

## Project 14.1c: Curves in Space

### Objective

The objective of this project is to illustrate some of the geometry of curves in space.

### Narrative

If you have not already done so, read Section 14.1 in the text.

In this project we introduce the command `spacecurve` in Maple's `plots` package. This command allows us to plot the graph of a parametrized space curve. Also, we illustrate how we can visualize curves in space as intersections of planes and other surfaces.

### Task

a) Type the command lines below into Maple in the order in which they are listed.

```
> # Project 14.1c: Curves In Space
> restart: with(plots):
> setoptions3d(scaling=constrained,orientation=[30,80],axes=normal):
```

b) Continue by typing the command lines below into Maple in the order in which they are listed. They illustrate a trefoil knot, and a “tubular” version of this knot (which gives a better idea of the knot’s geometry.)

```
> # A trefoil knot
> spacecurve([(2+cos(1.5*t))*cos(t), (2+cos(1.5*t))*sin(t), 2*sin(1.5*t)], t=0..4*Pi,
  color=blue, numpoints=100);
> tubeplot([(2+cos(1.5*t))*cos(t), (2+cos(1.5*t))*sin(t), 2*sin(1.5*t)], t=0..4*Pi,
  radius=0.25, numpoints=100);
```

c) Continue by typing the command lines below into Maple in the order in which they are listed. They illustrate a helix, and a right circular cylinder and a sinusoidal cylinder whose intersection is the helix.

```
> # A helix
> plot0 := spacecurve([cos(t), sin(t), 0.5*t], t=0..3*Pi, shading=none, color=red,
  thickness=2):
> display(plot0);
> plot1 := implicitplot3d(x^2+y^2=1, x=-1.5..1.5, y=-1.5..1.5, z=0..1.5*Pi,
  shading=none, color=green):
> display({plot0, plot1});
> plot2 := implicitplot3d(x = cos(2*z), x=-1.5..1.5, y=-1.5..1.5, z=0..1.5*Pi,
  shading=none, color=blue, grid=[10, 10, 20]):
> display({plot0, plot2});
> display({plot0, plot1, plot2});
```

At this point, make a hard-copy of your typed input and Maple’s responses. Then, ...

d) On the last graphic you created in part (c), label (by hand) the positive  $x$ -,  $y$ -, and  $z$ -coordinate axes, and the right circular cylinder and the sinusoidal cylinder with their equations. Finally, highlight (by hand) the helix.

Your lab report will be a hard copy of your typed input and Maple’s responses.

### Comments

Some other curves you might enjoy looking at include:

1. the toroidal curve:  $[(4+\sin(20*t))*\cos(t), (4+\sin(20*t))*\sin(t), \cos(20*t)]$  where  $t \in [0, 2\pi]$ ,
2. the twisted cubic:  $[t, t^2, t^3]$  where  $t \in [-2, 2]$ ,
3. the curve:  $[\sin(t), \sin(2*t), \sin(3*t)]$  where  $t \in [0, 2\pi]$ .

For the toroidal curve you might want to use the option `numpoints=200` to produce a smooth curve.