

Project 5.3a: The Fundamental Theorem of Calculus

Objective

To illustrate the Fundamental Theorem of Calculus.

Narrative

If you have not already done so, read Section 5.3 of the text. One of the key ideas you should come away from this section with is the Fundamental Theorem of Calculus. This result states that if f is a continuous function on the finite closed interval $[a, b]$ then

$$D_x \left(\int_{t=a}^x f(t) dt \right) = f(x)$$

for $x \in (a, b)$, and hence

$$\int_{x=a}^b f(x) dx = F(b) - F(a)$$

where F is any antiderivative for f (that is, $F'(x) = f(x)$).

Task

a) Type the command lines in the left-hand column below into Maple in the order in which they are listed. These commands are aimed at investigating the Fundamental Theorem of Calculus in the context of $\int_{x=1}^4 4/(x^2 + 1) dx$. The effect of each command is described in the right-hand column for your reference. Your lab report will be a hard copy of your typed input and Maple's responses (both text and graphics).

> # Project 5.3a: The Fundamental Theorem of Calculus	
> restart;	Clear Maple's memory.
> f := x -> 4/(x^2+1);	Let $f(x) = 4/(x^2 + 1)$.
> plot(f(x), x=-5..5, scaling=constrained);	Plot the graph of f .
> a := 1.0; b := 4.0;	Let $a = 1$ and $b = 4$.
> limit(sum(f(a+i*(b-a)/n)*(b-a)/n, i=1..n), n=infinity);	Find $\int_{x=a}^b f(x) dx$ thinking of it as an area.
> F := x -> int(f(t), t=1..x);	Let $F(x) = \int_{t=1}^x f(t) dt$. (We're defining F as an integral.)
> plot({f(x), F(x)}, x=1..4);	Plot the graphs of f and F from $x = 1$ to $x = 4$.
> D(F)(x);	The first part of the Fundamental Theorem of Calculus states that the derivative of F is f .
> F(b) - F(a);	Find $F(b) - F(a)$.

At this point, make a hard-copy of your typed input and Maple's responses. Then proceed with the following steps.

- Label the graphs of f and F in the plot you made in part (a) by hand.
- Plot and label the point $P(3, F(3))$.
- The value $F(3)$ represents an area. Shade in the area that corresponds to $F(3)$. (Be careful!)