MATH 163, Spring 2001 **Due Date:**____________ **Name(s):** ________________________

Project 2.2: Guessing Limits Numerically

Objective

To guess the limit of a function at a point numerically.

Narrative

If you have not already done so, read Section 2.2 in the text.

Prior to having theorems on limits at our disposal, there are two major issues surrounding the limit of a function at a point: The first is guessing what the limit is, if it even exists; this issue can often be approached either graphically or numerically. The second issue involves proving that the guess you made is correct; this issue involves using the formal definition of limit.

In this project we address the issue of guessing limits numerically. In Project 2.4 we address the issue of proving a guess is correct. In this project we also illustrate how to perform repeated computations efficiently in Maple using a "do loop".

Task

a) Type the command lines in the left-hand column below into Maple in the order in which they are listed. These commands will help you estimate $\lim_{x\to 0} \dfrac{1-\cos x}{x^2}$ numerically, if it exists. (*Note*: It's OK to type the entire first loop `for n from 1 to 6 do ... od:` on one line, and the entire second loop `for n from 1 to 6 do ... od:` also on one line.)

`> # Project 2.2:  Guessing Limits Numerically`	
`> restart;`	Clear Maple's memory.
`> f := x -> (1-cos(x))/x^2;`	Let $f(x) = (1-\cos x)/x^2$.
`> plot(f(x),x=-1..1);`	Plot the graph of f.
`> a := 0.0;`	We are interested in estimating $\lim_{x\to a} f(x)$.
`> f(a);`	What is $f(a)$?
`> for n from 1 to 6`	Let's look at the values of $f(x)$ for $x < a$.
`do`	This is the beginning of a "do loop".
`x := a-1/2^n:`	Let $x = a - \frac{1}{2^n}$.
`print(evalf(x), evalf(f(x)));`	Print the values of x and $f(x)$.
`od:`	This is the end of our "do loop".
`> for n from 1 to 6`	Now let's look at some values of $f(x)$ for $x > a$.
`do`	
`x := a+1/2^n:`	
`print(evalf(x), evalf(f(x)));`	
`od:`	

b) On the basis of this data, do you think $\lim_{x\to 0} \dfrac{1-\cos x}{x^2}$ exists? If so, what do you think it is (to 4 decimal places of accuracy)? Justify your answer.

Your lab report will be a hard copy of your typed input and Maple's responses (including both text and graphics), together with your written response.

Comments

1. You can *guess* whether or not $\lim_{x\to 0} \dfrac{1-\cos x}{x^2}$ exists, and if it does exist, what its value is, on the basis of numerical "evidence" (as we did in this project), but you cannot say *for sure* that you're correct: you can never perform more than a finite number of computations, and however close x is to 0, you may *miss* some critical behavior of $f(x) = \dfrac{1-\cos x}{x^2}$ that might affect your guess. It is because of this that we *must* turn to the formal concept of the limit.
2. Different rates of convergence can be achieved by replacing `1/2^n` by `1/n^2` (this produces a slower rate of convergence) or `1/n^n` (this produces a faster rate of convergence).
3. The physical limitations of your computer may limit the accuracy of your computations.
4. Maple has a built in command `limit(f(x),x=a)` that allows you to compute (some) limits automatically. (Variations on this command include `limit(f(x),x=infinity)` and `limit(f(x),x=-infinity)` for computing limits at $\pm\infty$, and `limit(f(x),x=a,left)` and `limit(f(x),x=a,right)` for computing left- and right-hand limits.) Since we are interested not just in what limits are, but how they are computed, we intentionally avoided using this command in this project.
5. At the end of the `do` loops in the above code, Maple will think that $n = 6$ and $x = a \pm 1/2^n$. (You can check this by entering the commands `n;` and `x;` after each loop.) This is important to know since if, subsequent to the appropriate `do` loop, you wanted to reuse n or x as a variable then you would have to redefine it as a variable using the command `n = ´n´` or the command `x = ´x´`.