Project 7.2: Logarithmic Identities and Inequalities

Objective

To investigate various logarithmic identities and inequalities.

Narrative

If you have not already done so, read Section 7.2* in the text.

Throughout this course we will make use of various logarithmic identities and inequalities. In this project we investigate these identities and inequalities. To set the stage for this project, note that the relationship between the graphs of $f(x) = \ln x$ and $g(x) = \log_{10} x$ (see the figure to the right) is that the graph of $g(x) = \log_{10} x$ is a compression of the graph of $f(x) = \ln x$ by a factor of $\dfrac{1}{\ln 10} \approx 0.4343$ in the y-direction. This is justified by the fact that

$$\log_{10} x = \frac{\ln x}{\ln 10} = \frac{1}{\ln 10} \ln x \approx 0.4343 \ln x.$$
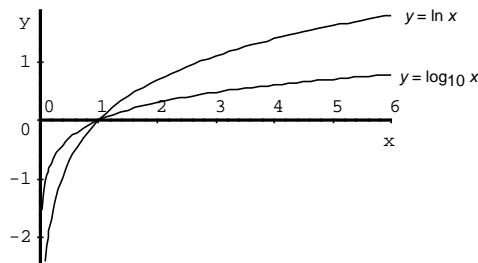
Also note that in Maple, the natural log function is written `ln(x)`, the natural exponential function is written `exp(x)`, and the common log function is written `log10(x)`.

Task

1. a) Type the command lines below into Maple in the order in which they are listed. These commands produce plots of the graphs of $f(x) = \ln x$, $g(x) = \ln(2x)$, $h(x) = \ln(x/2)$, and $k(x) = \ln(x^2)$ over the interval $[0, 6]$ on one set of axes.

    ```
    > # Project 7.2:  Logarithmic Identities
    > restart;
    > plot({ln(x),ln(2*x),ln(x/2),ln(x^2)},x=0..6);
    ```

    b) Continue by typing the command lines below into Maple in the order in which they are listed. These commands produce plots of the graphs of $f(x) = \ln x$, $g(x) = (x - 1)/x$, and $h(x) = x - 1$ over the interval $[0, 6]$ on one set of axes.

    ```
    > plot({ln(x),(x-1)/x,x-1},x=0.5..2);
    ```

    At this point, make a hard-copy of your typed input and Maple's responses (both text and graphics). Then, ...

2. a) Label the graphs of f, g, h, and k in the plot you created in part (a) of Task 1 by hand.

    b) Describe (in terms of vertical shifting, vertical stretching, and vertical reflecting — see Section 1.3 of the text) the relationship between the graphs of the following pairs of functions, and justify each answer with a logarithmic identity as we did in the Narrative. (*Hint*: Look at the properties of logarithms on p. 446 of the text — the relationships between the graphs of the indicated pairs of functions involve these properties.)

i) $f(x) = \ln x$ and $g(x) = \ln(2x)$,

ii) $f(x) = \ln x$ and $h(x) = \ln(x/2)$,

iii) $f(x) = \ln x$ and $k(x) = \ln(x^2)$.

3. a) Label the graphs of f, g, and h in the plot you created in part (b) of Task 1 by hand.

   b) What double inequality does the plot you created in part (b) of Task 1 suggest? (A *double inequality* is an inequality of the form $a \le b \le c$. See the Squeeze Theorem on p. 90.)

Comments

The double inequality discussed in this project will be useful later in our discussion of sequences and series.