Math 163, Spring 2001 **Due Date:**____________ **Name(s):** _______________________

Project 3.4: Ballistics

Objective

To illustrate an important application of differentiation to ballistics.

Narrative

If you have not already done so, do Project 3.8 Differentiation. In that project we illustrate how derivatives can be computed in Maple.

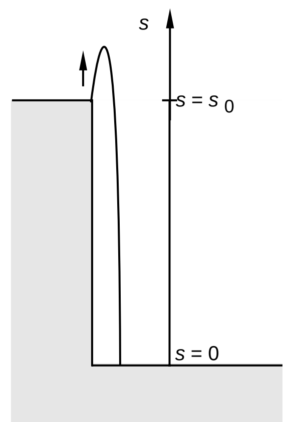
If a projectile is fired vertically upward with an initial velocity of v_0 m/sec from an initial position s_0 meters above the ground (see the figure to the right), then (neglecting air resistance) after t sec the projectile is

$$s = s(t) = -\frac{1}{2}gt^2 + v_0t + s_0$$

meters above the ground, where $g = 9.8$ m/sec^2 is acceleration due to gravity, and the velocity of the projectile, after t seconds, is

$$v = v(t) = D_t(s(t)) = -gt + v_0$$

meters per second. (If s is measured in feet ft and v is measured in ft/sec, then $g = 32$ ft/sec^2.)



Task

a) Type the command lines in the left-hand column below into Maple in the order in which they are listed.

`> # Project 3.4:  Ballistics`	
`> restart;`	Clear Maple's memory.
`> g := 9.8; t0 := 0; s0 := 100; v0 := 128;`	Let $g = 9.8$, $t_0 = 0$, $s_0 = 100$, and $v_0 = 128$. (In this project we'll be using metric units.)
`> s := t -> -0.5*g*t^2+v0*t+s0;`	Let the distance $s(t) = -\frac{1}{2}gt^2 + v_0t + s_0$.
`> plot(s(t),t=t0..20);`	Graph $s(t)$ for $t \in [t_0, 20]$. Observe that after 20 sec, the projectile is still in the air.
	Let's find when the projectile hits the ground.
`> solve(s(t)=0,t);`	Find when $s(t) = 0$. You should get two values: one negative and one positive. The positive value is the time at which the projectile hits the ground.
`> t1 := %[2];`	Let t_1 be the the *positive* value. (We're assuming here that the second value is positive; if it's the first value that's positive, type `t1 := %[1];` instead.)
`> plot(s(t),t=t0..t1);`	Graph $s(t)$ for $t \in [t_0, t_1]$.

`> v := D(s);`	Let the velocity $v(t) = D_t(s(t))$.
`> v(t1);`	Find the velocity of the projectile when it hits the ground.
`> t_smax := solve(v(t)=0,t);`	Find the time t_{smax} at which the velocity of the projectile is 0; t_{smax} is the time it takes the projectile to reach its maximum altitude $s(t_{smax})$.
`> v(t_smax);`	This just checks Maple's work: the result should be zero (or close to zero).
`> s(t_smax);`	Find the maximum altitude $s(t_{smax})$ of the projectile.
`> plot(v(t),t=t0..t1);`	Plot v as a function of t for $t \in [t_0, t_1]$.

At this point, make a hard-copy of your typed input and Maple's responses. Then ...

b) Label by hand the coordinate axes in the second graphic you produced. (One should be a t-axis, and the other an s-axis.) Plot and label the points $(t, s(t))$ for $t = t_0$, $t = t_1$, and $t = t_{smax}$ in this graphic.

c) Label by hand the coordinate axes in the third graphic you produced. (One should be a t-axis, and the other a v-axis.) Plot and label the points $(t, v(t))$ for $t = t_0$, $t = t_1$, and $t = t_{smax}$ in this graphic.

Your lab report will be a hard copy of your typed input and Maple's responses (both text and hand-labeled graphics).