

Project 8.7: Approximate Integration

Objective

To illustrate the Trapezoidal Rule and Simpson's Rule.

Narrative

If you have not already done so, read Section 8.7 in the text.

In this project we illustrate and compare Riemann sums, the Trapezoidal Rule, and Simpson's Rule by looking at how they approximate $\int_{x=1}^2 \frac{dx}{x} = \ln 2$. In this project we introduce the command `type`: `type(i,odd)` is true if i is odd and false if i is even, and `type(i,even)` is true if i is even and false if i is odd.

Task

1. Type the command lines in the left-hand column below into Maple in the order in which they are listed. The effect of each command is described in the right-hand column for your reference. This section of code provides Maple some initial information.

| | |
|---|--|
| <pre>> # Project 8.7: Approximate Integration > restart; > f := x -> 1/x; > NInts := 8; > a := 1.0; b := 2.0; dx := (b-a)/NInts; > Actual := int(f(x),x=a..b);</pre> | <p>Clear Maple's memory.</p> <p>Let $f(x) = 1/x$.</p> <p>Let the number of intervals $NInts = 8$.</p> <p>Let $a = 1, b = 2$, and $dx = (b-a)/NInts$.</p> <p>What is $\int_{x=a}^b f(x) dx$?</p> |
|---|--|

- a) Continue by typing the command lines in the left-hand column below into Maple in the order in which they are listed. These commands approximate our integral by Riemann sums.

| | |
|---|---|
| <pre>> # Riemann Sums > Riemann := 0.0; > for i from 1 to NInts do Riemann := Riemann + dx*f(a+i*dx) od; > plot(f(x),x=a..b); > abs(Actual-Riemann);</pre> | <p>Initialize <i>Riemann</i> to 0.</p> <p>Compute the Riemann sum.</p> <p>Draw the graph of f.</p> <p>The Riemann sum error.</p> |
|---|---|

- b) Continue by typing the command lines in the left-hand column below into Maple in the order in which they are listed. These commands approximate our integral by the Trapezoidal Rule.

| | |
|---|---|
| <pre>> # The Trapezoidal Rule > wt := i -> if i=0 then 1 elif i=NInts then 1 else 2 fi; > for i from 0 to NInts do wt(i) od; > Trap := 0.0; > for i from 0 to NInts do Trap := Trap + (dx/2)*wt(i)*f(a+i*dx) od; > plot(f(x),x=a..b); > abs(Actual-Trap);</pre> | <p>Define the weighting function wt.</p> <p>Check the weighting function.</p> <p>Initialize <i>Trap</i> to 0.</p> <p>Apply the Trapezoidal Rule.</p> <p>Draw the graph of f.</p> <p>The Trapezoidal Rule error.</p> |
|---|---|

c) Continue by typing the command lines in the left-hand column below into Maple in the order in which they are listed. These commands approximate our integral by Simpson's Rule.

```

> # Simpson's Rule
> wt := i -> if i=0 then 1           Define the weighting function wt.
      elif i=NInts then 1
      elif type(i,odd) then 4
      elif type(i,even) then 2 fi;
> for i from 0 to NInts do wt(i) od;   Check the weighting function.
> Simp := 0.0;                         Initialize Simp to 0.
> for i from 0 to NInts do
  Simp := Simp + (dx/3)*wt(i)*f(a+i*dx) od;  Apply Simpson's Rule.
> plot(f(x),x=a..b);                 Draw the graph of f.
> abs(Actual-Simp);                  Simpson's Rule error.

```

At this time, make a hard-copy of your typed input and Maple's responses. Then, ...

2. On each of the graphs you created in parts (a), (b), and (c) above, sketch and shade in the region whose area approximates $\int_{x=1}^2 \frac{dx}{x}$ by Riemann sums, the Trapezoidal Rule, and Simpson's Rule, respectively, when $n = 8$.
3. When $n = 8$, which method — Riemann sums, the Trapezoidal Rule, or Simpson's Rule — provides the best estimate of $\int_{x=1}^2 \frac{dx}{x} = \ln 2$? Which provides the worst?

Comments

In practice, many definite integrals cannot be evaluated in closed form, and techniques such as the Trapezoidal Rule and Simpson's Rule must be applied to find numerical approximations. Maple actually has built in commands that make finding such approximations quite easy. We did not use them in this project, however, since our goal was to illustrate how such methods work, not just how to use certain commands.