Project 13.7: Rectangular, Cylindrical and Spherical Coordinates

Objective

The objective of this project is to better acquaint you with cylindrical and spherical coordinates.

Narrative

If you have not already done so, read Section 13.7 in the text.

In this project we introduce the commands `cylinderplot` and `sphereplot` in Maple's `plots` package. The command `cylinderplot` allows us to plot poiints in cylindrical coordinates. The command `sphereplot` allows us to plot points in spherical coordinates.

Task

1. We begin by creating some graphics. (*Note*: Although a default set of display options is specified, you may — depending on the hardware you're using — need to modify them to get an acceptable hard-copy.)

   a) First we consider rectangular coordinates. Type the command lines below into Maple in the order in which they are listed.

   ```
   > # Project 13.7:  Rectangular, Cylindrical and Spherical Coordinates
   > restart:  with(plots):
   > setoptions3d(axes=normal,scaling=constrained,color=blue,orientation=[30,60]):
   > # Part a:  Rectangular Coordinates
   > plot3d([1,y,z],y=0..1,z=0..1);
   > plot3d([x,1,z],x=0..1,z=0..1);
   > plot3d([x,y,1],x=0..1,y=0..1);
   ```

   b) Next we consider cylindrical coordinates. Type the command lines below into Maple in the order in which they are listed.

   ```
   > # Part b:  Cylindrical Coordinates
   > cylinderplot([1,theta,z],theta=0..2*Pi,z=0..2);
   > cylinderplot([r,3*Pi/4,z],r=-1..1,z=0..2);
   > cylinderplot([r,theta,1],r=0..1,theta=0..2*Pi);
   ```

   c) Next we consider cylindrical coordinates. Type the command lines below into Maple in the order in which they are listed.

   ```
   > # Part c:  Spherical Coordinates
   > sphereplot([1,theta,phi],theta=0..2*Pi,phi=0..Pi/2);
   > sphereplot([rho,3*Pi/4,phi],rho=0..1,phi=0..Pi/2);
   > sphereplot([rho,theta,Pi/4],rho=0..1,theta=0..2*Pi);
   ```

   At this time make a hard-copy of your typed input and Maple's responses (both text and graphics). Then, ...

2. Label the graphics you produced as follows:

   a) On each graphic you produced in part (a) of Task 1, label the positive x-, y-, and z-coordinate axes.

   b) On each graphic you produced in part (b) of Task 1, label the positive x-, y-, and z-coordinate axes. On the first graphic, highlight by hand that part of the surface over which θ is between 0 and $\pi/2$, on the second highlight that part over which r is between 0 and 1, and on the third that part over which r is between 0.5 and 1 and θ is between 0 and $\pi/2$.

   c) On each graphic you produced in part (c) of Task 1, label the positive x-, y-, and z-coordinate axes. On the first graphic, highlight by hand that part of the surface over which θ is between 0 and $\pi/2$ and ϕ is between $\pi/4$ and $\pi/2$, on the second highlight that part over which ϕ is between $\pi/4$ and $\pi/2$, and on the third that part over which θ is between 0 and $\pi/2$.

Comments

1. The parameters θ and ϕ you were adjusting in Project 13.5b were the θ and ϕ spherical coordinates of the viewer!

2. The effect of replacing the 1 in the [x,y,1] command line of Task 1a by a function $z = z(x, y)$ of x and y is to graph $z = z(x, y)$ in rectangular coordinates over the square x=0..1,y=0..1. (As it stands, $z = z(x, y) = 1$.) You can obtain some interesting surfaces if, in the same vein, you replace:

   a) the 1 in the [1,theta,z] in Task 1b with a function $r = r(theta, z)$,

   b) the 3*Pi/4 in the [r,3*Pi/4,z] in Task 1b with a function $theta = theta(r, z)$,

   c) the 1 in the [r,theta,1] in Task 1b with a function $z = z(r, theta)$,

   d) the 1 in the [1,theta,phi] in Task 1c with a function $rho = rho(theta, phi)$,

   e) the 3*Pi/4 in the [rho,3*Pi/4,z] in Task 1c with a function $theta = theta(rho, phi)$,

   f) the 1 in the [r,theta,1] in Task 1c with a function $phi = phi(rho, theta)$.

   Try experimenting, and see what you get!