and Series

Objective

To investigate sequences and series using Maple.

Narrative

If you have not already done so, read Sections 12.1–12.2 of the text.
In this project we investigate:

1. the sequence $\{a_n\} = \{\frac{1}{2^n}\}$ and the associated series $\Sigma_{n=1}^{\infty} \frac{1}{2^n}$,
2. the sequence $\{a_n\} = \left\{\frac{1}{n(n+1)}\right\}$ and the associated series $\Sigma_{n=1}^{\infty} \frac{1}{n(n+1)}$, and
3. the sequence $\{a_n\} = \left\{\frac{n}{n+1}\right\}$ and the associated series $\Sigma_{n=1}^{\infty} \frac{n}{n+1}$.

To focus attention on our analysis, we use two procedures: `values` and `graphs`. Procedures are fragments of code which facilitate the repetition of tasks.

Tasks

1. Type the command lines in the left-hand column below into Maple in the order in which they are listed. These commands define the procedures `values` and `graphs`.

```
> # Project 12.2b:  Sequences and Series
> restart:  with(plots):
> values := proc(a,N)
    local M,n,s,pts,graphs:
    M := matrix(N+2,3,(Row,Col)->0):
    M[1,1] := n:  M[1,2] := a_n:  M[1,3] := s_n:
    s := n -> sum(a(i),i=1..n):
    for n from 1 to N do
        M[n+1,1] := n:  M[n+1,2] := evalf(a(n)):  M[n+1,3] := evalf(s(n)):  od:
    n := ´n´:
    M[N+2,1] := infinity:
        M[N+2,2] := evalf(limit(a(n),n=infinity)):
            M[N+2,3] := evalf(limit(s(n),n=infinity)):
    RETURN(eval(M)):
  end:
> graphs := proc(a,N)
    local apts,s,spts,n:
    apts := [seq([n,a(n)],n=1..N)]:
    s := n -> sum(a(i),i=1..n);
    spts := [seq([n,s(n)],n=1..N)]:
    plot({apts,spts},x=1..N,style=point);
  end:
```

2. a) Type the command lines in the left-hand column below into Maple in the order in which they are listed. These commands are aimed at studying the sequence $\{a_n\} = \left\{\frac{1}{2^n}\right\}$ and the associated series $\Sigma_{n=1}^{\infty} \frac{1}{2^n}$.

```
> values(n -> 1/2^n,10);
> graphs(n -> 1/2^n,10);
```

b) Repeat part (a) using the sequence $\{a_n\} = \left\{\frac{1}{n(n+1)}\right\}$ rather than $\{a_n\} = \left\{\frac{1}{2^n}\right\}$.

c) Repeat part (a) using the sequence $\{a_n\} = \left\{\frac{n}{n+1}\right\}$ rather than $\{a_n\} = \left\{\frac{1}{2^n}\right\}$.

At this time, make a hard-copy of your typed input and Maple's responses. Then, ...

3. For each of the (three) parts of Task 2:

a) connect the dots in the sequence $\{a_n\}$, and the dots in the sequence $\{s_n\}$ by hand,

b) label the sequence $\{a_n\}$ as "$\{a_n\}$", and the sequence $\{s_n\}$ as "$\{s_n\}$" by hand, and

c) next to the *limit* of the sequence $\{a_n\}$ write, "The sequence $\{a_n\}$ _____.", filling in the blank with "converges" or "diverges", and next to the *limit* of the sequence $\{s_n\}$ write, "The series $\Sigma_{n=1}^{\infty} a_n$ _____.", again filling in the blank with "converges" or "diverges".