**MATH 164, Spring 2001** **Due Date:**____________ **Name(s):** ________________________

# Project 12.10a: Maclaurin Series

### *Objective*

To investigate the approximation of a function by its Maclaurin series using Maple.

### *Narrative*

If you have not already done so, read Section 12.10 of the text.

In this project we investigate the approximation of $f(x) = \sin x$ and $f(x) = \ln(1+x)$ by their respective Maclaurin series expansions. In so doing we introduce two new commands:

| | |
|---|---|
| `taylor(f(x),x=a,n)` | finds the $n$th order Taylor series expansion of $f(x)$ about the point $x = a$ |
| `p := convert(g,polynom)` | truncates the Taylor series approximation $g(x)$ to the polynomial $p(x)$ |

### *Tasks*

1. a) Type the command lines in the left-hand column below into Maple in the order in which they are listed. These commands are aimed at producing several Maclaurin series approximations to $f(x) = \sin x$.

| | |
|---|---|
| `> # Project 12.10a:  Mclaurin Series` | |
| `> # Part a` | |
| `> restart;` | Clear Maple's memory. |
| `> f := x -> sin(x);` | Let $f(x) = \sin x$. |
| `> t2 := taylor(f(x),x=0,2); p2 := convert(%,polynom);` | Let $p_2(x)$ be the 2nd order Maclaurin series approximation of $f(x)$. |
| `> t4 := taylor(f(x),x=0,4); p4 := convert(%,polynom);` | Let $p_4(x)$ be the 4th order Maclaurin series approximation of $f(x)$. |
| `> t6 := taylor(f(x),x=0,6); p6 := convert(%,polynom);` | Let $p_6(x)$ be the 6th order Maclaurin series approximation of $f(x)$. |
| `> t8 := taylor(f(x),x=0,8); p8 := convert(%,polynom);` | Let $p_8(x)$ be the 8th order Maclaurin series approximation of $f(x)$. |
| `> plot({f(x),p2,p4,p6,p8},x=-2*Pi..2*Pi,y=-2..2);` | Plot $f(x)$, $p_2(x)$, $p_4(x)$, $p_6(x)$, and $p_8(x)$. |

b) Continue by typing the command lines in the left-hand column below into Maple in the order in which they are listed. These commands are aimed at producing several Maclaurin series approximations to $f(x) = \ln(1+x)$.

| | |
|---|---|
| `> # Part b` | |
| `> restart;` | Clear Maple's memory. |
| `> f := x -> ln(1+x);` | Let $f(x) = \ln(1+x)$. |
| `> t2 := taylor(f(x),x=0,2); p2 := convert(%,polynom);` | Let $p_2(x)$ be the 2nd order Maclaurin series approximation of $f(x)$. |
| `> t3 := taylor(f(x),x=0,3); p3 := convert(%,polynom);` | Let $p_3(x)$ be the 3rd order Maclaurin series approximation of $f(x)$. |
| `> t4 := taylor(f(x),x=0,4); p4 := convert(%,polynom);` | Let $p_4(x)$ be the 4th order Maclaurin series approximation of $f(x)$. |
| `> t5 := taylor(f(x),x=0,5); p5 := convert(%,polynom);` | Let $p_5(x)$ be the 5th order Maclaurin series approximation of $f(x)$. |

```
> plot({f(x),p2,p3,p4,p5},x=-2..2,y=-8..2);
```
Plot $f(x)$, $p_2(x)$, $p_3(x)$, $p_4(x)$, and $p_5(x)$ over a large range.

```
> plot({f(x),p2,p3,p4,p5},x=-1..1,y=-1..1);
```
Plot $f(x)$, $p_2(x)$, $p_3(x)$, $p_4(x)$, and $p_5(x)$ over a restricted range.

2. On the graphic you produced for part (a) of Task 1, label by hand the graphs of $f(x)$ and $p_2(x)$, $p_4(x)$, $p_6(x)$, and $p_8(x)$. Label the graph of $p_2(x)$ by "$p_2(x)$", for example.

3. On *both* graphics you produced for part (b) of Task 1:

   a) draw the line whose equation is $x = -1$ by hand, and

   b) label by hand the graphs of $f(x)$ and $p_2(x)$, $p_3(x)$, $p_4(x)$, and $p_5(x)$. Label the graph of $p_2(x)$ by "$p_2(x)$", for example.

### *Comments*

Note how well both power series approximate the given functions near the origin. Also note the differences between power series which converge over the entire real number line ($f(x) = \sin x$) and those that do not ($f(x) = \ln(1 + x)$).