Project 3.8: Differentiation

Objective

To investigate differentiation with Maple.

Narrative

If you have not already done so, read Section 3.8 in the text. One of the key ideas you should take away from Chapter 3 is that, as long as the limit exists, the value of

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

for any given value of x is the slope of the tangent line to the graph of f at x.

In this project we introduce the two commands:

    `D(f)`   the derivative f' of the function f (*Note*: `D(f)` is a function.)

    `D(f)(x)`   the expression $f'(x)$

Task

a) Type the command lines in the left-hand column below into Maple in the order in which they are listed.

```
> # Project 3.8:  Differentiation
> restart:                              Clear Maple's memory.
> f := x -> 1/(x^2+4);                  Let f(x) = 1/(x² + 4).
> (f(x+h)-f(x))/h;                      Set up the difference quotient.
> simplify(%);                          Simplify the difference quotient.
> limit(%,h=0);                         Find the limit of the difference quotient as h → 0.
> D(f);                                 Compute f' using Maple's D operator.
> plot({f(x),D(f)(x)},x=-4..4);         Plot the graphs of f and f'.
> f := x -> x^4+3*x^2+2;                Let f(x) = x⁴ + 3x² + 2.
> f1 := D(f);                           Let f1 denote f'.
> f2 := D(f1);                          Let f2 denote f''.
> f3 := D(f2);                          Let f3 denote f'''.
> plot({f(x),f1(x),f2(x),f3(x)},x=-2..2);  Plot the graphs of f, f', f'' and f'''.
```

At this point, make a hard-copy of your typed input and Maple's responses. Then, ...

b) on the first graphic you created in part (a), label the graphs of f and f', and highlight that part of the graph of f over which the tangent lines (to the graph of f) have positive slope, and that part of the graph of f' over which f' is positive.

c) on the second graphic you created in part (a), label the graphs of f, f', f'' and f'''.

Your lab report will be a hard copy of your typed input and Maple's responses (both text and hand-labeled graphics).

Comments

You can also refer to `D(f)(x)` in Maple by `diff(f(x),x)`.