Project 8.6: Techniques of Integration

Objective

To illustrate the strengths and weaknesses of Maple in evaluating integrals.

Narrative

If you have not already done so, read Section 8.6 in the text.

This project involves two tasks. The first illustrates how easily Maple integrates some expressions (such as partial fractions), and how Maple can be used to check some of the complicated intermediate work you might have to do by hand when doing your homework. The second illustrates that while Maple's ability to evaluate integrals is powerful, it is not so great that we can dispense with learning techniques of integration.

Tasks

1. Type the command lines in the left-hand column below into Maple in the order in which they are listed. These commands are aimed at finding $\int \dfrac{dx}{x^3 - x}$.

   ```
   > # Project 8.6:  Techniques of Integration
   > # Task 1
   > restart;                              Clear Maple's memory.
   > int(1/(x^3-x),x);                     Find ∫ dx/(x³−x).
   ```

`> restart;`	Clear Maple's memory.
`> int(1/(x^3-x),x);`	Find $\int \dfrac{dx}{x^3 - x}$.

   Maple handles this integral pretty fast. Let's see how Maple can be used to check some of the complicated intermediate work you might have to do by hand when doing your homework. Continue by typing the command lines in the left-hand column below into Maple in the order in which they are listed.

`> convert(1/(x^3-x),parfrac,x);`	Find the partial fraction decomposition of the integrand $1/(x^3 - x)$.
`> int(%,x);`	Integrate the result.
`> diff(%,x);`	Check our answer by differentiating it.
`> simplify(%);`	Simplify to make comparison with the original integrand easier.

2. a) Type the command lines in the left-hand column below into Maple in the order in which they are listed. These commands are aimed at finding $\int \dfrac{\arcsin x}{(1 - x^2)^{3/2}} \, dx$.

`> # Task 2`	
`> restart;`	Clear Maple's memory.
`> MyInt := int(arcsin(x)/((1-x^2)^(3/2)),x);`	Let $MyInt = \int \dfrac{\arcsin x}{(1 - x^2)^{3/2}} \, dx$.

   Note that Maple can't evaluate $\int \dfrac{\arcsin x}{(1 - x^2)^{3/2}} \, dx$: it would have if it could have in response to the third command line you typed!

   At this time, make a hard-copy of your typed input and Maple's responses. Then, ...

   b) use the substitution $x = \sin u$ to evaluate $\int \dfrac{\arcsin x}{(1 - x^2)^{3/2}} \, dx$ by hand.

Comments

The moral of Task 1 is that Maple is a very powerful tool when it comes to techniques of integration. The moral of Task 2 is that as powerful as Maple is, there are (and always will be, some fairly simple) integrals it cannot evaluate, and hence we cannot rely on it (to the extent that we can avoid learning techniques of integration).